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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

STABILITY AND STIFFNESS OF CELLULAR COFFERDAMS

By Karl Terzaghi,[1] M. Am. Soc. C. E.

Synopsis

The incentive for preparing this paper was given by observations made during and after the construction of a cellular cofferdam which constitutes the side-walls and the middle wall of a shipway. A digest of these and various other observations on cellular cofferdams indicated that the theories on which the design of these cofferdams is usually based give the designer an inadequate conception of the factors that determine the stability of cofferdams.

The paper is divided into three parts. The first deals with past experiences with cellular cofferdams in general, the deflection of the crests of cofferdams, and the permeability of the sheet-pile enclosures. In the second part current methods for designing cellular cofferdams on rock, sand, and clay foundations are reviewed, examined, and revised. The third part contains conclusions. Discussion is specifically limited to cellular cofferdams whose cells are filled with sand or sand and gravel.

Part I. Practical Considerations

Experiences with Cellular Cofferdams.—The first cellular cofferdam was built in 1908 or 1909 at Black Rock Harbor, Buffalo, N. Y. (1).[2] It consisted of seventy-seven prismatic cells, whose horizontal dimensions were 30 ft by 30 ft. It rested on a rock foundation. Since all the walls were straight, it was inevitable that the longitudinal walls should bulge badly between the cross-walls. In one of the cells, the inner wall, with an unsupported height of about 30 ft, bulged 3.45 ft between cross-walls; yet it is reported that the inward movement of the crest of the wall at the cross-walls nowhere exceeded about 1 in.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **February 1, 1945.**

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[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix I).

In 1910 a cellular cofferdam was built in connection with the raising of the battleship *Maine* in the harbor of Havana, Cuba (2). It consisted of twenty cylindrical cells interconnected by short arcs as shown in Fig. 1(*a*). The subsoil consisted of a stratum of soft silt and mud resting on medium soft clay. The cells were filled with clay. It was expected that the dam would be self-supporting. However, while the salvage crews were pumping inside the cofferdam, the inward deflections increased so alarmingly that it was decided to

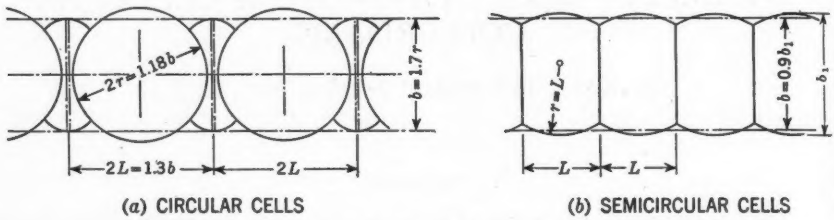

FIG. 1.—PRINCIPAL TYPES OF CELLULAR COFFERDAMS

increase the stability of the structure by adding an inner berm. During the last stage of excavation and pumping it was even considered necessary to brace the cofferdam against the hull of the *Maine*.

In spite of the difficulties encountered, some engineers realized that the basic idea of the design of the cofferdam for the *Maine* was sound. Therefore, many cofferdams of a similar type have since been constructed. They are known as cellular cofferdams with circular cells.

In 1915 or 1916 the U. S. Army Engineers repeated the Black Rock Harbor experiment by building a similar cofferdam in Troy, N. Y. (3). Since nature gave a drastic hint at Black Rock Harbor that straight-length walls were not appropriate, the cross-walls were connected by arcs. The success of this dam initiated the development of cellular cofferdams with semicircular cells, Fig. 1(*b*).

Some engineers have expressed the opinion that cellular cofferdams are less reliable than other types because several important cellular cofferdams have failed. The design of such cofferdams on a base other than rock requires more judgment and experience than does the design of a double-wall cofferdam with a broad inner berm on a similar base. Hence failures of cellular cofferdams due to faulty design may be more frequent than similar failures of double-wall cofferdams; yet failures of this category cannot be ascribed to an inherent defect of the type of construction.

The only inherent weakness of the cellular cofferdams is the vulnerability of the locks. One or several cells of a cofferdam may burst while all the others remain intact. An individual cell may fail either because of a defect in the material of one of the locks or because a sheet pile was driven out of a lock without the incident being noticed; yet the number of failures due to both causes combined is so small, and, on an average, the costs of remedying the consequences of the incident are so unimportant that the hazard is far more than compensated for by the economic advantages. This statement is cor-

roborated by the fact that large construction organizations have used the cellular cofferdam type many times.

Since cellular cofferdams are both economical and durable, they can also be used to great advantage as parts of permanent structures such as shipways, quay walls, breakwaters, and the like, provided the foundation conditions are satisfactory and suitable material for filling the cells can be secured at low cost.

Deflection of Cellular Cofferdams.—Experience indicates that the sheet-pile walls of cellular cofferdams can stand a horizontal deflection of the crest of the cofferdam equal to more than 10% of its height without injury to the locks. On the landward section of the cellular cofferdam, built about 1916 at the site for a lock at Brown's Landing on the Cape Fear River, North Carolina (4), the top of four cells moved in a distance of 7 ft 6 in. (25% of the free height of the cofferdam) because the pin connection of diagonal tie rods attached to the cross-walls failed; yet the published description of the incident indicates that the locks of these cells remained intact.

The deflection of a cellular cofferdam with given dimensions depends, quite obviously, to a large extent on the elastic properties of the fill in the cells. As a consequence, the relation between the unbalanced horizontal pressure on a

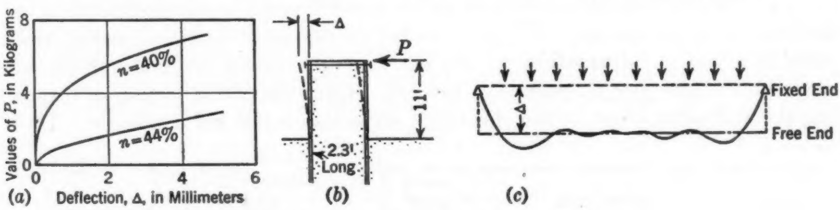

Fig. 2.—Deflection of Cellular Cofferdams

cofferdam and the corresponding deflection is very similar to the relation between stress and strain in soils in general. The strain in soils increases roughly with the square of the stress and the deflection of the crest of the dams increases roughly with the square of the unbalanced horizontal pressure. Both the strain in soils and the deflection of the crest of cofferdams increase at slowly decreasing rates under constant stress or pressure conditions.

The decisive influence of the elastic properties of the fill on the horizontal deflection of a cofferdam was demonstrated experimentally by I. A. Rimstad with tests on a small-scale model of a double-wall, sheet-pile cofferdam (5). In one of the tests the space between the two walls was filled with loose sand (porosity $n = 44\%$) and in the second with the same sand in a compacted state ($n = 40\%$). In Fig. 2(*a*) the ordinates represent the unbalanced horizontal force and the abscissas the corresponding deflection of the crest of the dam.

In many instances the ends of a cellular cofferdam are tied to rigid structures or to cross-walls which prevent the horizontal pressure from producing a deflection of the ends. A cofferdam between two such ends may be considered as a thick, rectangular slab whose lower edge is fixed and whose vertical edges are freely supported. The upper edge is free. The theory of the horizontal

deflection of such slabs under the influence of water pressure—which leads to the conclusion that the upper edge would deflect as indicated in Fig. 2(*c*)—is beyond the scope of this paper. Each half of the section between the two deflection maxima resembles the graphic representation of damped vibrations. The peculiar shape of the deflection curve is due to the resistance of the wall against bending in horizontal planes. If the distance between the end supports is not greater than a few times the height, the two maxima are absent, the elastic line is roughly parabolic, and the maximum deflection is very much smaller than the deflection Δ indicated in Fig. 2(*c*).

From the foregoing, it is evident that the deflection of a cofferdam with a given cross section acted upon by a given system of horizontal forces depends not only on the elastic properties of the cells but also, to a certain extent, on the ratio between the length and the height of the dam. Since a cellular cofferdam consists of two very different materials, steel and soil, its elastic properties can be compared to those of a composite construction material such as reinforced concrete. These properties are chiefly governed by those of the soil in the cells, but the steel walls are likely to stiffen the structure very considerably (see Part II).

The elastic properties of a given soil in a cell depend to a large extent on the method of filling the cell. There is no laboratory procedure that would yield reliable advance information on the state of the soil in the cell and on the corresponding elastic properties of the soil. Quantitative information regarding the stiffening effect of the sheet-pile walls also is not yet available. However, experience indicates that the crest of a cellular cofferdam really deflects approximately as theory leads one to expect. Fig. 3 represents the deflection of the crest of the cofferdam for the land section of the piers between West 44th Street and West 47th Street on the east side of the Hudson River in New York, N. Y. Similar curves were obtained from deflection observations on a very much wider cofferdam between West 47th Street and West 52d Street (6). In both instances the inside of the cofferdam was supported by a rock berm which was provided in the original design. In spite of the presence of this berm, the crest of both dams assumed the characteristic W-shape shown in Fig. 2(*c*) at an early stage of unwatering.

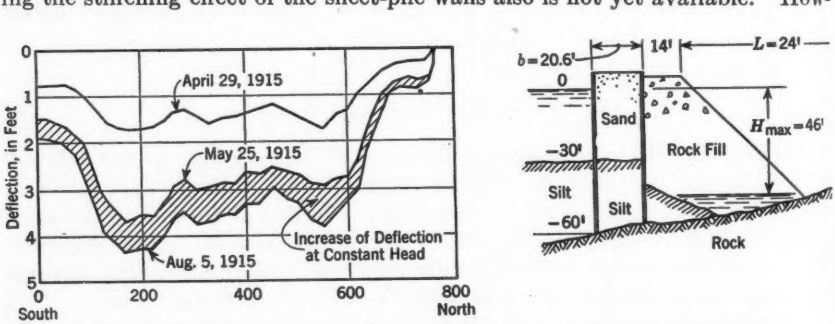

FIG. 3.—DEFLECTION OF COFFERDAM CREST FOR THE LAND SECTION OF HUDSON RIVER PIERS (H = 46 FT)

The same general tendency was noticed while the water was pumped out of two full-length submerged shipways whose walls consisted of self-supporting cellular cofferdams of the diaphragm type (7). The deflection of two of the longitudinal (east–west) walls is represented in Fig. 4. The reason for the excessive deflection near the west end of the wall in Fig. 4(*a*) will be discussed subsequently. The deflection of the two short walls was roughly parabolic,

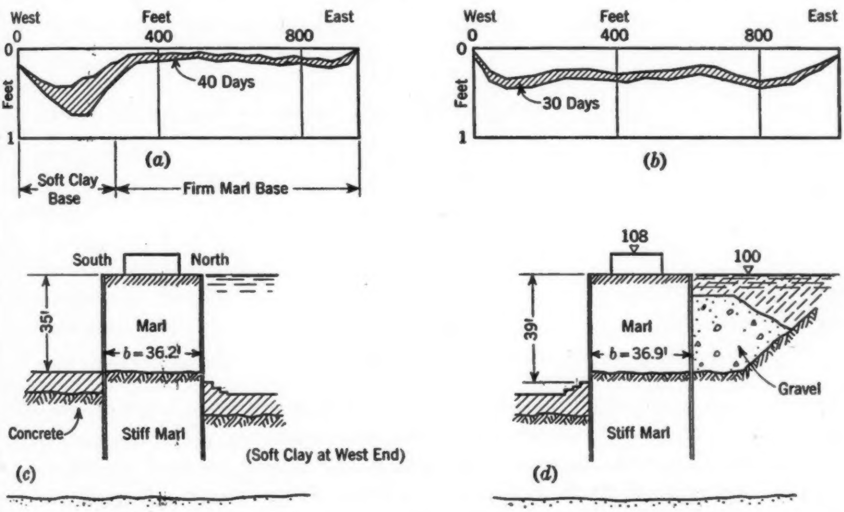

FIG. 4.—DEFLECTION OF A STEEL SHEET COFFERDAM FOR A SUBMERGED SHIPWAY

and, in accordance with theory, the greatest deflection of these walls was very much smaller than that of the long ones; yet there was no difference between the east–west sections and the north–south sections except in length.

Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures.—The water that percolates across a cellular cofferdam must pass through two different mediums—the rows of sheet piles and the soil between the sheet piles. The passage through each medium causes a loss of head, disclosed by a difference in the position of the line of saturation on either side of the obstacle against flow. If the cells of a cofferdam are filled with fairly clean sand and gravel, the loss of head is almost exclusively caused by the outer row of sheet piles. All engineers who have reported on experiences with cellular cofferdams filled with sand and gravel emphasized the fact that they had no trouble with water. Many single-wall cofferdams consisting of a row of steel sheet piles have leaked so badly that it was necessary to stop the flow through the locks by artificial means such as washing cinders or sawdust into the leaky locks. Hence, the consistently favorable reports on pumping between cellular cofferdams suggest that the relative watertightness of these dams is primarily due to the tension in the locks. Since there is tension only above the level of the base of the fill in the cells, that part of the rows of sheet piles below the level of the base is likely to be much more permeable than the upper part.

Favorable experiences with pumping from the space within cellular cofferdams filled with sand and gravel have led many engineers to believe that the practical implications of the permeability of the rows of sheet piles are negligible under any circumstances and that a cellular cofferdam with weep holes constitutes a mass of soil enclosed between one impermeable wall and one permeable wall. This conception may be justified under certain conditions, but it is unwarranted under others and it can lead to false conclusions.

Fig. 5 represents vertical sections through a cellular cofferdam, backed by water. The cofferdam rests on an impermeable base and the inner row of sheet piles is provided with weep holes. If the cofferdam is filled with clean, coarse sand (Fig. 5(*a*)), the quantity of water that leaks through the joints of

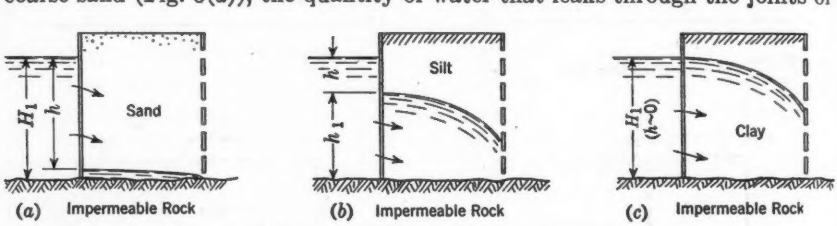

FIG. 5.—CELLULAR COFFERDAMS ON ROCK FOUNDATIONS

the outer wall is very small. Hence, only the bottom of the cell will be covered with water. If the cells are filled with fine, silty sand (Fig. 5(*b*)), the quantity of water that flows through the outer wall is still smaller because the fill constitutes one more obstacle against the passage of the water. Nevertheless, the line of saturation in the cell should occupy a high position, because the permeability of the fill is so low that the discharge even of a small quantity of water requires a steep hydraulic gradient. Finally, if the cells are filled with clay (Fig. 5(*c*)), the loss of head caused by the passage of the water though the outer row of sheet piles is negligible compared to the loss of head in the clay. As a consequence, the water percolates through the dam as if the outer row were nonexistent.

In practice the condition illustrated by Fig. 5(*a*) has rarely been encountered. The cells of the cofferdam for building the Troy dam and locks were filled with sand and gravel (3); yet the line of saturation started at the outside a short distance below water level. The average slope of the line was 1 (vertical) on 2 (horizontal). This observation suggests that the silt content in the fill of the cells was sufficient to establish conditions almost identical to those illustrated by Fig. 5(*c*). The cells of the cofferdam at Pickwick Landing (Tennessee Valley Authority) in Tennessee were filled with sand and gravel, and the inner wall of the cofferdam was provided with weep holes (8). Nevertheless, the line of saturation started at the outside of the dam only a few feet below the free water level, and it was almost horizontal over three fourths of the width of the cells. In spite of these experiences and observations, the customary methods of computing the tension in the locks of the sheet piles are based on the tacit assumption that the weep holes suffice to drain the fill in the cells almost completely, provided the fill consists of sand and gravel.

The cofferdams illustrated by Fig. 5 rest on an impermeable rock foundation. The seepage across a cofferdam with such a foundation merely affects the tension in the locks. If a cofferdam rests on a soil foundation, the effect of the seepage on the subsoil must also be taken into consideration. Fig. 6 shows three vertical sections through cofferdams resting on the same kind of soil as that with which the cells are filled. Because of the permeability of the subsoil, the water which seeps across the outer row of sheet piles combines with that which detours the buried part of the outer row and rises through the

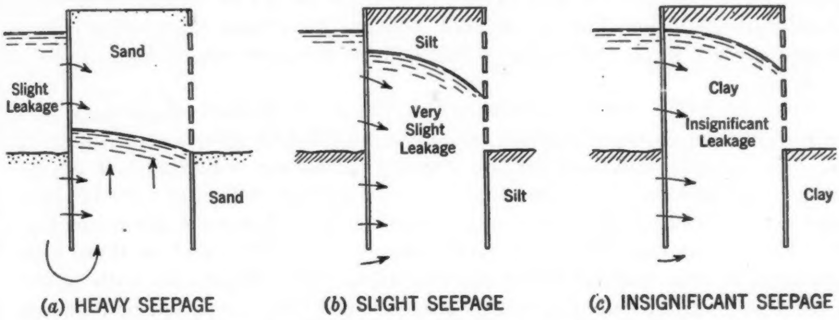

FIG. 6.—CELLULAR COFFERDAMS ON A SOIL FOUNDATION

subsoil. This additional flow raises the line of saturation to a position slightly above that shown in Fig. 5. Otherwise the flow diagrams are identical.

The conditions illustrated by Fig. 6(*c*) existed in the cofferdam around the hull of the *Maine* in Havana. Since the flow toward the pumps across the cofferdam with a total length of about 1,000 ft was less than 4 gal per sec, the engineers who worked on the dam were surprised by the high position of the line of saturation. They seem to have expected conditions like those illustrated by Fig. 6(*a*).

The water that passes through openings in the locks of the buried part of the inside row of sheet piles flows through the soil along the inside of this row in a vertical upward direction toward the surface adjoining the inside of the cofferdam. The mechanical effects of such a flow through clay are usually so insignificant that they cannot be noticed, because clay has a great resistance against scour. On the other hand, if the sheet piles were driven into fine silt, which has almost no resistance against scour, conditions are different. Because of the percolation through the locks, "boiling" may start inside the sheet-pile cutoff in silt at a small fraction of the head which would be required to produce the same effect in the same material on the inside of a perfectly impermeable cutoff. Hence, in connection with cofferdams on silt the permeability of the sheet-pile walls is a factor that must be considered.

PART II. DESIGN OF CELLULAR COFFERDAMS

General Considerations.—Cellular cofferdams may fail by bursting of the cells, by sliding on their base, by the shear deformation created by an inward tilt, or by a failure of their foundation.

Practically all the failures by bursting which occurred during or immediately after filling the cells can be traced back to the driving out of the lock. There seems to be no record to indicate that the cells of a cofferdam burst during application of the overturning moment, unless the cofferdam was in an advanced state of tilting, accompanied by a deflection of the crest of 20% or 25% of the height. There seems to be no record of a failure by sliding. All the failures by overturning that have come to public attention, such as the failures of several cellular cofferdams on the Mississippi River (6) and of the one in Grand Coulee (9) have been preceded by boiling or piping. Since the foundation conditions have a decisive influence on design, the cellular cofferdams on rock, sand, and cohesive soil will be discussed under separate subheadings.

Notation.—The letter symbols in this paper are defined where they first appear, in the text or by diagram, and are assembled for reference in Appendix II. To simplify equations for the stability of cellular cofferdams, it is customary and justified to assume that the curved inner and outer rows of sheet piles are replaced by fictitious straight ones spaced a distance b apart (see Fig. 1). For cofferdams with circular cells (radius r) $b = 1.7\, r$, and for those with semicircular cells (outside width b_1) $b = 0.9\, b_1$. The lengthwise walls of the ideal substitute for cofferdams with circular cells (Fig. 1(*a*)) will be assumed to be interconnected by straight cross-walls. These substitute cross-walls are arranged in pairs between the cells. The space between the two walls of each pair is assumed to be empty, but very narrow. The distance between two consecutive pairs is $2\, L$. The cells are assumed to be filled with sand unless some other material is specified in the text.

Cellular Cofferdams on Rock.—The customary methods for designing cellular cofferdams on rock were well described in 1934 by Raymond P. Pennoyer, Assoc. M. Am. Soc. C. E. (10). Rules that can be considered representative of current methods have been published by the Carnegie-Illinois Steel Corporation (11). According to these rules, the sliding resistance on the base per unit of length of the dam is given by the formula,

$$F_b = b\, H\, \gamma \tan \rho \qquad (1)$$

and the corresponding factor of safety of a cofferdam, acted upon by a horizontal force P per unit of length, with respect to sliding is

$$G_s = \frac{b\, H\, \gamma \tan \rho}{P} \qquad (2)$$

Since the force which tends to displace the dam by sliding also produces an overturning moment, the unit pressure on the base of the dam increases from a minimum at the outer toe to a maximum at the inner one as indicated by the plain curve cd in Fig. 7(*a*). The area abdc is equal to the product $b\, H\, \gamma$ in Eq. 1. This equation is based on the assumption that the cofferdam does not fail by sliding until the shearing stress at every point of the base of the dam is equal to the normal stress at that point multiplied by the coefficient of friction, $\tan \rho$. Hence the distribution of the shearing stresses at the instant of failure should be given by the dash-dotted line ef whose ordinates

are equal to those of curve cd times tan ρ. By the laws of mechanics it can be shown that the shearing stress at the outer toe must be almost equal to zero and at the inner toe it cannot be greater than the friction between the fill and the inner row of sheet piles per unit of area of the sheet piles at the

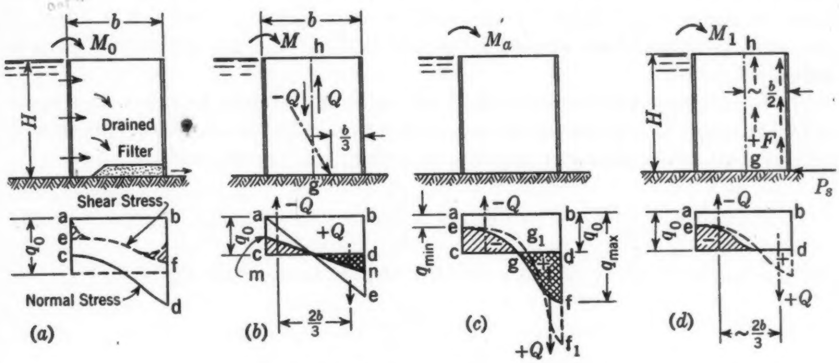

Fig. 7.—Cellular Cofferdams on Rock

elevation of the base. Hence, the shear-stress line is as indicated by the upper boundary of the shaded areas and the real sliding resistance is equal to the area abef $= b\,H\,\gamma \tan \rho$ reduced by the shaded areas. However, the amount of reduction is not sufficiently great to require consideration.

Since γ in Eq. 2 represents not the submerged but the real unit weight of the fill in the cells, the equation is not valid unless the fill in the cells is almost completely drained. However, as stated, there is no record to indicate that this condition ever existed. If the line of saturation is at an average height H_s above the base of the dam, the corresponding sliding resistance F_{bs} is lower than F_b, Eq. 1.

The pressure that produces friction is equal to the difference between the total pressure and the water pressure on the base. Since the unit weight of the fill in a saturated state is but slightly greater than the unit weight γ of the fill in a moist state, the shearing resistance on the base of a fill saturated to a height of H_s above the base is:

$$F_{bs} = b\,(H\,\gamma - H_s\,\gamma_w)\tan\phi \qquad (3)$$

On the cofferdams at Troy and at Pickwick Landing the value H_s was almost equal to H. Nevertheless, the dams did not fail by sliding. Failure was probably prevented by the roughness of the rock surface which compensated for the effect of incomplete drainage. If the rock surface is rough, the coefficient of sliding of the fill on the base is much closer to the coefficient of internal friction of the fill than to the customary value of 0.5 which represents the coefficient of friction between sand and fairly smooth rock. The sliding resistance of a cofferdam on rough rock is further increased by the resistance against the sliding of the lower edge of the inner row of sheet piles on the rock surface. Since the sheet piles of the inner row are pressed down by wall friction, this supplementary resistance may be very considerable. However,

if the rock surface is smooth, these factors do not apply, and hence the incomplete drainage of the fill may lead to failure by sliding. Therefore, if the rock surface is not very rough, Eq. 2 should not be used unless the designer takes care that the cells are really drained. This can be done by covering the bottom of the cells with a filter layer which communicates through weep holes with the space surrounded by the dam as shown in Fig. 7(*a*). The effect of the drainage provisions should be supervised on the job by means of telltale pipes.

In formulating the condition that the cofferdam should be safe with respect to overturning, it is generally assumed that the cofferdam can be regarded as a gravity wall. The moment required to overturn a gravity wall is

$$M_{max} = \frac{b^2}{2} H \gamma \qquad (4)$$

and the factor of safety of the wall with respect to overturning is

$$G_s = \frac{M_{max}}{M} \qquad (5)$$

If a perfectly elastic gravity wall with a rectangular cross section rests on a perfectly rigid base, the overturning moment M_a required to shift the point of application of the resultant pressure on the base from the center point to the inner boundary of the middle third is, approximately,

$$M_a = \frac{1}{3} M_{max} = \frac{b^2}{6} H \gamma \qquad (6)$$

If the overturning moment is equal to M_a, the unit pressure on the base increases from zero at the outer edge to $2\,H\,\gamma$ at the inner edge as indicated by the straight line ae in Fig. 7(*b*), and the corresponding factor of safety is

$$G_s = \frac{M_{max}}{M_a} = 3 \qquad (7)$$

If a cofferdam is acted upon by an overturning moment M, Eq. 7 is satisfied, provided the average width b of the dam is equal to

$$b_m = \sqrt{\frac{6\,M}{H\,\gamma}} \qquad (8)$$

Because of Eq. 7, a cofferdam with width b_m (Eq. 8) is considered safe, provided it is safe with respect to sliding. This conclusion is open to serious objections.

According to the theory on which Eqs. 4 and 6 are based the distribution of the unit pressure over the base of a cofferdam is determined by a straight-line law as indicated by the straight line mn in Fig. 7(*b*). As soon as the overturning moment M becomes equal to M_a, Eq. 6, the unit pressure at the outer edge of the cofferdam should become equal to zero (pressure area abe); yet it is obvious that this condition cannot exist in a dam whose cells are filled with sand. Even if arching develops above the outer part of the base of the fill to the fullest possible extent, the pressure on the outer part of the base must have

a considerable positive value q_{min}, which cannot decrease in spite of an increase of the overturning moment. Hence, the most unequal distribution of the pressure on the base that can possibly develop under the influence of an overturning moment M_a, Eq. 6, must be like that shown by the curve ef in Fig. 7(*c*). Since q_{min} cannot decrease, an increase of the overturning moment and the corresponding increase of the force Q in Fig. 7(*c*) to Q_1 must cause the neutral point g to shift toward the right. The pressure line egf moves into the position eg_1f_1. The base of the cross-shaded pressure triangle dfg decreases, but its area increases from Q to Q_1. Hence the toe pressure q_{max} increases not in simple proportion but roughly with the square of the overturning moment, and the cofferdam would fail by overturning while the point of application of the resultant pressure on the base is still fairly close to the middle third, provided the locks could stand the tension due to the rapidly increasing side pressure at the toe.

However, before this failure occurs, the cofferdam is likely to fail from another cause which has never received any attention. To demonstrate the existence of this cause in simple terms, the customary assumption is made that the distribution of the pressure on the base of the dam is such as shown by the straight line mn in Fig. 7(*b*). The shearing stresses that act at the surface of contact between the fill in the cells and the sheet piles prior to the application of the overturning moment will be disregarded. In Fig. 7(*b*) the shaded areas indicate the stresses on the base due to the overturning moment M. If Q is the total force represented by each triangle, one can write $M = \frac{2\,b}{3}\,Q$; or

$$Q = \frac{3\,M}{2\,b} \qquad (9)$$

Equilibrium requires that the total shearing force on the neutral plane gh per unit of length of the dam should be equal to Q. The shearing resistance on this plane, S' per unit of length of the dam, is equal to the earth pressure P_e times the coefficient of internal friction $\tan\phi$ of the fill. The earth pressure per unit of area of a vertical section through the fill at depth Z is equal to the vertical unit pressure, $\gamma\,Z$, in the fill at that depth multiplied by an empirical constant C known as the coefficient of earth pressure

$$p_e = \gamma\,C\,Z \qquad (10)$$

The earth pressure on gh per unit of length of the dam is

$$P_e = \tfrac{1}{2}\,\gamma\,C\,H^2 \qquad (11)$$

and the shearing resistance S' is

$$S' = P_e \tan\phi = \tfrac{1}{2}\,\gamma\,C\,H^2 \tan\phi \qquad (12)$$

To the value S' must be added the friction in the locks, because no failure by shear along gh, Fig. 7(*b*), can occur without simultaneous slippage in the lock in this plane. If p_e, Eq. 10, is the unit earth pressure on gh at depth Z, the tension in the locks of an arc with radius r at depth Z is

$$t = r\,p_e = r\,\gamma\,C\,Z \qquad (13)$$

Total tension in a lock with length H is

$$T = \tfrac{1}{2}\,\gamma\, C\, H^2\, r \qquad (14a)$$

and the total resistance against slippage in the lock is

$$T f = \tfrac{1}{2}\,\gamma\, C\, H^2\, r\, f \qquad (14b)$$

wherein f is the coefficient of lock friction. Since the rows of sheet piles intersect the **Y**-piles at an angle of 120°, the tension in the locks of the cross-walls and the corresponding lock friction is equal to that in the arcs. The cofferdams with circular cells contain two cross-walls per cell and the length of each cell is $2\,L$; those of the diaphragm type contain one cross-wall per cell and the length of each cell is L. Hence, the resistance to shearing along gh contributed by the lock per unit of length of either type of cofferdam is

$$S'' = \frac{T f}{L} = \tfrac{1}{2}\,\gamma\, C\, H^2 \frac{r}{L} f \qquad (15)$$

To obtain the total average shearing resistance S on gh per unit of length of the cofferdam, S'' must be added to S', Eq. 12, whence,

$$S = S' + S'' = \tfrac{1}{2}\,\gamma\, C\, H^2 \left(\tan\phi + \frac{r}{L} f\right) \qquad (16)$$

The ratio $\frac{r}{L}$ is close to or equal to unity. Hence, approximately,

$$S = \tfrac{1}{2}\,\gamma\, C\, H^2 (\tan\phi + f) \qquad (17)$$

The corresponding factor of safety G_s with respect to failure by shear along gh is equal to the ratio between the shearing resistance S (Eq. 17) and the shearing force on gh which is equal to Q (Eq. 9) or

$$G_s = \frac{S}{Q} = \gamma \frac{b\, H^2}{3\, M} C\, (\tan\phi + f) \qquad (18)$$

If one face of the cofferdam with height H is acted upon by water pressure, the overturning moment is

$$M_0 = \frac{1}{2}\,\gamma_w\, H^2 \frac{H}{3} = \frac{1}{6}\,\gamma_w\, H^3 \qquad (19a)$$

and

$$G_s = 2\,\frac{\gamma\, b}{\gamma_w\, H}\, \mathrm{C}\, (\tan\phi + f) \qquad (19b)$$

The customary theories of cofferdams are based on the assumption that every vertical section through the fill in the cells is acted upon by the active Rankine pressure, P_A per unit of width of the section, involving

$$C = \tan^2\left(45 - \frac{\phi}{2}\right) \qquad (20)$$

To visualize the practical implications of Eqs. 19*b* and 20, it is assumed that $\phi = 30°$, $f = 0.3$, $\gamma = 110$ lb per cu ft, $\gamma_w = 62.5$ lb per cu ft, and $b = 0.85\,H$.

Evaluating Eq. 5 on the basis of these data it is found that the factor of safety with respect to overturning should be 3.8. In reality, it is certainly not greater than about 2. However, the factor of safety with respect to shear, Eq. 19*b*, is only $G_s = 2 \frac{110}{62.5} \frac{0.85\,H}{H} 0.577^2\,(0.577 + 0.3) = 0.87$. In other words, the cofferdam would fail by shear along gh, Fig. 7(*b*), although even the real factor of safety with respect to overturning is still excessive for a temporary structure. Replacing $\phi = 30°$ by $\phi = 34°$ and $\phi = 40°$, the values of $G_s = 0.82$ and 0.74 instead of 0.87 are obtained.

The average width *b* of the cellular cofferdams with circular cells which were built on a rock foundation is not greater than about 0.85 *H*. The width of some of them was somewhat smaller; yet none of them failed by shear along gh, Fig. 7(*b*). The coefficient of friction on this plane cannot be greater than $\tan \phi$. Therefore, the normal pressure on this plane, and, as a consequence, the horizontal earth pressure in the entire fill in the cells, must be considerably greater than the active Rankine pressure involving

$$C > \tan^2\left(45° - \frac{\phi}{2}\right) \qquad (21)$$

Comparing Eq. 17 with Eq. 12, it can be noted that the lock friction increases the average coefficient of shearing resistance along gh (Fig. 7(*b*)) from $\tan \phi$ to $(\tan \phi + f)$. The value of f is likely to be fairly close to 0.3. If $\phi = 30°$ and $\tan \phi = 0.58$, the lock friction increases the factor of safety with respect to failure by shear by about 50%. Hence, the stabilizing influence of the lock friction is by no means negligible.

Because of the lock friction a cellular cofferdam may be as much as 50% more stable than a double-wall cofferdam with equal width and height, but without cross-walls. If the inner side of a cellular cofferdam is supported by a berm whose height is almost equal to the height of the dam, both the cross-walls and the inner row of sheet piles are acted upon by equal earth pressures from both sides. This condition practically eliminates the tension in the locks, but it also eliminates the lock friction. The lock friction becomes fully operative at a very much smaller deflection of the cofferdam than does the shearing resistance of the fill in the cells. Therefore, a cellular cofferdam with a free inside wall (self-supporting cellular cofferdam) is likely to deflect less than a cofferdam reinforced by an inner berm. In this connection, it may be mentioned that the crest of one of the two cofferdams with rock-fill berms between West 44th Street and West 49th Street in New York deflected more than 4 ft (Fig. 3) and the other one more than 6 ft. This is far in excess of the usual deflection of the crest of self-supporting cellular cofferdams.

On some cofferdams, the locks were lubricated to facilitate pulling the sheet piles. From a functional point of view, it would be preferable to increase the lock friction by artificial means.

Finally the possibility of a failure of cofferdams by a bursting of the locks must be considered. The tension per unit of length of a lock is given by Eq. 13. The greatest value which Z in this equation can assume is H. For

$Z = H$, t becomes equal to

$$t_1 = r\,\gamma\,C\,H \quad \text{(22a)}$$

The standard method for computing the greatest tension per unit of length in a lock is based on the assumption that C in Eq. 22*a* is equal to the coefficient of the active Rankine pressure, $\tan^2\left(45° - \frac{\phi}{2}\right)$, whence

$$t_1 = r\,\gamma\,H \tan^2\left(45° - \frac{\phi}{2}\right) \quad \text{(22b)}$$

The sheet piles are chosen so that t_1 is equal to or smaller than the allowable lock tension which in turn is equal to the failure tension divided by about 2.5. Hence, the factor of safety with respect to a failure by tension in the locks is assumed to be about 2.5. However, the discussion of the resistance of the cellular cofferdams against shear has led to the conclusion that the factor C in Eq. 22*a* is considerably greater than $\tan^2\left(45° - \frac{\phi}{2}\right)$. Hence, the factor of safety with respect to a failure of the locks by tension is smaller than 2. Furthermore, the method of computation represented by Eq. 22*b* fails to consider that the application of the overturning moment may increase the pressure in the fill at the inner toe. According to the standard concept, the application of the overturning moment should change the initial pressure area abdc, Fig. 7(*b*), for a cofferdam with the customary dimensions to about abe which involves an increase of the soil pressure at the inner toe by roughly 100%. This would reduce the factor of safety to less than unity and the cells would fail by bursting. In reality the lower boundary of the pressure area for the base of a cofferdam is not straight as indicated by the lines mn and ae in Fig. 7(*b*) but curved as shown by the line ef in Fig. 7(*c*). Hence the real increase of the lock tension due to the overturning moment is still more important.

Finally, Eq. 22*b* involves the tacit assumption that the fill in the cells is almost completely drained. If the line of saturation is close to the top of the fill as it was in the cellular cofferdams at Troy and at Pickwick Landing, the cells not only should burst; they should explode.

In contrast to this conclusion, experience has demonstrated the following fact: If the locks are strong enough to stand the lateral pressure of the fill before an overturning moment is applied, they are also strong enough to stand the subsequent changes produced by the overturning moment, provided the cofferdam is safe in every other respect. Hence, the increase of the lock tension due to the overturning moment cannot possibly be important. As a matter of fact, there is no evidence that the real factor of safety with respect to a failure by bursting of the cells of any one of the successful cellular cofferdams was appreciably smaller than 2; yet equilibrium requires that the overturning moment be fully counterbalanced by a couple on the base of the dam. Hence the question arises: "What is the nature of the forces which constitute this couple?"

To answer this vital and obvious question, the interaction between the fill in the cells and the adjoining sheet piles must be considered. If there were no

initial shearing stresses at the surface of contact between the fill and the inner row of sheet piles, the vertical unit pressure in the fill adjoining the inner row at any depth Z below the top of the cells would be γZ, and at the inner edge of the base it would be γH. Any increase of the vertical pressure in the fill beyond these values involves a shortening of the height of the fill, and the shortening is resisted by the friction between the fill and the sheet piles. Since an insignificant slip along the surface of contact between fill and sheet piles is sufficient to mobilize the full sliding resistance along this surface, the unit pressure on the inner part of the base cannot increase beyond γH until the shearing stresses on the inner contact face become equal to the full value of the friction between sand and sheet piles. At that stage the pressure on the base of the fill is about as shown by the area abde in Fig. 7(d). On the inside (right-hand side) of the neutral plane gh the unit pressure on the base of the fill is still γH, but the vertical contact faces of the fill are acted upon by vertical friction forces, F_1 per unit of width of the sheet piles or Q per unit of length of the dam. Since the sand pressure per unit of width is equal to $\frac{1}{2} \gamma H^2 C$, the friction force F_1 is

$$F_1 = \tfrac{1}{2} \gamma H^2 C \tan \delta \quad \text{(23)}$$

The total perimeter of one cell with length $2L$ of a cellular cofferdam with cylindrical cells, Fig. 1(a), on one side of the neutral plane gh, is $(2L + b)$ or

$$l = 1 + \frac{b}{2L} \quad \text{(24)}$$

per unit of length of the dam. Hence,

$$Q = l F_1 = \tfrac{1}{2} \gamma H^2 C \left(1 + \frac{b}{2L}\right) \tan \delta \quad \text{(25a)}$$

Since $\frac{L}{b}$ is roughly equal to 0.65 or $\frac{b}{2L} = 0.77$,

$$Q = 1.77 \times \tfrac{1}{2} \gamma H^2 C \tan \delta \quad \text{(25b)}$$

The sum of all the vertical reactions produced by the overturning moment is equal to zero. Therefore, the sum of the forces acting on the fill on the outside (left-hand side) of the neutral plane gh must be equal to $-Q$ per unit of length of the dam. Assuming that the lever arm of the couple constituted by the forces $+Q$ and $-Q$ is roughly equal to $\frac{2b}{3}$, the overturning moment M_1 required to establish the state of stress on the base of the fill, indicated by the area abde in Fig. 7(d), is

$$M_1 = \tfrac{2}{3} b Q = 1.18 b \times \tfrac{1}{2} \gamma H^2 C \tan \delta \quad \text{(26a)}$$

The overturning moment produced by the water pressure on one side of the dam is given by Eq. 19a:

$$M_0 = \tfrac{1}{6} \gamma_w H^3 \quad \text{(26b)}$$

and the ratio m between the required moment M_1 and the acting overturning

moment M_0 is

$$m = \frac{M_1}{M_0} = 3.54 \frac{b}{H} \frac{\gamma}{\gamma_w} C \tan \delta \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The average width of the base of cofferdams on rock which are built to withstand full water pressure is usually about 0.85 H. Assuming $\gamma = 110$ lb per cu ft, $\gamma_w = 62.5$ lb per cu ft, $C = 0.4$, and $\tan \delta = 0.4$, $m = 0.85$. This value, $m = 0.85$, indicates that about 85% of the total overturning moment M_0 is carried by the couple $+ Q$ and $- Q$. In other words the unit pressure at the inner edge of the base of the fill does not begin to increase until about 85% of the total overturning moment M_0 is applied. The increase of the unit pressure due to the remaining 15% of the overturning moment is well within the margin of safety. This explains the empirical fact that the cells of a cofferdam with the customary dimensions burst either before the overturning moment is applied or not at all. However, once the overturning moment becomes equal to 1.2 M_1 or 1.3 M_1, a further increase of the moment must cause as rapid an increase of the soil pressure at the inner toe as it would from the very outset under the fictitious conditions illustrated by Fig. 7(c). Hence, after the overturning moment becomes greater than 1.2 M_1 or 1.3 M_1, the danger of a failure due to the bursting of the cells increases rapidly.

The maximum lock tension is further reduced by the forces which counteract an inward movement of the lower edge of the inner row of sheet piles. According to Eq. 22a the lock tension should increase in simple proportion to the depth below the top of the cells. Assuming that the elongation of the sheet-pile walls increases in simple proportion to the lock tension, the vertical section through the inner row of sheet piles should be fairly straight at any stage of filling and pumping, and its lower edge should move inward. However, experience indicates that the inner row of sheet piles bulges at an elevation $\frac{H}{4}$ or $\frac{H}{3}$ above the level of the base and that the lower edge of the sheet piles does not move. Such a deformation can occur only if the distribution of the tension along a lock is similar to that indicated by the area acc_1 in Fig. 8. Since there is little doubt that the earth pressure increases from the top of the cells toward the bottom as indicated by the pressure area aa_1c, equilibrium requires that the lower edge of the sheet piles be acted upon by a horizontal force P_s. The origin of this force is obvious. The force F_1 (Eq. 23) presses the lower end of the sheet piles to the rock surface and the pressure produces a frictional resistance, P_s. Fortunately for the cofferdam, the force P_s acts precisely at the point where the lateral earth pressure is greatest. Hence, the greatest lock tension is considerably smaller than that which would correspond to the soil pressure at the base. At the lower edge of the sheet piles where, according to theory, the lock tension should be greatest, it is equal to zero.

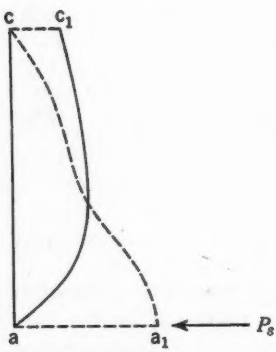

FIG. 8

To make the statement of the fundamental assumptions complete, it should be mentioned that throughout the discussion the empirical factor C in Eq. 10 was tacitly assumed to have the same value for every point in the fill of the cells. This is certainly not more than a very crude approximation. First, by analogy to the conditions in the backfill of a retaining wall it is very probable that the value C for the central part of the fill is much greater than that for the fill adjoining the inner row of sheet piles. Second, if C were a constant along every horizontal section through the inner row, the arcs should remain circular. However, the observations described subsequently show that this conclusion is not even approximately correct. The laws according to which the value C varies within the fill of the cells are still unknown. Hence, an attempt to evaluate the greatest lock tension on a purely theoretical basis would appear rather futile.

The preceding leaves no doubt that two of the three customary equations for the design of cellular cofferdams (Eqs. 8 and 22*b*) are inadequate. The third, Eq. 1, is based on the assumption that the fill in the cells is almost completely drained; yet this formula has been advocated without a statement of the important fact that weep holes alone may not suffice to drain the cells although they may be filled with sand or sand and gravel. So far the equations have not done any harm because they have fortunately been used on the basis of very conservative values for the soil constants. However, some day an ambitious designer may conceive the idea of determining the soil constants by laboratory tests and using the values furnished by the tests. The result could be fatal for the cofferdam.

The equations derived by the writer contain an empirical factor C (ratio between horizontal and vertical soil pressure in fill of cells) whose value could be determined only by actually measuring both the horizontal and the vertical soil pressures at different points in the fill of the cells in full-sized cellular cofferdams before and after the application of the overturning moment. So far no such observations have been made. On the basis of the known behavior of cellular cofferdams, the writer estimates that the value C for the middle part of the fill in the cells ranges between 0.4 and 0.5. Along the inner row of sheet piles it may be considerably smaller. In no event could C be determined by laboratory tests.

In the preceding paragraphs an attempt was made to analyze the conditions that determine the stability of cellular cofferdams on rock. The analysis disclosed the existence of a potential source of failure by shear along vertical planes (Eq. 19*b*) which has been ignored in the past, although it is much more likely to take place than failure due to overturning. It also demonstrated the decisive influence of the lock friction and of the friction between the fill and the walls of the cells on the strength and stability of the dam (Eqs. 17 and 22*a*). This opens up at least a theoretical possibility for increasing the stability or for reducing the dimensions of cellular cofferdams by artificially increasing the roughness of the surface of the steel. Since all the equations contain the empirical factor C whose value, for the time being, can only be guessed at, the equations cannot yet be used directly in connection with the design of cellular cofferdams. Pending further development, it seems safest

to proceed on the basis of past experience as explained in the following paragraphs.

The average width b_0 of all the cellular cofferdams built to date on a rock foundation to sustain nothing but one-sided water pressure was about equal to 0.85 H. The water pressure that acts on the outside of the cofferdam per unit of length of the dam at the instant of overtopping will be designated by P_0 and the corresponding overturning moment is equal to M_0, Eq. 19*a*. Experience has shown that cofferdams with such dimensions are fully adequate; yet the preceding numerical computations have demonstrated that their factor of safety with respect to failure by shear along the neutral plane is certainly not greater than 1.5. Hence, the ratio, $\frac{b_0}{H} = 0.85$, may be considered an acceptable basis for design. To keep the factor of safety of a cofferdam with respect to sliding (Eq. 2) unaltered while increasing the unbalanced horizontal pressure from P_0 to $P = n_1 P_0$, it is necessary to increase its width from b_0 to $\frac{b_0 P}{P_0} = n_1 b_0$. To maintain the same condition with respect to a failure by shear along a vertical plane (Eq. 18), while increasing the overturning moment from M_0 to $M = n_2 M_0$, it is necessary to increase the width from b_0 to $\frac{b_0 M}{M_0} = n_2 b_0$. Hence, if a cofferdam with height H on a rock foundation is acted upon by a horizontal force $P = n_1 P_0$ producing an overturning moment $M = n_2 P_0$, it should have an average width,

$$b = n\,b_0 = 0.85\,n\,H \qquad (28a)$$

in which n is the greater one of the two values n_1 and n_2.

According to the assumption illustrated by Fig. 1(*a*), the radius r of circular cells is $r = 0.59\,b$, whence,

$$r = 0.59\,b = 0.59\,n\,b_0 = 0.59 \times 0.85\,n\,H = 0.50\,n\,H \qquad (28b)$$

The outside width b_1 of cellular cofferdams with semicircular cells (Fig. 1(*b*)) is 1.11 b; thus:

$$b_1 = 1.11\,b = 1.11\,n\,0.85\,H = 0.94\,n\,H \qquad (28c)$$

The distance L between the cross-walls of such dams is 0.65 b ($\pm$), whence,

$$L = 0.65\,n\,0.85\,H = 0.55\,n\,H \qquad (28d)$$

Since the arcs intersect the cross-walls at the **Y**-piles on an angle of 120°,

$$r = L = 0.55\,n\,H \qquad (28e)$$

The maximum lock stress is given by Eq. 22*b*. The writer is not aware that any designer has ever used a value of much less than 0.4 for C in Eq. 22*a*. As a result of this wise practice, there have been no serious failures due to the bursting of cells. Therefore, the writer suggests that the greatest lock tension in the walls of cells filled with sand, or sand and gravel, be expressed by means of the empirical equation,

$$t = 0.4\,\gamma\,r\,H \qquad (28f)$$

Eq. 28*f* does not create the illusion that it rests on a scientific basis and leaves no margin for misinterpretation. If the cells are filled with a soil with a high clay content, the resistance of the soil against sliding is likely to be deficient. If the surface of the rock is not very rough, adequate provisions should be made for draining the fill in the cells and the state of drainage should be ascertained at different stages of pumping within the cofferdam by measuring the elevation of the water level in telltale pipes whose lower ends are located at different depths below the crest—for instance, $\frac{H}{4}$, $\frac{H}{2}$, and $\frac{3\,H}{4}$. It is also advisable to observe the deflection of the crest of the dam during, and after, the time when the space between the cofferdams is being pumped out.

The cross-walls of some of the older cellular cofferdams, such as those of Troy (3), Cape Fear River (4), and St. Louis Intake (12) were reinforced by diagonal tension rods or tension beams. These members increased the stiffness of the cofferdams very considerably. This was demonstrated impressively on the Cape Fear River, where the failure of the pin connection of the tie rods in four cells caused the top of these cells to advance a distance up to 7.5 ft, whereas the deflection of all the other cells was unimportant. To utilize the tie-rod principle to full advantage, it would be necessary to make the pin connections strong enough to permit stretching of the tie rods well beyond the yield point. Otherwise, the pin connections fail before the shearing resistance of the fill in the cells is fully active.

Lubrication of the locks may facilitate pulling the sheet piles, but it reduces, very considerably, the strength and stiffness of the cofferdam.

Cellular Cofferdams on Deep Sand Strata.—A review of current methods of designing cofferdams on sand reveals an amazing situation. The design of permanent storage dams on sand is governed by the necessity of eliminating the danger of the formation of boils at the downstream toe. All other requirements are of secondary importance and can easily be satisfied. In contrast to this sound practice, in all the theories pertaining to cofferdams on sand, such as those of H. Krey (13), K. Hager (14), Mr. Rimstad (5), and Bureau of Public Roads (15), the water is replaced by a continuous horizontal load equivalent to a water pressure on the outside of the dam and the other far more dangerous mechanical effects of the water are not even mentioned. The relative frequency of the failure of cofferdams on sand suggests that this unnatural mental attitude toward the problem is not limited to the writers of theories.

The mechanical effect of seepage on the stability of cofferdams had not received any adequate attention in engineering literature until it was treated by Lazarus White and Edmund A. Prentis, Members, Am. Soc. C. E. (6), in 1940. This work contains the first vigorous and well-documented protest against the prevailing indifference regarding the effect of seepage on the stability of cofferdams on sand. The following paragraphs contain a brief review of the mechanics of the formation of boils and of the means for preventing this dangerous phenomenon.

Fig. 9(*a*) is a vertical section through a prismatic element of a mass of sand with a volume V and a length l. The water percolates through the sand in

the direction of the dashed arrow. Let γ' = submerged unit weight of the sand; γ_w = unit weight of the water; h = loss of hydraulic head associated with flow of water through distance l; and $i = \frac{h}{l}$, the corresponding hydraulic gradient.

In Fig. 9(a) the loss of head h is given by the vertical distance between the water levels in two standpipes whose lower ends are at the center of the entrance

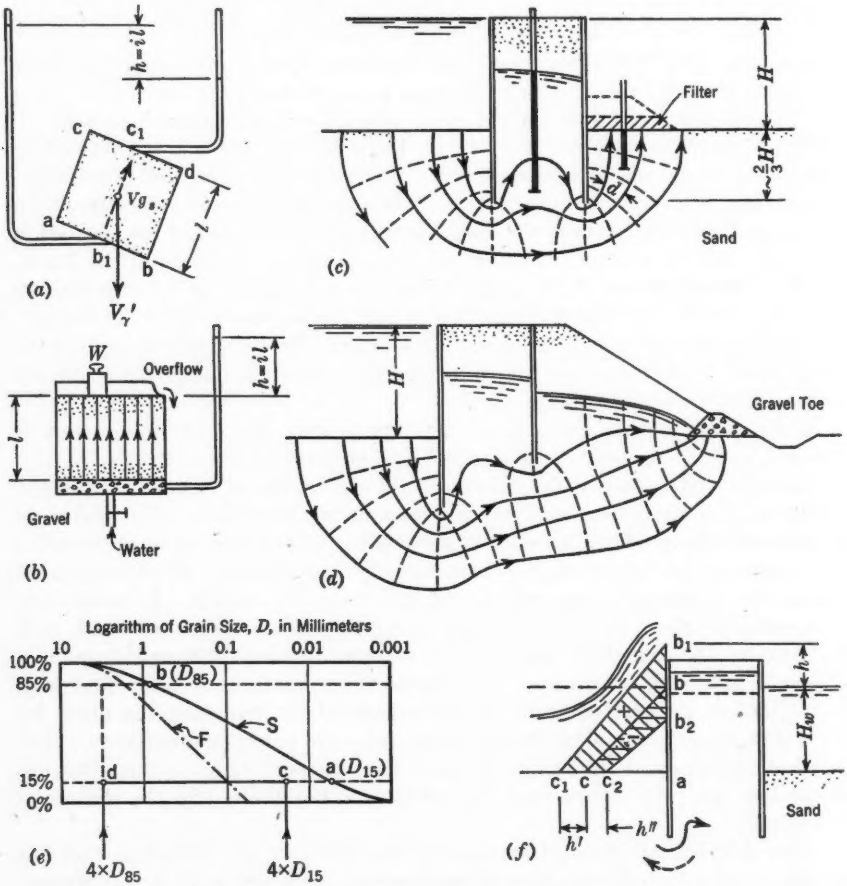

FIG. 9.—CELLULAR COFFERDAMS ON DEEP STRATUM

and the exit surface of the prismatic element. A loss of head h reduces the water pressure per unit of area of a plane section at a right angle to the direction of the flow by $\gamma_w h$. Hence, the percolating water adds to the submerged weight, $\gamma' V$, of the prismatic element (Fig. 9(a)), an unbalanced pressure $\gamma_w h$ per unit of area of its base ab acting in the direction of the dashed error. Since the length of the element in the direction of the flow is l, the unbalanced

force is equal to

$$g_s = \gamma_w \frac{h}{l} = \gamma_w i \qquad (29a)$$

per unit of area and unit of length, in other words per unit of volume. This force represents the seepage force. If the water flows through a mass of sand vertically upward, the seepage force g_s acts in a direction opposite to that of gravity. If $g_s = \gamma_w i_1 = \gamma'$—

$$i_1 = \frac{\gamma'}{\gamma_w} \qquad (29b)$$

the resultant of the hydrostatic pressures counterbalances the force of gravity, and the sand should behave like a liquid (16). To verify Eq. 29b, a cylindrical vessel (Fig. 9(b)) was filled with sand, and water was allowed to percolate through the sand in an upward direction. The lines along which the water particles travel are marked by arrows. As long as the hydraulic gradient i was smaller than i_1, Eq. 29b, the sand grains were motionless and the permeability of the sand was unaltered. However, at the instant when i became equal to i_1, the sand started to boil and its permeability increased abruptly. Before the test a brass weight W was placed on the surface of the sand. As i approached the value i_1, the weight settled progressively, and when i became i_1 the weight disappeared in the boiling sand.

Fig. 9(c) is a vertical section through a cellular cofferdam on sand. The design of such a dam must satisfy three independent conditions: (a) The sand along the outer face of the cofferdam should be protected adequately against erosion; (b) the dam should be able to withstand the overturning moment produced by the water pressure on its outer face; (c) the subsoil should be able to carry the pressure on the base of the dam in spite of the tendency of the seepage forces to liquefy the sand at the inner toe. The practical implications of surface erosion have been presented expertly by Messrs. White and Prentis (6). In this paper only conditions (b) and (c) will be discussed; and for the sake of clarity they will be considered independently.

In the discussion of the stability of cofferdams on a rock foundation, illustrated by Figs. 7(c) and 7(d), it was shown that the inner row of sheet piles is pressed down by a friction force F_1, Eq. 23, per unit of length of the row. If the sheet piles move down under the influence of this force, the cofferdam is likely to fail by bursting of the locks. To prevent such a failure of a cofferdam on sand, the sheet piles of the inner row should be driven deep enough into the sand to withstand the vertical load F_1 (Eq. 23) per unit of length of the row of sheet piles without perceptible penetration. This condition is scarcely satisfied unless the sheet piles are driven to a depth of at least $\frac{2}{3} H$ into the sand.

To plan appropriate measures for the prevention of boiling, it is necessary to consider the flow of water through the subsoil of the cofferdam. In Figs. 9(c) and 9(d) the flow lines are marked by arrows. They were traced in such a manner that the quantity of water v_d which percolates between two flow lines is the same for every pair of flow lines. If this condition is satisfied, it is easy to realize that the spacing d between the flow lines indicates the distribution of the seepage force g_s (Eq. 29a) throughout the subsoil of the cofferdam.

The quantity of water that flows between two adjoining flow lines per unit of the distance between these flow lines is $\frac{v_d}{d}$. It is called the rate of percolation. According to Darcy's law the rate of percolation is equal to

$$\frac{v_d}{d} = k\,i \qquad (30)$$

in which k is a constant, the coefficient of permeability. Hence, $i = \frac{v_d}{d\,k}$ and

$$g_s = i\,\gamma_w = \frac{v_d\,\gamma_w}{d\,k} = \frac{1}{d} \text{ times a constant} \qquad (31)$$

According to Eq. 31, the seepage force g_s is inversely proportional to the spacing of the flow lines. As shown in Fig. 9(*a*), it acts at every point in the direction of the tangent to the flow line through this point.

The flow lines in Figs. 9(*c*) and 9(*d*) were constructed on the assumption that the rows of sheet piles are perfectly impermeable and that the sand is perfectly homogeneous. The short horizontal lines in Fig. 9(*c*) represent the lower ends of observation wells. The height to which the water would rise in each well is represented by a black column. Methods of constructing the flow lines and determining the height of the rise of the water in the wells have been discussed elsewhere (17). These methods also permit a determination of the seepage pressure g_s (Eq. 29*a*), for every point of the subsoil.

The water that percolates through the soil beneath the cofferdam escapes from the soil in a vertical upward direction as it does in the vessel shown in Fig. 9(*b*). The seepage velocity and the corresponding seepage force that tends to lift the sand are greatest next to the toe of the dam. Therefore, the floor of the excavation adjoining the toe constitutes an area of potential boiling. This will be called the danger zone. Boiling eliminates the resistance of the sand against an inward movement of the buried part of the inner row of sheet piles. It reduces the resistance of the inner row of sheet piles against penetration under the influence of the force F_1 (Eq. 23), and the cofferdam fails by toppling inward. The failure is either preceded or followed by a bursting of the cells. Several failures of this type have occurred on the Mississippi River (6).

The formation of boils can be prevented by two different methods: (*a*) By covering the danger zone with a loaded, inverted filter; and (*b*) by increasing the length of the path of percolation by means of a berm with a wide base. The inverted filter which constitutes the essence of method (*a*) is indicated in Fig. 9(*c*) by dashed lines. Since the filter material is much more permeable than the protected soil, the presence of the filter has no influence on the shape of the flow net, but the load that acts on the filter counteracts the vertical component of the seepage forces which tend to lift the sand beneath the danger zone.

Method (*b*) is illustrated by Fig. 9(*d*). The berm can be made of dredged material. Although the downstream ends of the flow lines in Fig. 9(*d*) are not vertical, they are spaced fairly closely, producing a tendency of the toe of the berm to slump due to backward erosion. This can be prevented by a gravel toe.

If an inverted filter made out of clean, coarse gravel or broken stone is used to cover the surface of the danger zone in a mass of sand, the sand is washed into the interstices of the gravel. One piece of gravel after another sinks into the sand and finally the filter ceases to function. Therefore, the filter material should satisfy two independent conditions. It should be coarse enough to permit free discharge of the seepage water, and, at the same time, its voids should be small enough to exclude the possibility of an invasion of finer soil particles from below. By experimenting with different subsoils and filter materials, the writer arrived at the empirical rule illustrated by Fig. 9(*e*). The abscissas of this diagram represent the logarithm of the grain size and the ordinates the percentage of the total weight composed of grains smaller than the size denoted by the abscissa. Curve S is the grain-size curve for the soil which should be protected. Abscissas of points c and d represent four times the grain size D_{15} and D_{85}, corresponding to the points a and b on the grain-size curve S. If the grain-size curve F for a filter material intersects the line dc, it can be considered suitable.

When building a cofferdam on an alluvial deposit, suitable filter material can usually be obtained by screening the soil which needs to be protected and by wasting the finest fractions. By wasting 50% of the soil represented by curve S in Fig. 9(*e*), one obtains the material represented by curve F which satisfies all the requirements. A filter bed on a flooded surface can be constructed only by means of one of the methods which are used for pouring concrete under water. If the filter material is simply dumped into the water, the coarsest particles settle first, and the result of the operation is a stratified layer which does not satisfy the essential requirements. The functioning of the filter should be supervised by means of observation wells whose lower ends are a short distance below the base of the filter. If the filter functions properly, the water should not rise in the wells more than a few inches above the base of the filter.

In 1923 the writer equipped the foundation for a permanent storage dam on sand and gravel with a filter designed on the basis of Fig. 9(*e*), and in 1925 he repeated the procedure on another dam on sand. Since both experiments were successful, he used the filter principle in the design of many dams and all the filters functioned as anticipated. On the basis of these experiences he recommends the design illustrated by Fig. 9(*c*) as an alternative to the customary berm type shown in Fig. 9(*d*), provided the rules for grading and constructing the filter are strictly adhered to and the functioning of the filter is conscientiously supervised.

The preceding comment furnishes a basis for the design of cellular cofferdams on sand. The width of the dam can be determined by Eq. 28*a*. The maximum lock tension is computed by Eq. 19(*b*). Boiling can be prevented by either one of the methods illustrated by Figs. 9(*c*) and 9(*d*). If a filter is used, the sheet piles should be driven to a depth $\frac{2\,H}{3}$ or until they bear on a hard stratum. The final choice between the two methods depends on the estimated cost and on the width of the space available for constructing the cofferdam. It is obvious that the unit price for the filter layer is much higher

than that for a berm made of dredged material and the necessity for expert supervision during and after construction is a distinct liability in connection with temporary structures; but the berm requires much more space than the filter, as indicated in Figs. 9(*c*) and 9(*d*).

Figs. 9(*c*) and 9(*d*) represent two different ways for preventing boils by means of berms. If the local conditions do not permit the construction even of a narrow berm, the filter can be established at the base of the fill in the cells, in the position occupied by the filter in the cofferdam shown in Fig. 7(*a*). If an untried method is to be considered, it is necessary to investigate the mechanical efficacy of the proposed method by means of a flow net. The flow net can be constructed (17) or it can be obtained experimentally (6). Some engineers prefer the experimental method. The graphical method is cheaper, more expedient, and somewhat more accurate, but the experimental method carries more conviction with those engineers who are not accustomed to scientific reasoning.

The results obtained by either method are strictly valid only if the rows of sheet piles are perfectly impermeable and if the natural sand stratum is perfectly homogeneous. In reality neither one of the two conditions is satisfied and the resulting error can be very important. Hence, whatever method of investigation is used, it is imperative to discover by means of observation wells on the job whether the flow of seepage is at least approximately similar to that anticipated by the designer. If the discrepancy is on the unsafe side, supplementary measures of protection should be adopted during construction.

Fig. 9(*f*) is a vertical section through a breakwater. During storms breakwaters are subject to a pulsating water pressure. While the waves pound against the outside of the breakwater during a storm, the hydrostatic pressure on the outside of the breakwater varies between the values represented by the pressure triangles ab_1c_1 and ab_2c_2. The hydrostatic pressure at the outer toe oscillates between $[\gamma_w (H_w + h')]$ and $[\gamma_w (H_w - h'')]$, in which H_w is the depth of the water adjoining the breakwater before the storm. The values h', h'', and h in Fig. 9(*f*) depend on H_w, the wave length, and the height of the waves. A theory for estimating the wave pressure on the uppermost part of the exposed face of the breakwater was presented in 1935 by the late D. A. Molitor, M. Am. Soc. C. E. (18). A comparison between computed and observed values of wave pressure was made by T. L. Condron, M. Am. Soc. C. E. (19). Equations for computing the wave pressure over the entire exposed face were derived by M. Sainflou in 1928 (20). Since the excess hydrostatic pressure on the outside of the breakwater has the character of a pulsating force, it produces vibrations in both the structure and the subsoil. Because of the vibrations, the settlement is likely to be much more important than that produced by a static system of forces under similar circumstances (17). If the sand on which the breakwater rests is loose, the vibrations may cause a spontaneous settlement associated with temporary liquefaction of the sand whereupon the breakwater would suddenly sink into the sand as if the sand were a liquid. Thus, if the sand is loose, the construction of the breakwater should be preceded by compacting the sand beneath and on both sides of the breakwater by artificial means to a depth which is equal to at least 1.5 times the

height H. This can be accomplished, for instance, by driving sand piles or by the "vibroflotation method" (21).

Cellular Cofferdams on Clay.—The only attempt to design a cofferdam on clay on a purely theoretical basis was made by H. Epstein (22), who computed stability on the assumption that the cofferdam can be regarded as a simple gravity wall. The highly controversial aspects of this assumption have already been discussed herein under the heading, "Cellular Cofferdams on Rock." In estimating the required depth of penetration of the sheet piles, the vertical forces transmitted from the fill in the cells through side friction to the sheet piles, and through the sheet piles to the foundation, have not been considered. The computations themselves involve an unwarranted generalization of the Rankine-Résal equations. For these reasons, the analysis fails to furnish a satisfactory basis for the design of a cofferdam on a clay foundation.

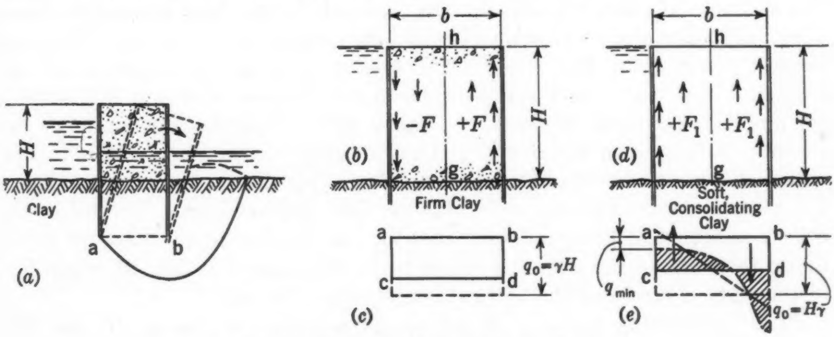

FIG. 10.—CELLULAR COFFERDAMS ON CLAY

The first requirement for the stability of a cofferdam on clay is that the clay stratum should be strong enough to sustain the weight of the cofferdam. The ultimate bearing capacity of an undisturbed, homogeneous clay stratum depends only on the average cohesion c of the clay. This statement has been confirmed repeatedly by experience—most recently by the failure of a large footing on soft clay (23) reported in 1942. Fig. 10(a) is a vertical section through a cellular cofferdam that rests on a homogeneous clay stratum of great depth. Before the cofferdam is acted upon by one-sided water pressure, it represents a continuous uniformly loaded footing whose base ab adheres to the supporting clay. If the upper surface of the clay were at the elevation of the base of ab, the failure load (in excess of the pressure which acted on ab before the construction of the cofferdam) would be equal to 5.7 c (17). The layer of clay above the level of ab in Fig. 10(a) increases the bearing capacity slightly. Hence, if the weight of the cofferdam, γH per unit of area, is equal to 5.7 c, the clay would be able to carry slightly more than the weight of the cofferdam. However, subsequent application of a one-sided water pressure would cause the cofferdam to fail by inward tilting as indicated by the dashed lines in Fig. 10(a). From the expression, $\gamma H = 5.7\, c$, the value,

$$H = 5.7 \frac{c}{\gamma} \qquad (32)$$

is obtained. The height of the cofferdam should not exceed 0.7 H. Otherwise, the cofferdam would fail on account of a failure of its foundation although it may be safe in every other respect. The value of γ is usually about 110 lb per cu ft, and the value of c for soft clay ranges between 400 and 1,000 lb per sq ft. The corresponding values of H are 20 ft and 50 ft. The danger of a foundation failure can be disregarded only if the clay is stiff or if the sheet piles are driven into a stratum of sand or stiff clay. A method of evaluating the average cohesion c of natural clay strata has been described elsewhere (24).

The stability of a cellular cofferdam itself depends to a considerable extent on the compressibility of the soil between the base of the fill in the cells and the level of the lower edge of the sheet piles. The following investigation of the influence of the compressibility of the base of the fill in the cells on the stability of a cellular cofferdam is based on the simplifying assumption that the fill obeys Hooke's law and that the intensity of the vertical friction forces which act on the rows of sheet piles is independent of the distance from the neutral plane, gh in Fig. 10(b). The second assumption is justified by the empirical fact that a very small displacement between sand and wall suffices for mobilizing the full frictional resistance (25). A further displacement has no more influence on the shearing stresses at the contact face. It is further assumed that the sheet piles are driven deep enough into the clay to sustain the vertical forces without noticeable vertical displacement. The fill in the cells is assumed to be completely drained, and the horizontal pressure of the fill on the walls of the cells is assumed to be determined by Eq. 10, regardless of the state of stress on the vertical boundaries of the fill.

The overturning moment M can be divided into two parts, M_s and M_f. One part, M_s, is carried directly by the sheet-pile enclosure of the cells without the assistance of the shearing resistance of the fill. The other one, M_f, is carried by the fill.

In the following computations the sheet-pile enclosures of the cells are considered prismatic boxes with lengths 2 L and widths b. In Fig. 1(a), these boxes are represented by dash-dotted lines. The lower edge of the boxes is fastened to the ground by the buried part of the sheet piles. The shearing strength of the walls along the locks is equal to the lock friction. Application of an overturning moment increases the pressure at the lower edge of the boxes on the inside of the neutral plane gh (Fig. 10(b)) by Q_p per unit of width of the sheet piles, and it reduces the pressure on the outside by the same quantity. The sum of all the vertical pile reactions on one side of the neutral plane of one cell is $Q_p\,(2\,L + b)$. At the instant of the first slip in the locks, this force must be equal to the friction $2\,T\,f$ (Eq. 14b) in the two locks located in the neutral plane of the cell. Hence, $Q_p\,(2\,L + b) = 2\,T\,f = 2 \times \frac{1}{2}\,\gamma\,H^2\,r\,C\,f$; or

$$Q_p = \tfrac{1}{2}\,\gamma\,H^2\,C\,f\,\frac{r}{L + 0.5\,b} \qquad (33)$$

The couple formed by the equal and opposite sheet-pile reactions on the two sides of the neutral plane of one cell is $2\,M_s\,L = 2\,L\,Q_p\,b + 2\,Q_p\,\frac{b}{2} \times \frac{b}{2}$;

whence,

$$M_s = Q_p \frac{b}{L}(L + 0.25\,b) = \tfrac{1}{2}\,\gamma\,H^2\,C\,r\,f\,\frac{b}{2}\,\frac{L + 0.25\,b}{L + 0.5\,b} \qquad (34a)$$

Introducing the customary values for cofferdams with circular cells, $b = 0.85\,H$, $L = 0.55\,H$, and $r = 0.50\,H$:

$$M_s = 0.3\,\gamma\,H^3\,C\,f \qquad (34b)$$

The value M_s is independent of the compressibility of the base of the fill in the cells, because it depends only on the lock friction. On the other hand, the second part, M_f, of the resisting moment depends on the compressibility of the base, because the compressibility has a decisive influence on the stress conditions at the surfaces of contact between fill and sheet piles.

After a cell has been filled with sand all the vertical boundaries of the fill are acted upon by side friction in an upward direction, because the fill settles due to its own weight while the sheet piles are stationary. At that stage (that is, prior to the application of an overturning moment), the distribution of the pressure on the base of the fill is approximately uniform as indicated in Fig. 10(*c*) by the horizontal line cd and the total pressure is equal to the difference between the weight of the fill and the total side friction.

If the fill rests on an unyielding base such as firm clay, the application of an overturning moment reverses the direction of the friction forces on the vertical boundaries of the outer part of the fill, as indicated in Fig. 10(*b*), because the height of the outer part of the fill increases slightly while the height of the sheet-pile enclosure remains unchanged. The greatest value which the friction on the vertical boundaries can assume is F_1 per unit of length of the row of sheet piles. The value of F_1 is given by Eq. 23. The displacement between fill and sheet piles required to increase the friction forces F to their maximum value F_1 is very small. Hence the distribution of the pressure on the base of the fill remains approximately uniform until F becomes equal to F_1.

As long as the distribution of the pressure on the base of the fill is uniform, requiring $F < F_1$, the cofferdam can fail only by shear along the neutral plane. As soon as the unit pressure on the inner side of the base starts to increase ($F = F_1$), the cofferdam may also fail by a bursting of the locks of the inner row of sheet piles. In order to find out which one of these two types of failure is to be expected, compute the overturning moment required to produce a failure by shear and determine the corresponding friction value F. If the value thus obtained is smaller than F_1 the cofferdam fails by shear before the lock tension starts to increase appreciably.

The following computation is based on the assumptions illustrated by Figs. 10(*b*) and 10(*c*): The neutral plane passes through the center line of the crest of the cofferdam, the friction forces F which act on the vertical contact faces of the fill on either side of the neutral plane are equal and opposite, and the distribution of the pressure on the base of the fill is approximately uniform. On these assumptions the cofferdam fails as soon as the total friction force on the vertical boundaries of the fill on one side of the neutral plane, $F\,(2\,L + b)$ per cell of a cofferdam with circular cells, becomes equal to the total shearing resistance, $2\,L\,S'$, on the neutral plane of the fill in one cell. The value of S'

is given by Eq. 12. Since $2\,L\,S'$ must be equal to $F\,(2\,L + b)$,

$$F = \tfrac{1}{2}\,\gamma\,H^2\,C\tan\phi\,\frac{L}{L + \frac{b}{2}} \qquad (35a)$$

The greatest value that F can assume is

$$F_1 = \tfrac{1}{2}\,\gamma\,H^2\,C\tan\delta \qquad (35b)$$

The value $\tan\phi\,\frac{L}{L + \frac{b}{2}}$ in Eq. 35*a* is commonly smaller than the value $\tan\delta$ in Eq. 35*b*. Hence, the cofferdam is likely to fail by shear before the friction between fill and sheet piling is fully mobilized. The overturning moment for one cell at which failure occurs is $2\,L\,M_f = 2\,F\,L\,b + 2 \times F\,\frac{b}{2} \times \frac{b}{2}$. Hence,

$$M_f = F\,b\left(1 + \frac{b}{4\,L}\right) = \tfrac{1}{2}\,\gamma\,H^2\,C\,b\,\frac{1 + \frac{b}{4\,L}}{1 + \frac{b}{2\,L}}\tan\phi \qquad (36a)$$

per unit of length of the cofferdam. Assuming $b = 0.85\,H$, $L = 0.55\,H$, and $\phi = 34°$, $\tan\phi = 0.675$,

$$M_f = 0.22\,\gamma\,H^3\,C \qquad (36b)$$

The total overturning moment M_t required to produce the failure is equal to the sum of M_s (Eq. 34*b*) and M_f. Assuming f in Eq. 34*b* to be equal to 0.3 the value—

$$M_t = \gamma\,H^3\,C\,(0.09 + 0.22) = 0.31\,\gamma\,H^3\,C \qquad (37)$$

—is obtained.

Since the shearing force on the outside contact faces of the fill in the cell is most likely smaller than that on the inside, the cofferdam must be expected to fail at an overturning moment of less than M_t. Whatever the critical value of the overturning moment may be, the stability of the cofferdam is higher than that of a cofferdam with similar dimensions on a rock foundation, because the outer row of sheet piles is anchored in the subsoil. The anchorage increases the stability of the dam. Therefore, it is safe to design the cofferdam on the basis of the same semiempirical rules that have been proposed for the design of cellular cofferdams on a rock foundation.

The sheet piles are acted upon by the sum of the vertical forces Q_p (Eq. 33) and F (Eq. 35*a*). The depth of sheet-pile penetration is determined by the condition that the sheet piles should be able to sustain these forces without noticeable displacement. The bearing capacity and the skin friction of the sheet piles can be determined by pulling tests on test sheet piles on the job.

If a cofferdam rests on a base that continues to yield under the influence of the weight of the fill in the cells while the overturning moment is applied, the fill slides along the sheet piles in a downward direction. Because of friction, all the contact faces of the sand are acted upon by a force F_1 per unit of width acting in an upward direction as indicated in Fig. 10(*d*) by arrows. The

pressure on the base of the fill is equal to the difference between the weight of the fill and the full side friction (pressure area abcd in Fig. 10(*e*)). The overturning moment causes the cofferdam to tilt inward, but, since the sand continues to slide along the sheet piles in a downward direction, the tilt cannot alter the shearing stresses on the vertical contact faces. This fact constitutes a vital difference between the conditions for the stability of the cofferdams on a firm base (Fig. 10(*b*)) and on a consolidating base (Fig. 10(*d*)). Since the shearing stresses on the vertical boundaries of the fill in the cells have a constant value, no increase of the overturning moment beyond M_s (Eq. 34*b*) is conceivable without a corresponding change of the pressure on the base of the fill in the cells. In Fig. 10(*e*), the change of the pressure on the base, caused by the overturning moment, is indicated by shaded areas.

Hence as soon as the overturning moment becomes greater than M_s, Eq. 34*a*, the unit pressure on the inner part of the base increases rapidly and a failure due to bursting of a lock of the inner row of sheet piles becomes imminent, irrespective of the state of stress on the neutral plane. On account of the increase of the pressure on the inner part of the compressible base, an increase of the overturning moment beyond M_s should be associated with a conspicuous inward tilt of the cofferdam.

Only three cellular cofferdams on a soft, highly compressible base have come to the writer's attention. One of them is the cofferdam for raising the battleship *Maine*; the second is the western part of the middle wall of the shipways described by Mr. Jansen (7); and the third is a bulkhead adjoining one of these shipways.

Fig. 11(*a*) is a simplified vertical section through the cofferdam around the *Maine*. The cofferdam consisted of circular cells with a radius of 25 ft. The stratum between the base of the fill in the cells at El. − 35 and the surface

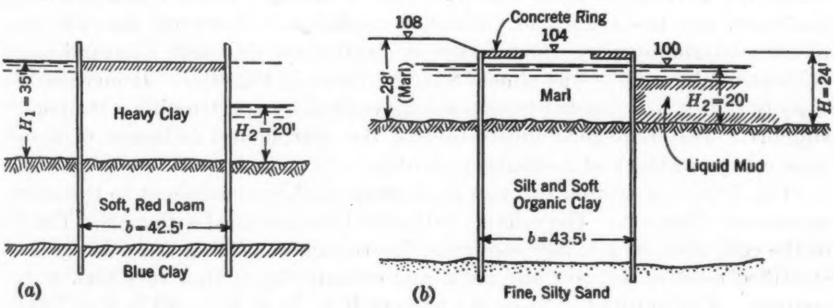

FIG. 11

of the clay at El. − 65 was described as a red loam with shells (26). The cells were filled with heavy clay excavated by means of a hydraulic dredge. The progressive consolidation of the clay beneath the base of the fill in the cells was demonstrated by the fact that the wooden drain boxes which were built into the fill of the cells settled 4 ft to 5 ft (26). As soon as the water level inside the cofferdam was lowered 15 ft, the inward tilt of the cofferdam in-

creased so considerably that it was decided to increase the stability of the dam by constructing an inner berm. At that stage the overturning moment produced by the unbalanced water pressure was about 370 kip-ft per ft. To correlate the known behavior of the cofferdam with the results of the preceding analysis, the overturning moment M_s, which the sheet-pile enclosure of the cells can stand without the assistance of the shearing resistance of the fill in the cells, will be computed.

To determine the moment M_s, it is necessary to estimate the lock friction. If P_t denotes the difference between the horizontal pressure on the inside and that on the outside of the inner row of sheet piles of the cofferdam per unit of width of the sheet piles, the total friction in one lock is $P_t r f$. Using the same procedure which led to Eq. 34*b*, the formula,

$$M_s = P_t r f \frac{b}{L} \times \frac{L + 0.25\, b}{L + 0.50\, b} \qquad (38)$$

is obtained. For the cofferdam around the *Maine*, the writer assumes $\gamma = 110$ lb per cu ft, $\gamma' = 50$ lb per cu ft, $\gamma_w = 63.5$ lb per cu ft, and $C = 1.00$. The corresponding value of M_s is 490 kip-ft per ft. The deflection of the cofferdam became conspicuous as soon as the overturning moment became equal to 371 kip-ft per ft $= 0.76\, M_s$.

In Fig. 4(*c*) is shown a section through the middle wall of the shipways described by Mr. Jansen (7). The cells were filled with a stiff, calcareous clay known as marl. The height H of this cofferdam is 35 ft, $r = L = 24.3$ ft, $\gamma = 120$ lb per cu ft, and $\gamma_w = 63.5$ lb per cu ft. Assuming $C = 1$, the value M_s (Eq. 38) is equal to 625 kip-ft per ft. When the shipway south of the wall was pumped out, the unbalanced water pressure on the north side of the wall produced an overturning moment $M = 456$ kip-ft per ft $= 0.73\, M_s$. At that stage, the average deflection of that part of the cofferdam which rested on stiff marl was less than 2 in.—which is negligible. However, the deflection of the westernmost part which rested on a soft, dark clay with a natural water content of about 70% was almost 8 in. as shown in Fig. 4(*c*). It increased so rapidly that it was decided to stop the movement by constructing a temporary support. This is a good illustration of the detrimental influence of a soft base on the stability of a cellular cofferdam.

Fig. 11(*b*) is a section through the cellular bulkhead adjacent to the aforementioned shipways. The cells are cylindrical and are filled with marl. The fill in the cells rests on a highly compressible stratum consisting of an irregularly stratified mass of silt, soft and dark, and organic clay with a very high water content. Evaluating Eq. 38 on the basis of $H = 24$ ft, $H_1 = 20$ ft, $\gamma = 120$ lb per cu ft, $\gamma' = 57$ lb per cu ft, $C = 1.00$, and considering, in addition, the side pressure of the liquid mud on the river side, with a submerged unit weight of 34 lb per cu ft, the value $M_s = 178$ kip-ft per ft was obtained.

The bulkhead is acted on by the active earth pressure of a mass of marl whose surface is at El. 108. The marl was excavated in a borrow pit by means of a hydraulic dredge, transported through pipe lines and deposited in such a manner that it displaced the river mud adjoining the land side of the bulkhead and forced part of the mud out into the river through a temporary gap in the

bulkhead. The overturning moment produced by the active earth pressure of the marl was estimated on the assumption that the angle of internal friction of the marl is zero and that the cohesion is between 250 and 300 lb per sq ft. Thus, a moment M between 116 lb-ft per ft and 155 lb-ft per ft (0.65 to 0.88 M_s) was obtained.

During the construction of the backfill, the crest of the dam moved out to distances as great as 7 in. During the following 1.5 years the deflection increased by amounts as great as 15 in. which indicates that the overturning moment is fairly close to the maximum value which the cofferdam can safely stand. To prevent a distortion of the cells, a massive concrete ring was poured on the surface of the fill in every cell. Because of the consolidation of the fill and of the base of the fill, the concrete rings settled along the waterfront as much as 2 ft. On the land side the settlement is less than one half of that on the water side, which indicates that the pressure on the base increased from the land toward the water side. However, the overturning moment was still considerably smaller than M_s (Eq. 38).

By the theoretical analysis presented herein, the cofferdams represented in Fig. 11 should have shown no signs of distress until the overturning moment M became equal to M_s (Eq. 38); but all of them started to deflect quite alarmingly as soon as M became greater than about 0.75 M_s. Hence, it seems that the evaluation of M_s involves a considerable error on the unsafe side. The error may be due to the real lock tension being smaller than the estimated value, to the coefficient of lock friction being less than 0.3, or to both causes combined. Because of this error, cellular cofferdams on a soft base should be designed so that the computed resisting moment M_s is at least equal to 1.5 times the overturning moment.

If the construction of a cellular cofferdam on a clay foundation involves lowering the water level on one side of the cofferdam, the effect of the seepage pressure on the foundation of the cofferdam and on the subsoil of the unwatered area must be considered independently. In many instances it is possible to eliminate undesirable hydrostatic pressure conditions by simple drainage. The discussion of the drainage devices is beyond the scope of the paper.

Part III. Conclusions

Eleven conclusions seem indicated from the foregoing:

1. The traditional assumption that a cellular cofferdam can be regarded as a gravity wall leads to erroneous conclusions regarding the distribution of the soil pressure on the base of the dam.

2. Because of the friction between the fill and the sheet-pile walls of the cells, the increase of the soil pressure at the inner part of the base of a cellular cofferdam with conventional dimensions due to the overturning moment produced by the water pressure is negligible, provided the base is firm.

3. The lock friction in the cross-walls adds materially to the stability of the cofferdam, and it reduces very considerably the deflection of the crest. This fact constitutes an important advantage of the cellular cofferdam over the double-wall cofferdam.

4. The assumption that the weep holes in the inner wall of cells filled with sand suffice to drain the cells is not necessarily justified.

5. The lateral earth pressure of the fill in the cells on the sheet-pile enclosure of the cells is not even approximately equal to the Rankine pressure. Therefore, the greatest lock tension can only be estimated on a semiempirical basis.

6. The required depth of penetration of the sheet piles of cofferdams on a sand or a clay foundation is determined by the condition that the bearing capacity of the sheet piles per unit of width should be equal at least to 1.5 times the shearing force which acts per unit of width between the fill and the length rows of sheet piles at maximum overturning moment. Since the vertical forces that act on the sheet piles can be estimated in advance, the required depth of sheet-pile penetration can be determined by means of pulling tests on test sheet piles in the field.

7. Cellular cofferdams on a sand foundation require, in addition to adequate sheet-pile penetration, a protection against the formation of boils. Protection can be achieved with either a loaded, inverted filter or a plain sand berm with a broad base. If an inverted filter is used, its functioning requires supervision by means of telltale pipes.

8. Rows of steel sheet piles in contact with, or embedded in, fine silt or clay do not intercept the flow of seepage.

9. The principal dimensions of cellular cofferdams on stiff clay can be determined by means of the same semiempirical equations that have been proposed in this paper for the design of cellular cofferdams on a rock foundation.

10. If the fill in the cells of a cofferdam rests on a compressible stratum which continues to consolidate after the overturning moment has been applied, a relatively small moment suffices to produce a very unequal distribution of the pressure on the base of the fill in the cells. Such a cofferdam should not be expected to carry more than about 0.7 times the overturning moment M_s (Eq. 38). The degree of stability of the cofferdam is practically independent of the strength of the fill in the cells.

11. The shearing resistance on vertical sections through the fill in the cells and the lock friction can only be estimated. Since the stability of every cellular cofferdam depends to a large extent on these quantities, no theoretical refinements in the design of such dams are warranted and no equation pertaining to such dams can be used safely unless its approximate validity has been repeatedly demonstrated by construction experience. Therefore, the principal sources for increased knowledge on cellular cofferdams are accurate field observations.

APPENDIX I

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APPENDIX II

Notation

The following letter symbols, used in this paper, conform essentially with "Soil Mechanics Nomenclature,"[3] adopted by the Society's Board of Direction in 1941:

b = width; width of equivalent cofferdam with straight walls:

b_1 = outside width of the real cofferdam (Figs. 1(*a*) and 1(*b*));

b_0 represents an empirical value for the width b of equivalent cofferdams whose outside is acted upon by water pressure only;

b_m (Eq. 8) = value of b for a cofferdam with a theoretical factor of safety $G_s = 3$ with respect to overturning;

C = coefficient of earth pressure, an empirical constant;

c = cohesion; average cohesion of clay per unit area;

D = diameter;

d = distance between flow lines (Fig. 9);

F = total frictional resistance on a surface of contact, per unit width:

F_b = value of F on the base of a cofferdam (Eq. 1);

F_{bs} = value of F at the base of a cofferdam which is completely saturated to a height H_s above the base;

F_1 = resistance against sliding along surface of contact between fill and sheet piles, per unit of width of the row of sheet piles;

f = coefficient of lock friction;

G_s = factor of safety;

g = a force per unit of volume; g_s = seepage force in Eq. 29*a*;

H = height of a cofferdam:

H_s = average elevation of line of saturation above base of a partially saturated cofferdam;

$H_w \doteq$ depth of water adjoining a breakwater, between storms;

[3] *Manual of Engineering Practice No. 22*, Am. Soc. C. E., 1941.

H_1 = elevation of level of water adjoining the outside of a cofferdam, if the highest water level is below the crest of the dam (Fig. 5);

H_2 = elevation of level of water adjoining the inside of a cofferdam, with reference to base of dam (Fig. 11);

H_{max} = greatest vertical distance between level of water on two sides of a cofferdam (Fig. 3);

h = loss of hydraulic head that is caused by flow of water through distance l in a mass of sand (Fig. 9(a)) or through the locks of a row of sheet piles (Fig. 5); also maximum temporary excess head on wind side of a breakwater during a storm (Fig. 9(f)):

h_1 = hydrostatic head on the inside of the outer row of sheet piles of a cofferdam (Fig. 5(b));

h' = hydrostatic excess head at heel of breakwater while a wave breaks (Fig. 9(f));

h'' = drop of head at heel of breakwater after a wave has spent itself;

i = hydraulic gradient h/l (Fig. 9(a)); $i_1 = \frac{\gamma'}{\gamma_w}$ (Eq. 29b) = hydraulic gradient in the case of upward seepage at the instant when boiling begins;

k = a constant = coefficient of permeability;

L = half the spacing of circular cells in cofferdams (Fig. 1(a)), or the distance between cross-walls in cellular cofferdams of the diaphragm type (Fig. 1(b));

l = length of a cubical element of a mass of sand (Fig. 9);

M = overturning moment per unit length per unit of length of a cofferdam:

M_a = the moment required to move the point of application of the resultant pressure on the base of a gravity wall from the center to the inner boundary of the middle third (Eq. 6);

M_f = the part of an overturning moment carried by the fill within the cells of a cofferdam on stiff clay;

M_s = the part of an overturning moment carried by sheet-pile enclosures of the cells of a cofferdam on clay without the assistance of the shearing resistance of the fill;

M_t = total overturning moment required to produce failure of a cofferdam on stiff clay;

M_0 = overturning moment produced by the water pressure on the outside of a cofferdam after the outside water level has arrived at the level of the crest of the dam (Eq. 19a);

M_1 = overturning moment required to increase the pressure on the inner part of the base of a cofferdam on a rock foundation (Fig. 7(d));

M_{max} = moment required to overturn a gravity wall;

m = ratio between M_1 and M_0 (Eq. 27);

n = the greater of two values n_1 and n_2; $n_1 = \frac{P}{P_0}$ and $n_2 = \frac{M}{M_0}$;

P = resultant of all the horizontal forces which act on a cofferdam, per unit of length of the dam:

P_A = active Rankine pressure on a row of sheet piles or on the neutral plane of a cofferdam, per unit of width;

P_e = earth pressure on a row of sheet piles or on the neutral plane of a cofferdam, per unit of width;

P_s = horizontal force which prevents the inward movement of the lower edge of the inner row of sheet piles of a cofferdam on a rock foundation per unit of width of the row (Fig. 8);

P_t = difference between pressure which acts on inside and outside of inner row of sheet piles of cofferdam on clay, per unit of width of sheet piles;

p = pressure per unit of area; p_e = earth pressure per unit of area of a vertical surface, at depth Z below top of fill;

Q = equal and opposite vertical forces per unit of length of a cofferdam constituting a couple which is equal to the total overturning moment or to part of it; Q_p = vertical force per unit of width of sheet-pile enclosure of cells of a cofferdam on clay constituting a couple equal to the overturning moment M_s per unit of length of the dam;

q = pressure per unit of area of base of a cofferdam:

q_0 = average unit pressure, equal to weight of fill in the cells divided by area of its base;

q_{min} = smallest value which the unit pressure at outer edge of the base of cells filled with sand can assume;

r = radius of circular cells or arcs;

S = average shearing resistance per unit of width of neutral plane of a cofferdam:

S' = that part of S which is due to shearing resistance of fill in cells;

S'' = that part of S which is due to resistance against slippage in locks;

T = total tension in one lock;

t = lock tension per unit length at depth Z; t_1 = greatest unit tension in a lock (Eq. 22*b*);

V = volume;

v_d = quantity of water which flows through space between two adjoining flow lines per unit of time and unit of length of the dam;

W = weight;

Z = depths measured from the tops of cells;

γ = unit weight of moist fill in cells; γ' = submerged unit weight of fill in cells;

γ_w = unit weight of water;

Δ = deflection;

δ = angle of friction between substances such as the fill in a cell and sheet piles;

ρ = angle of friction for sliding; and

ϕ = angle of internal friction.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

AMPLIFIED SLOPE DEFLECTION

By Ralph W. Stewart,[1] M. Am. Soc. C. E.

Synopsis

Slope deflection, as presented in current textbooks, is encumbered by the necessity of: (*a*) Reversing the signs of certain moments in writing equations; (*b*) Studying the significance of double signs (namely, $\mp$ and $\pm$) which are set up in the standard equations; and (*c*) a set of four standard equations when two are ample.

The purpose of this paper is to eliminate these difficulties, to present a complete physical interpretation of the components of slope-deflection equations, and to show how to use these components in such a manner that, for many problems, the algebraic work required will be only one half that required by the procedure taught in current texts.

Definition of Symbols and Other Data

The abbreviation FEM, as used in this paper, denotes "fixed-end moment" —the moment that would exist at an end of a beam if both ends were fixed against rotation. The location of the FEM and the identification of the span to which it pertains are indicated by a subscript. Thus, FEM_{AB} means the FEM at end A of span AB; FEM_{DC}, the FEM at end D in span CD, etc.

The abbreviation FHEM means the fixed-end moment at one end of a beam when the opposite end is hinged, subscripts being used as for FEM.

The symbol Δ is used herein to represent the angle between the end tangents of a beam due to the application of a moment at one end when the other end is hinged. The reason for the selection of this symbol is that it is in common use in treatises on railway curves (notably transition spirals)[2] to indicate the angle between the end tangents of a curve. It measures the amount of curvature in the elastic curve of a flexed beam due solely to the end moment to which it is related.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by February 1, 1945.

[1] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

[2] "The Railway Transition Spiral," by A. N. Talbot, McGraw-Hill Book Co. Inc., New York, N. Y. (6th Ed., 1927, out of print).

The symbol θ is used to represent the angular change in direction, due to flexure, of the neutral axis of a beam at a support. Angles Δ and θ are identified as to location by subscripts.

Symbol A_o is the area of the $\frac{M}{E\,I}$-diagram $\left(\text{or } \frac{M}{I}\text{-diagram}\right)$ of a span when the span is considered as simply supported—that is, free from end restraints except simple supports acting at right angles to the beam.

A traverse of the elastic curves of a structure is a series of straight lines which delineate all the θ-angles and Δ-angles, and also, when necessary or desirable, the A_o-angles.[3] The lines of a traverse which pass through supports are tangent to the elastic curve at the supports.

Four important properties of an elastic curve traverse are:

(1) Each Δ-angle and each A_o-angle is located where the center of gravity of the $\frac{M}{E\,I}$-diagram appurtenant to such angle is projected upon the neutral axis of the member.

(2) Each Δ-angle or A_o-angle is numerically equal to the area of its $\frac{M}{E\,I}$-diagram. This follows because the Δ-angle or A_o-angle and the area of the appurtenant $\frac{M}{E\,I}$-diagram each measure the curvature in the beam due to the same moment.

(3) In any triangle formed by the lines of a traverse and the unsprung axis of a beam the angles are proportional to the opposite sides. The altitude of a traverse triangle for any ordinary beam is so small that the angles may be taken as equal to either their sines or their tangents. It is usually convenient in making computations to consider the tangent of an angle as representing its magnitude.

The basic stiffness of a beam is the magnitude of the end moment necessary to produce a unit Δ-angle, or the ratio of an end moment to the Δ-angle it produces. If a Δ-angle can be evaluated from the geometry of an elastic curve traverse it is converted into an end moment by multiplying it by the basic stiffness of the member. A tapered member will have a different basic stiffness for each end.

(4) Property (3) leads to the theorem that any Δ-angle multiplied by the appurtenant beam stiffness equals the adjacent end moment, and any end moment divided by the stiffness equals the appurtenant Δ-angle.

It is probably unnecessary to state that an elastic curve traverse, if correctly used, will close geometrically the same as a land survey. It should be mentioned that with amplified slope deflection, which gives geometrical values to the components of slope-deflection equations, the deflections of a structure are easily computed as a by-product of the stress analysis.

Signs.—In slope-deflection equations those moments and joint movements which produce or tend to produce clockwise rotation at the joint for which

[3] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521 (see also references therein for demonstrations of the use of the elastic curve traverse).

an equation is written are taken as positive; the reverse, negative. In a traverse computation which progresses along a series of angles, the angles which turn the alinement of the traverse toward the right may be taken as positive; and those turning it toward the left, negative.

Demonstration Problems

Let member AB in Fig. 1 be a beam of constant section. Its basic stiffness will be $\frac{2\,E\,I}{l}$ since this is the ratio of an end moment to the area of the $\frac{M}{E\,I}$-diagram which it produces. Figs. 1(a), (1b), and (1c) show, respectively, the $\frac{M}{E\,I}$-diagrams for the simply supported condition, the condition of restraint

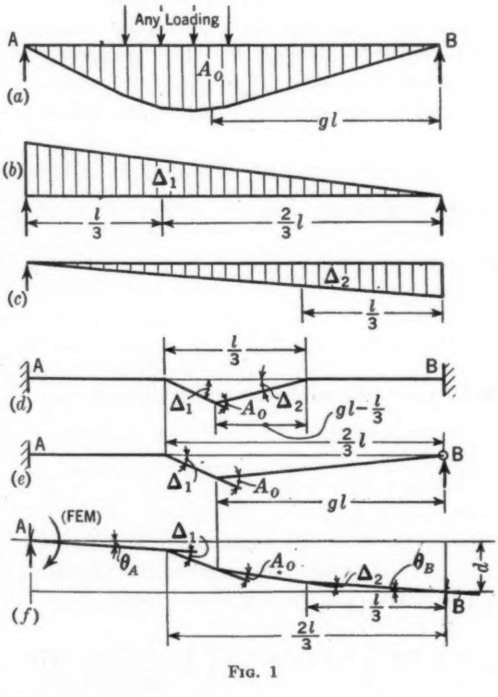

Fig. 1

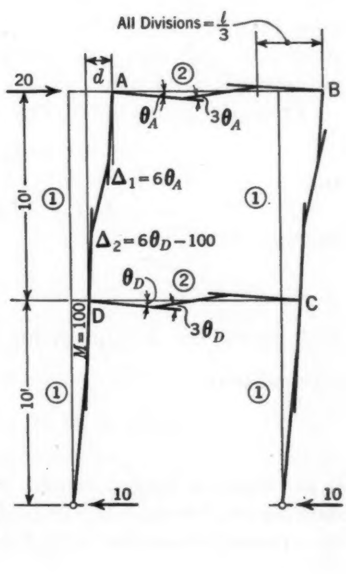

Fig. 2

at the left end only, and the condition of restraint at the right end only, each moment acting along the full length of the beam.

Fig. 1(d) represents a traverse of the elastic curves of this beam with both ends fixed so that the θ-angles are zero. From traverse property (3) the angle Δ_1 will be equal to

$$\Delta_1 = \left[\left(g\,l - \frac{l}{3}\right) \div \frac{l}{3}\right] A_o = (3\,g - 1)\,A_o \ldots\ldots\ldots\ldots (1)$$

From traverse property (4) multiply Eq. 1 by the stiffness of the beam $\frac{2\,E\,I}{l}$ and obtain

$$\text{FEM}_\text{A} = \frac{2\,E\,I\,A_o}{l}\,(3\,g - 1)\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Fig. 1(*e*) is the traverse with the left end of the beam fixed and the right end hinged. From traverse property (3),

$$\Delta_1 = (g\,l \div \tfrac{2}{3}\,l)\,A_o = \tfrac{3}{2}\,g\,A_o\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

and from property (4) of the traverse

$$\text{FHEM}_\text{A} = \frac{3\,E\,I\,A_o\,g}{l}\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Now consider the beam as one of a series of continuous spans so that the ends are subject to restraint less than fixation and some joint rotation can occur. Also, let the right end deflect a distance d. The traverse will then take the form of Fig. 1(*f*), which is drawn for loading and restraint conditions such that both Δ-angles, under slope-deflection sign rules, relate to positive end moments.

From the geometry of Fig. 1(*f*) the following equations may be written:

$$\theta_A + \Delta_1 - A_o - \Delta_2 - \theta_B = 0\ldots\ldots\ldots\ldots\ldots\ldots(5a)$$

and

$$\theta_A\,l + \tfrac{2}{3}\,l\,\Delta_1 - g\,l\,A_o - \tfrac{1}{3}\,l\,\Delta_2 = d\ldots\ldots\ldots\ldots\ldots\ldots(5b)$$

Solving,

$$\Delta_1 = A_o\,(3\,g - 1) - 2\,\theta_A - \theta_B + 3\,\frac{d}{l}\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Multiplying the Δ-value in Eq. 6 by the beam stiffness and letting $K = \frac{1}{l}$ it is found that

$$M_{AB} = \text{FEM}_\text{AB} - 2\,E\,K\left(2\,\theta_A + \theta_B - 3\,\frac{d}{l}\right)\ldots\ldots\ldots\ldots(7a)$$

If the beam is hinged at end B the angle Δ_2 will become zero, the traverse showing as a straight line from A_o to the right end. Solving for this condition by a procedure similar to that used to derive Eq. 7*a* it is found that:

$$M_{AB} = \text{FHEM}_\text{AB} - 2\,E\,K\left(1.5\,\theta_A - 1.5\,\frac{d}{l}\right)\ldots\ldots\ldots\ldots(7b)$$

Note that Eqs. 7 are the conventional slope-deflection equations found in the various textbooks except that all but two[4] of the textbooks, which the writer has consulted, reverse the sign of the fixed-end moments and enter the second term as positive. This leads to considerable confusion in the practical use of the equations. It is well known to all users of the slope-deflection method that the final moment at the end of a beam for which the moments are not

[4] "Statically Indeterminate Structures," by John I. Parcel and G. A. Maney, John Wiley & Sons, Inc., New York, N. Y., 2d Ed. 1936; also "Analysis of Statically Indeterminate Structures," by Clifford D. Williams, International Textbook Co., Scranton, Pa., 1943.

dominated by joint movements is less than the fixed-end moment. The form in which Eqs. 7 are written may be expressed by the following rule:

> "The moment at A in a beam AB is equal to the moment which would occur at A if the beam were fixed, less the relaxation in the moment due to the joint rotations, plus the moment due to the deflection of a support."

Under this rule the fixed-end moment is entered with its true sign. Note also that if the $\frac{d}{l}$-term is removed from the parentheses it will also have its true sign. This is logical as both the FEM and the deflection d create clockwise moment at end A in Fig. 1(*f*). The joint rotations θ reduce this moment and are therefore negative. However, amplified slope deflection will operate with either the direct or the reversed use of signs so any analyst can use his own system.

The important feature of amplified slope-deflection developed by this paper is that, in any slope-deflection equation written in the form of either Eq. 7*a* or Eq. 7*b*, the expression inside the parentheses is the value of a Δ-angle in an elastic curve traverse. This is true for all special conditions such as tapering members and "semi-rigid" riveted or welded joints at intermediate points along members. The use of this property of the parenthetical expression in a slope-deflection equation offers short-cut solutions for many problems. For convenience in wording this parenthetical term will be called the "Δ-term."

A demonstration of a solution using the geometrical values of Δ-terms is presented in Fig. 2, which shows a symmetrical two-story frame subjected to a lateral force. The numerals in circles near the centers of the members show their relative stiffness values. From property (3) of the traverse the Δ-angles in the beams will be $3\,\theta_A$ and $3\,\theta_D$, as shown. (Note: In the conventional slope-deflection equation for beam AB the parenthetical term is $(2\,\theta_A + \theta_B)$. Since, due to symmetry, θ_A and θ_B are equal, this equals $3\,\theta_A$.) If a conventional slope-deflection formula like Eq. 7*a* were written for member AD, the Δ_1-term would be $2\,\theta_A + \theta_D - 3\,\frac{d}{l}$. This is inconvenient as it contains three unknown quantities. Amplified slope deflection offers a simpler value for Δ_1 containing only one unknown quantity derived as follows:

From traverse property (4)—

$$M_{AB} = 2\,(3\,\theta_A) = 6\,\theta_A \qquad \text{(8a)}$$

Balancing moments about A—

$$M_{AD} = 6\,\theta_A \qquad \text{(8b)}$$

and from traverse property (4)—

$$\Delta_1 = 6\,\theta_A \qquad \text{(8c)}$$

In actual practice this procedure is shortened to

$$\frac{2}{1}\,(3\,\theta_A) = 6\,\theta_A \qquad \text{(9)}$$

For joint D the sum of the column moments must balance the beam moment.

The bottom column moment is one half the lateral force times the column height, which is equal to 100. The beam moment is $6\,\theta_D$. Subtracting the lower column moment from M_{DC} and observing that the relative stiffness of the upper column is 1:

$$M_{DA} = \Delta_2 = 6\,\theta_D - 100 \qquad (10)$$

Column AD carries one half the lateral force and the sum of its end moments will be equal to half the lateral force in the structure multiplied by the column height, from which

$$6\,\theta_A + 6\,\theta_D - 100 = 100 \qquad (11)$$

Next, write the equation for traverse angle closure in member AD as follows:

$$\theta_A + 6\,\theta_A - (6\,\theta_D - 100) - \theta_D = 0 \qquad (12)$$

Eqs. 11 and 12 can be solved simultaneously with a little less than half the algebraic work necessary if the conventional slope-deflection equations involving three unknown quantities were used. Solving, $\theta_A = 9.524$; $\theta_D = 23.81$; $M_A = 57.14$; and $M_{DA} = 42.86$; $M_{DC} = 142.86$.

If a pin connection is introduced at one end of a beam DC the unsymmetrical frame so formed will require six simultaneous equations for its solution by conventional slope deflection. Amplified slope deflection will reduce the number of equations to three.

If tapering members are involved the Δ-angle points will not be at the one-third length points. Their locations, and also the basic stiffnesses of the members, must be determined by a method described by the writer elsewhere[5] unless tables, which give coefficients for these beam constants, are available.

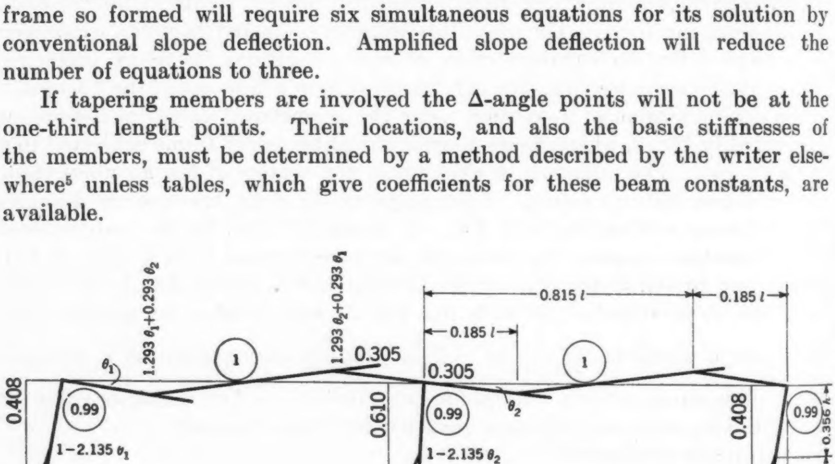

Fig. 3

Fig. 3 shows the solution of a symmetrical two-span frame[6] in which the beams are connected to the columns with semi-rigid joints; also the columns have semi-rigid connections to the foundations. This moves the Δ-points

[5] *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1200.

[6] *Ibid.*, Vol. 107 (1942), p. 1029, Fig. 19(*c*).

away from the one-third length points and alters the basic stiffness of each member. The beam and column properties of Fig. 3 were computed by a method demonstrated elsewhere.[7] Next, the lateral deflection of the tops of the columns was, for convenience, taken as 0.312 which is the coefficient for the distance between the column Δ-angles.

All beams and column Δ-angles were then computed by the procedure used in discussing Fig. 1. Two slope-deflection equations were then written—one to express the condition of moment balance at the top of an end column and one to balance moments at the top of the center column.

The solution of these two simultaneous equations for the two unknown θ-angles enabled the moments to be evaluated and written on the frame as shown by large numerals.[7]

In conventional slope deflection the number of simultaneous equations required to solve a given problem is definite. In amplified slope deflection the number may be varied some in accordance with the analyst's judgment. In fact, all of the problems presented in this paper can be solved by using the elastic curve traverse in a manner which entirely avoids the use of simultaneous equations and does not use successive approximations.

One caution should be observed in the use of relative stiffness values. If all members have the same E-value and are of constant section the value of its $\frac{I}{l}$-ratio may be used to express the relative stiffness of each member. However, if some of the members in a structure have tapered sections and some have constant sections, it is necessary to use $\frac{2\,I}{l}$ for the stiffness factor of the constant section members. This is why Eq. 7*b* is written in the form shown with the factor 2 outside the parentheses.

Conclusions

If the slope-deflection equations are considered to be functions of only end slopes and deflections, their usefulness is substantially less than if the fact is recognized that they also are functions of Δ-angles which measure the changes in direction of the elastic curve within the beam. By recognizing and using all the angle components in slope-deflection equations, the computation labor required to use them is reduced by half for certain problems. Simplification of sign rules and a change in form and reduction in the number of standard equations also facilitate the work.

[7] *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1028.

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DISCUSSIONS

STORAGE AND THE UNIT HYDROGRAPH

Discussion

BY HOWARD M. TURNER

HOWARD M. TURNER,[33] M. AM. SOC. C. E.[33a]—The first part of this interesting paper is a discussion of the channel storage in a reach of a river. The author compares the storage in this case with that in a reservoir, indicating the difference in the behavior, due to the fact that the relation between outflow and storage is not clear on account of the slope. The author cites an assumed case showing how a sharp diminution in inflow "requires" an increase in outflow at the outlet end of the reach. The writer has never seen any such effect. There is a sudden decrease in inflow at the outlet of many water power plants at 5 o'clock or 6 o'clock every afternoon for months during the year when there is no waste. The writer has examined many hydrographs taken at gaging stations some distance downstream from such stations and has never seen any evidence of any rise attributable to the cause given by the author. He admits the possibility, but believes that the author's statement is much too categorical in stating that it is "required" under such circumstances. There may be a slightly rising outflow because the high discharge existing at the upper end has not yet become stabilized at the lower end—that is, water is still going into storage and this rise will continue for a period of time after the power station shuts down due to the time lag between the two points. The outflow then drops off in the regular recession curve as the storage drains off. The difficulty in the author's contention is his statement (see heading, "Valley Storage") that the additional storage must be released "during the period of inflow reduction." This increased storage that must flow out may be released over a longer period of time.

The writer has analyzed several of these conditions and, by using the storage curve determined from the recession side of the hydrograph (that is, the storage as a function of the outflow), he has obtained very satisfactory agreements with an actual hydrograph by routing such power station discharges

NOTE.—This paper by C. O. Clark was published in November, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by James S. Sweet, and Otto H. Meyer; March, 1944, by L. K. Sherman, Gordon R. Williams, and Ray K. Linsley, Jr.; May, 1944, by E. F. Brater, L. C. Crawford, Robert E. Kennedy, and Victor H. Cochrane; and June, 1944, by Franklin F. Snyder.

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[33a] Received by the Secretary May 27, 1944.

through this storage with time allowance for travel.[34] In two of the three cases considered, the hydrograph downstream did show higher flow than that upstream, but this was due to the flow from intervening drainage and the inaccuracy of computing the power station discharges from the kilowatt-hour output and the difference disappeared when the inflow hydrograph was adjusted to give the same total runoff. There was no evidence in these cases of any rise which the author states is required by sudden reduction of inflow to the reach. If the author has any actual examples showing this effect, it is hoped that he will present them as it is an extremely interesting and, as he states, a somewhat new conception.

The author then describes his distinction between storage as a function of the outflow and storage as a function of the inflow. The writer believes Mr. Clark has oversimplified this theory. Actually, storage is a function of neither inflow nor outflow alone, depending as it does on topography and the level of the water surface, which in turn are dependent upon slope. Therefore, any difference between the author's two concepts of storage is dependent on a difference in changing slope which with the topography of the banks affects the volume of storage. In this case a more exact statement would be that, in the case of the unsteady flow of a rising or falling river stage, the changing slope affects the storage.

The writer finds it difficult to accept some of the statements made in regard to the storage affected by the inflow. In Fig. 2 the author shows curves for routing a simple sine curve with different storages based entirely on inflow ($x = 1.0$) through to a curve based entirely on outflow ($x = 0.0$). In the case of the former the flow with storage is greater than without storage. Where $x = 0.5$, the hydrograph is very little changed. It is not until x equals less than 0.5 that the storage reduces the inflow, and not until $x = 0$ does the outflow hydrograph intersect the inflow hydrograph at the peak of the former, which is generally held as a basic requirement between inflow and outflow hydrographs.

As would naturally occur if these curves are correct, the curve of outflow with $x = 1.0$ intersects the curve of inflow at the peak of the former and thus the storage increases the peak discharge from the outlet of the reach by a very great amount. The writer gives $x = 0.3$ to 0.5 as the usual range of values. If this condition is valid, there will be many cases in which there will be little if any diminution of peak due to storage. The writer does not question the conclusions reached with the author's equations, but he has never seen any indication of such effects and therefore distrusts the original equation. He believes that it is based on too simple a function of the storage compared to that actually existing in any normal river channel.

The second part of the paper develops a method of constructing a unit hydrograph from an instantaneous hydrograph. In this case the author uses storage as a function of outflow alone ($x = 0$). The writer is not certain whether the author considers the condition to be somewhat different for a given reach than for the storage of the entire river system, since the storage

[34] "The Flood-Hydrograph and Valley-Storage," by Howard M. Turner, *Transactions*, Am. Geophysical Union, 1943, Pt. II, p. 613.

used in this section of the paper does not seem to correspond with his previous statement regarding what he has found to be that on most rivers.

The author's process consists of using storage based on the actual flood hydrograph at the beginning of the recession curve and routing through it a time-area concentration curve which is in effect an inflow hydrograph built from an analysis of the time required for the water from the different parts of the drainage area to reach the outlet. In doing this the same storage relation is used for the rising stage of the hydrograph as for the falling stage.

The writer was much interested in this procedure as it is based on the same principles that were used by him and Mr. Burdoin.[31] Using their method, a rational inflow hydrograph was constructed from the duration of the rainfall, the concentration time was obtained from the deflection point on the recession curve of the hydrograph, and the total flood volume was routed through storage computed from the recession curve to obtain the outflow or flood hydrograph. The author extends these principles somewhat further adding the feature of the instantaneous hydrograph.

The author's method differs with regard to the storage which he determines from only one point on the hydrograph—the inflection point on the descending limb. This assumes a straight line from this point down to zero, an assumption which may not always be correct. (On the Appomattox River at Petersburg, one of the examples cited, the curve is not a straight line all the way down.) Mr. Clark draws an instantaneous inflow hydrograph for a given river by actually figuring the time-area relation instead of using, as the writer did, a general one based on shape. This makes a much more flexible arrangement than the writer's and gives a more accurate inflow hydrograph. Such accuracy may not always be necessary, however, when the rainfalls over the area for different storms are considered. (For example, the Appomattox River at Petersburg has had other floods which present different characteristics. The one of March, 1932, with a peak of 8,000 cu ft per sec and about the same length of base shows no depression at the top of the hydrograph.) The writer found that an assumption of a rectangular shape gave satisfactory results in many cases that varied considerably from a rectangle.

The author's method, however, would permit extending the time-area concentration inflow not only to cover the shape of a given area with assumed uniform rainfall but also to include varying rates of rainfall on different parts of this area.

The author presents a new treatment of the base flow which, in the particular case of the Appomattox River, gives much better results than would the customary method—that is, a straight line at the base of the hydrograph, which, if used, gives a computed hydrograph less close to the actual one than the author's. This hardly can be taken as a proof that the author's method is any better since the entire matter of the rainfall distribution is unknown, as the author states. (Rainfall at Hopewell amounting to 23% of the total occurred during the two days, April 28 and 29, in the middle of the flood.) There are many unknowns in the behavior of the ground storage during a flood.

[31] "The Flood Hydrograph," by Howard M. Turner and Allen J. Burdoin, *Journal*, Boston Soc. of Civ. Engrs., Vol. XXVIII, No. 3, July, 1941.

Unanswered are such questions as how the water gets into the ground with a rapidly rising river, how far back the rise in ground-water level continues before the river begins to drop again and the storage flows back into the river, and how much of this drainage from storage in the ground may be due to normal ground-water flow backed up by the rising level and thus not originating from the flood rainfall. The outflow hydrograph will also be much affected by the assumption as to the amount of flood appearing as ground-water flow assumed by the author as 30% of the total. The author suggests that this percentage, taken from records of another river, must vary with floods of different magnitude. To use 30% for an instantaneous hydrograph for all floods may lead to error in the case of much greater or much smaller floods. The author should give some examples of the proportion of ground-water runoff which his method discloses in small floods compared to very large floods. The writer has found that with all these possible differences, the simple method of drawing a straight line across the base of the hydrograph is sufficiently accurate for general use in spite of its theoretical limitations.

The author is to be congratulated for devising an additional tool to use in the attempt to analyze the steps between the rainfall, inflow hydrograph, and the flood hydrograph.

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DISCUSSIONS

RIGIDITY AND AERODYNAMIC STABILITY OF SUSPENSION BRIDGES

Discussion

BY H. REISSNER, BLAIR BIRDSALL, HARDY CROSS, AND FREDERICK C. SMITH, HERMANN FRIEDLAENDER, AND GEORGE S. VINCENT

H. REISSNER,[49] ESQ.[49a]—The publication of this paper gives a welcome opportunity for a discussion of the dynamics and particularly the aerodynamics of suspension bridges.

Part 1 is a treatment of free oscillations determined by means of the principle of work developed by Lord Rayleigh[50] for the purpose of finding approximate values of natural frequencies of a beam system of varying cross section to simplify the integration of differential equations with variable coefficients. Probable deflection curves (characteristic functions) could thus be introduced that would satisfy the boundary conditions but not necessarily the differential equations.

When the coefficients of the differential equation of the equilibrium condition are constant, the Rayleigh method becomes superfluous and the only difference (sometimes a convenient difference) is that the frequency equation appears as a quotient of integrals instead of as a determinant.

In the application of his method, the author for simplicity omits the cases of continuous or fixed-end girders and of variable stiffness and divides the suspension bridge systems in two classes—those for which the change ΔH of the cable thrust H is zero and those for which it is not zero—without explaining the significance of this division. The fact is that, as long as the horizontal inertia forces and the cable extension are neglected, the values of the frequencies are not influenced by ΔH and the value of ΔH remains indeterminate for symmetric oscillations and must be zero for antisymmetric oscillations.

If this change ΔH is to be determined, the horizontal inertia forces and the local horizontal displacements of the cable must be considered as demon-

NOTE.—This paper by D. B. Steinman was published in November, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by Edward Adams Richardson; March, 1944, by R. K. Bernhard; May, 1944, by J. M. Robertson, and C. H. Gronquist; and June, 1944, by F. B. Farquharson.

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[49a] Received by the Secretary March 31, 1944.

[50] "The Theory of Sound," by John William Strutt, Baron Rayleigh, Vol. 1, Macmillan & Co., London, 1926, p. 287.

strated by the writer elsewhere.[51] The resulting differential equations become of the sixth order, because the cable must satisfy two conditions of horizontal displacements on the saddles, and because the girder must satisfy four equations for the vertical displacements and bending moments at the supports. The change ΔH is then no longer constant along the cable.

For the case of symmetrical oscillations in a structure with straight backstays, Mr. Steinman expresses ΔH in terms of the elastic extension. This is only necessary for the case of a one-segment oscillation, which special case has such a high frequency that it practically never occurs.

On the other hand, symmetrical multisegment oscillations in structures with straight backstays are possible without extension of the cable and in these cases the influence of extension (cable stretch) is negligible.

For the case of nonparticipating side spans and the case of the simply-supported girder, the writer has applied the Rayleigh method (Part 1 of the paper) to oscillations,[51] but only because consideration of the horizontal inertia forces, of the varying slope angle γ of the cable and of the local condition of inextensibility, introduces variable coefficients. The effect of inextensibility of a cable element is expressed by

$$\frac{d\eta}{dx} = -\frac{8f}{l^2} \times \frac{d\xi}{dx} \quad \text{......................(80)}$$

in which ξ denotes the horizontal displacement of any point of the cable. In this case the solution of the differential equation by a series development leads to extremely slow convergence and the Rayleigh method is indicated.

The Rayleigh method has been applied to the case of a two-segment oscillation by the writer[51] and elsewhere to the case of a three-segment and a four-segment oscillation by E. Sternberg in cooperation with the writer.[52]

The following expressions, taken from those analyses, show the influence of the aforementioned terms, which are usually neglected:

For $\eta_2 = a_m \sin \frac{2\pi x}{l}$:

$$N_2 = \frac{1}{2}\sqrt{\frac{g}{2f}}\sqrt{\frac{1 + 6.16\left(\frac{f}{l}\right)^2 + 315\,\frac{E\,I\,f}{w\,l^4}}{c_t + c_c\,1.12}} \quad \text{..........(81a)}$$

for $\eta_3 = a_m\left(\sin\frac{\pi x}{l} - 3\sin\frac{3\pi x}{l}\right)$:

$$N_3 = \sqrt{\frac{2.05}{4}}\sqrt{\frac{g}{2f}}\sqrt{\frac{1 + 5.33\left(\frac{f}{l}\right)^2 + \frac{700\,E\,I\,f}{w\,l^4}}{c_t + c_c\,1.05}} \quad \text{........(81b)}$$

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[51] "Oscillations of Suspension Bridges," by H. Reissner, *Journal of Applied Mechanics*, Vol. 10, No. 1, March, 1943, pp. A-23 to A-32 (preprinted and presented at the June, 1942, meeting in Cambridge, Mass., of the Applied Mechanics Div. of the A.S.M.E.).

[52] "Special Cases of the Oscillations of Suspension Bridges," by E. Sternberg, Master of Science thesis presented to the Illinois Inst. of Technology in Chicago, in 1942.

and, for $\eta_4 = a_m \sin \frac{4 \pi x}{l}$:

$$N_4 = \sqrt{\frac{4.05}{4}} \sqrt{\frac{g}{2 f}} \sqrt{\frac{1 + 5.52 \left(\frac{f}{l}\right)^2 + 1{,}250 \frac{E I f}{w l^4}}{c_t + c_c\, 1.07}} \qquad (81c)$$

in which c_t and c_c are the respective relative contributions of truss and cable weight computed so that $c_t + c_c = 1$.

For the Tacoma Narrows Bridge, the numerator of the last factor in Eqs. 81 has the following relative magnitudes:

n	Numerator, Eqs. 81	N, in cycles per minute
2	1 + 0.038 + 0.01183	7.95
3	1 + 0.0328 + 0.0228	11.6
4	1 + 0.0364 + 0.0468	16.3

The second terms in this numerator give the influence of the horizontal inertia forces, and the third (last) terms, the influence of the girder stiffness. It is noteworthy that the girder stiffness for the two-segment oscillation contributes less than one third of the effect of the horizontal inertia forces of the cable in the case of the relative girder stiffness of the Tacoma Narrows Bridge.

These additional terms are less than 4% and the values without them fit very well into the approximate formula, Eq. 17*b*, although the writer would prefer to write this formula in consistent units—that is, in the form,

$$P \approx \frac{1}{n} \sqrt{\frac{32 f}{g}} \qquad (82)$$

which is true for any system of units.

The foregoing data demonstrate that the free oscillations of suspension bridges which have no redundant constraints, such as stays, can be computed by simple formulas. In fact, they can be computed by formulas which assume that a pre-stressed suspension bridge acts like a mathematical pendulum of length,

$$L = \frac{32}{\pi^2 n^2} f (1 + \epsilon) \qquad (83)$$

in which ϵ is a small quantity depending on the sag-to-span ratio, the cable and girder mass ratio, and the girder stiffness, as may be seen from Eqs. 81.

Under the heading (see Part 1), "Oscillations with Longitudinal Motion of Span," the girder is assumed to execute a free horizontal oscillation regardless of its fixing or friction at the pier supports. The reason for this assumption is not clear. The case in which there is horizontal interference between cable and girder should be treated like a system with redundant members. There is a possibility (worthy of investigation) that the short waves observed running along the girder of the Tacoma Narrows Bridge were started from a combined compression and bending effect caused by such interference.

Example 3, on torsional oscillations, needs clarification. In the cables the masses are concentrated, and only in the girder-roadway system are they

distributed transversely. Hence, the transition in frequency from bending oscillation to torsional oscillation should be given by

$$N_t = N \sqrt{\frac{1}{c_t\,(2\,r/b)^2 + c_e}} \quad \text{.................. (84)}$$

For instance, for a weight of 2,113 lb per ft of girder roadway and of 747 lb per ft of cable, $c_t = 0.74$ and $c_e = 0.26$. Using a radius of gyration ratio of $\frac{r}{0.5\,b} = \frac{2}{3}$ the numerical result becomes (in cycles per minute): $N_t = 1.305\,N = 7.95 \times 1.305 = 10.4$, whereas Eq. 21*b* would give $1.5\,N = 11.9$.

The fact that the observed frequency ($N = 12$) was somewhat higher than the frequency (10.4) determined by the more exact formula is probably explained by a smaller value of the radius of gyration r, by the torsional elastic stiffness of the stringers and the concrete of the roadway, and by the transverse force components of the hangers.

The author's analysis of partly stable vertical oscillations due to the negative range of the lift characteristic of the cross section and his recommendation that such faulty cross sections be investigated and avoided (Part 2) deserve the full attention of bridge designers. On the other hand, his ideas and explanations concerning the generation of destabilizing phase differences by means of a phase factor β and by a disturbance or an impulse traveling across the width of the roadway, are not clear.

In principle the problem of the origin of bridge oscillations, which accumulate energy out of a uniform wind flow, is very hard to handle owing to the rugged and diversified shape of the cross sections. The rational derivation of a flow pattern around these aerodynamically irregular sections, when producing a phase difference of negative damping, appears extremely difficult. The writer may mention that even the so-called flutter theory of the potential flow around perfectly streamlined airfoils governing the rhythm of the circulation by their sharp trailing edges is considered as one of the most intricate problems of aerodynamics. This makes it very doubtful that a valid analysis can be deduced from the mere geometric concept of a phase factor.

The University of Washington in Seattle and the California Institute of Technology have made pertinent experiments[53,54] on models supported on springs in such a way that the effect of aerodynamic damping could be observed separately from that of structural damping. These experiments were made for such a wide variety of cases and with such careful attention to the theory of similarity that one may be justified in using them as the only available and reliable basis for a semi-empirical, semi-analytical formula. An expression for the logarithmic decrement, δ_a, as presented in the Carmody Report,[3,53,54] can

[53] "Failure of the Tacoma Narrows Bridge: Report of the Special Committee of the Board of Direction," *Proceedings*, Am. Soc. C. E., December, 1943, p. 1578.

[54] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, Chapter IV, p. 96.

[3] *Ibid.*, Appendix VI, p. 19.

be written in any arbitrary consistent units in the form,

$$\delta_a \frac{w}{\rho\, g\, b^2}\left(\frac{r}{b}\right)^2 = -\,2.05\left(\frac{V}{b\,\omega} - 0.958\right) \ldots\ldots\ldots\ldots\ldots (85)$$

in which $\omega = N \times 2\,\pi$. From Eqs. 84 and 85, it follows that the critical velocity V_a (without structural damping) is given by

$$V_a = 0.958\, b\, \omega = 0.958\, \pi\, b \sqrt{\frac{g}{2\, f\left[c_t\left(\frac{2\,r}{b}\right)^2 + c_c\right]}} \ldots\ldots (86)$$

For critical velocities greater or less than that expressed by Eq. 86, the logarithmic decrement becomes

$$\delta_a = 2.05\, \frac{\rho\, g\, b}{w\, \omega}\left(\frac{b}{r}\right)^2 \times (V_a - V) \ldots\ldots\ldots\ldots (87)$$

Eq. 87 is a simple, dimensionally accurate expression which has been confirmed experimentally. Until the development of a completely rational analysis leading to the derivation of the logarithmic decrement, designers must be content with it.

From Eq. 86, it follows that, for the case of aerodynamic damping alone, the only way to increase the critical velocity is to increase the product of the width of the bridge floor and the natural frequency. As the practical frequency variation is very small, the critical velocity can be raised only by increasing the width of the floor, b. The relation between this width and the aerodynamic critical velocity is demonstrated by Table 9. The low value of b and of the

TABLE 9.—CRITICAL WIND VELOCITY (NO STRUCTURAL DAMPING, SEE EQ. 86)

Bridge	Width b (ft)	Height h (ft)	Distribution of mass	Critical Velocity	
				Ft per sec	Miles per hr
Golden Gate	90	475	0.75	60.1	41.0
George Washington	106	325	0.64	98.5	67.1
Tacoma Narrows	39	232	0.75	37.2	25.3
Bronx-Whitestone	74	200	0.75	76.2	52.0

critical velocity for the Tacoma Narrows Bridge is particularly significant. For the conditions assumed in Eq. 84, c_t was found equal to 0.74. The writer has assumed the value of 0.75 in Table 9 except in the case of the George Washington Bridge, which has four cables and no stiffening girders.

Mr. Steinman seems to distinguish between two kinds of self-excited torsional oscillations, one as treated by the Carmody Report (defined by Eq. 87 and Table 9) and the other as defined by his torque coefficient, $C_t\,\alpha$, and the phase factor β. On the other hand, the Carmody Report[3,55,56] states

[55] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, p. 129.

[56] *Ibid.*, Appendix VIII, p. 23.

that curves like those in Fig. 6 represent a stable torque diagram such as might be derived by observations on an ordinary wind vane. A positive torque for a positive angle occurs only when the cross section is streamlined. Such a cross section may be called statically unstable. No self-excited oscillations appear with it but only an inclined equilibrium position which, with insufficient elastic resistance, may lead to overturning.

The Carmody Report[3] was the first to reveal the fact that in suspension bridges it is the structural damping rather than the structural elastic stiffness that contributes to dynamic stability. This is a contribution to a necessary new treatment. To maintain its economic and esthetic advantages, the suspension bridge must necessarily be much more flexible than other forms of truss frame bridges—that is, it must have a lower natural frequency. Therefore, its margin of safety over that furnished by aerodynamic stability must be provided by its internal frictional resistance. The utilization of this important knowledge, however, is impeded by the wide variation in frictional properties of materials and structures.

Experiments tend to show that the logarithmic decrement does not change with frequency but with amplitude. Eq. 35 incorporates this fact although Mr. Steinman does not discuss how this logarithmic decrement conforms with the equation of motion; nor does he cite the experiments on which it is based. Information about damping coefficients or logarithmic decrements of bridge structures—in terms of stiffness $E\,I$, depth d, span l, or stress σ—is certainly scarce if not nonexistent. For reasons of dimensional consistency, the author introduces the square root of the amplitude, thus giving an infinite slope of increase of the decrement toward zero amplitude and a steep slope for small amplitude. The writer doubts the correctness of this assumption. All published material available to the writer indicates that the decrement increases slowly with corresponding increases in stress up to the elastic limit. In fact, such an expression can be derived by starting from the concept of a quadratic velocity damping for which R. Grammel[57] in a specially simple case could show that the logarithmic decrement for small oscillations becomes independent of the frequency and proportional to the amplitude.

To linearize this procedure which is necessary to overcome the mathematical difficulties and justified for small oscillations the structural damping factor d_s may be assumed to be proportional to the average velocity in the form,

$$d_s = \frac{k}{g}\, a\, \omega \qquad (88)$$

in which a is the maximal amplitude of the girder, g the acceleration of gravity, and k is a dimensionless constant which expresses the functional characteristics of the members and joints of the structure, and the changes of amplitude along the length of the span.

The combination of aerodynamic and structural damping as formulated in Eq. 89 following will lead to the interpretation of a as the amplitude which is not

[57] "Neuere Versuche über elastische Hysterese," by R. Grammel, *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 58, 1914, p. 1600.

increasing any more. Such an effect of structural damping has also been stated by Louis G. Dunn.[54]

In 1942[51] the writer evaluated the available observations on aerodynamically critical velocities V_a and the total wind velocity V_c (condensed in Eq. 85 and in the actual observations of the critical velocity of the Tacoma Bridge) by means of the equations of motion. For this purpose constant viscous damping coefficients were used—that is, it was assumed that both decrements were inversely proportional to the frequency, which is correct only for aerodynamic damping. This assumption leads to approximate values of V_c only when applied to bridges with natural frequencies not too different from the frequency of the Tacoma Bridge. This same objection can be raised against the representation, sometimes used, of friction by an additional elastic force with an imaginary factor accounting for the phase difference in an energy-consuming resistance. Furthermore, such a method does not seem rational because it cannot be transformed into expressions involving functions of real quantities.

There seems to be no reason why the viscous damping factor, although constant for given dimensions of a system, cannot be made dependent on the frequency and the amplitude in such a way as to satisfy experimental determinations of logarithmic decrements.[58]

Neglecting the horizontal inertia forces and the cable slope, the equation of motion in its usual form is as follows:

$$\frac{w}{g}\psi\frac{\partial^2\eta}{\partial t^2} - H\frac{\partial^2\eta}{\partial x^2} + E\,I\,n_t\frac{\partial^4\eta}{\partial x^4} + \frac{\partial}{\partial t}\left(d_a\,V\,\eta + d_s\,E\,I\,n_t\frac{\partial^4\eta}{\partial x^4}\right) = 0 \quad (89)$$

In Eq. 89, in addition to the notation of the paper, n_t is a factor indicating an eventual torsional stiffness of the wind bracing (for a tubular cross section) and of the bridge floor; d_a is a coefficient of aerodynamic damping; d_s is a coefficient of structural damping (see Eqs. 85 and 88); and

$$\frac{w}{g}\psi = m_1 = m_{\text{girder}}\left(\frac{2\,r}{b}\right)^2 + m_{\text{cable}} \quad (90)$$

In the last term of Eq. 89 the structural damping moment is assumed to be proportional to the time rate of change of the elastic counter forces,

$$\frac{\partial}{\partial t}\left(E\,I\,n_t\frac{\partial^4\eta}{\partial x^4}\right)b \quad (91)$$

Solving Eq. 89 in the usual manner and observing Eq. 87, the logarithmic decrement becomes

$$\delta_a + \delta_s = 2.05\,\frac{\rho\,g\,b}{w\,\omega}\left(\frac{b}{r}\right)^2(V_a - V) + k\,a\,\frac{E\,I\,n_t}{w\,\psi\,l^4} \quad (92)$$

The condition for zero decrement now becomes

$$V_c = V_a + k\,\frac{a}{b}\,\frac{E\,I\,n_t}{\psi\,l^4}\,\frac{\omega}{\rho\,g\left(\frac{b}{r}\right)^2} \quad (93)$$

[58] "Damping Formulas and Experimental Values of Damping in Flutter Models," by R. P. Coleman, *Technical Notes No. 751*, National Advisory Committee for Aeronautics, February, 1941.

The only observations of critical wind velocities available with all necessary dimensions and weight data are those in connection with the Tacoma Narrows Bridge.[59] Using these observations one can discuss the significance of Eq. 93 by inserting the observed simultaneous values of $V_e = 62$ ft per sec, of the circular frequency $\omega = \frac{2\pi n}{60} = 1.25$, and the observed model value of $V_a = 37.2$ ft per sec from Table 9 into Eq. 93. Eq. 93 contains the product of the friction coefficient k and the ratio $\frac{a}{b}$ which latter is a measure of about half the maximum angle of twist, at which the bridge by the structural damping increasing with amplitude reaches a state of steady oscillation—that is, provided that the stresses in this state are not at the breaking limit as in the case of the Tacoma Bridge.

Solving for this dimensionless quantity $\frac{k\,a}{b}$ one obtains (compare Eq. 93):

$$\frac{k\,a}{b} = (V_e - V_a)\frac{l^4}{E\,I\,n_t\,\omega}\rho\,g\,\psi\left(\frac{b}{r}\right)^2$$

$$= 24.8\,\frac{2{,}800^4 \times 0.00238 \times 32.2 \times 0.59 \times 9}{3.46 \times 10^{10} \times 1.25} = 1.435 \times 10^4 \ldots\ldots (94)$$

With this value the following examples may be computed:

(1) For the Tacoma Bridge what increase of road width b and girder stiffness $E\,I$ would be necessary to raise the critical wind speed from 62 ft per sec to 150 ft per sec (for the same angle of twist)?

Choosing, as suggested by the Tacoma committee, $b = 52$ ft (instead of 39) and doubling the girder depth with the same effective flange area one obtains from Eq. 94: $V_e = 37.2 \times 1.3 + 24.8 \times 4 = 147.5$ ft per sec. If it is required that the tangent of the twisting angle for the steady state is only one fourth of the value observed at the Tacoma Bridge the allowable critical velocity would be $V_e = 48.3 + 24.8 = 73.1$ ft per sec.

(2) What is the critical velocity for the Golden Gate Bridge with the value of critical twisting angle of the Tacoma Bridge? From Table 9 and from the report of the Board of Direction,[60] the following values are used:

$$V_a = 60.1 \text{ ft per sec};\quad E\,I = 118.9 \times 10^{10} \text{ lb ft}^2;\quad n_t = 1,$$

$$\omega = 0.773 \text{ sec}^{-1};\quad \psi = 0.59;\quad l = 4{,}200 \text{ ft};\quad b = 90 \text{ ft};\quad \frac{b}{r} = 3.$$

Eq. 94 gives $V_e = 60.1 + 109.9 = 170$ ft per sec $= 116.5$ miles per hr. If only a quarter of the original value of $\frac{a}{b}$ is allowed, then $V_e = 60.1 + 27.5 = 87.6$ ft per sec = 60 miles per hr. It may be mentioned that the highest wind velocity of 110 ft per sec, acting on the Golden Gate Bridge was observed

[59] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, pp. 21–27.

[60] "Failure of the Tacoma Narrows Bridge: Report of the Special Committee of the Board of Direction," *Proceedings*, Am. Soc. C. E., December, 1943, p. 1566, Table 1.

by R. G. Cone, M. Am. Soc. C. E., on February 9, 1938,[61] and that the bridge attained steady deflections of which the magnitude could not be measured.

(3) Another opportunity for checking Eq. 94 is afforded by the Bronx-Whitestone Bridge. Here the data in feet and pounds are:

$$E\,I\,n_t = 8.5 \times 10^{10};\quad \omega = 1.19;\quad \psi = 0.59;$$

$$l = 2{,}300;\quad b = 74;\quad \text{and}\quad \frac{b}{r} = 3.$$

With the value of $V_a = 76.2$ from Table 9, Eq. 94 yields $V_e = 76.2 + 100.0 = 176.2$ ft per sec = 121 miles per hr and with one quarter of the twisting angle $\frac{a}{b}\,V_e = 76.2 + 25 = 101.2$ ft per sec.

Concerning the aim to increase the structural stiffness and friction (n_t in Eq. 89 and k in Eq. 88), Mr. Steinman's suggestion in Fig. 9 seems very appropriate. Geometrically rigid cross frames are used, particularly at the quarter points of the spans, running the entire way from and between the cables to the lower truss chords and the transverse roadway girders. In fact, such transverse bracings would force the horizontal wind bracing to resist the twisting of the bridge by means of the bending stiffness and the bending friction in its horizontal plane.

A combination of this device with diagonal stays running from the top points of the cross frames and in the planes of the cables and hangers to the piers at the bases of the pylons, and equipped with vibration absorbers, should be very efficient against all kinds of oscillations with a minimum of redundant members. An indication of a high damping effect of such stays has been found by the experiments of F. B. Farquharson,[53] M. Am. Soc. C. E.

Blair Birdsall,[62] Assoc. M. Am. Soc. C. E.[62a]—The profession is indebted to the author for his presentation of the results of an exhaustive study of the problem arising from the failure of the Tacoma Narrows Bridge and the more or less unstable behavior of several other structures. Many interesting and helpful new tools are offered for the use of the designer. Although the validity of these formulas is strikingly confirmed by comparison with observed oscillations, it is undoubtedly true, as stated by the author, that many gaps remain to be filled and that the empirical deductions require more thorough consideration.

For instance, a minimum limit of 1:100 for the ratio of depth of truss or girder to span may be more conservative than necessary. This is evident in the case of the Golden Gate Bridge. A decrease in cable sag from 11.1% to 10% of span would have increased K per cable well above the minimum value of 120 selected by the author and in this case the ratio of truss depth to span is 1:168.

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[61] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, Appendix IX.

[62] Asst. Chf. Engr., Bridge Div., John A. Roebling's Sons Co., Trenton, N. J.

[62a] Received by the Secretary June 21, 1944.

Thus, it is to be hoped that this paper will be the beginning of a series of discussions and additional papers through which the Society will receive the benefit of the vast amount of research and investigation in progress on this subject.

The writer wishes to offer a few remarks on the "spring constant" or "coefficient of rigidity," designated by the author as K. From the derivation of K, it appears perfectly proper to consider this constant as a measure of the resistance of the span to deformations caused by loads applied uniformly along segments of the span: K per cable is a measure of resistance to loads that have a uniform intensity per foot of horizontal cable span; and K per foot is a measure of the resistance to loads that have a uniform intensity per square foot. However, since the present problem involves the aerodynamic behavior of suspension bridges, an attempt should be made to find a constant which will be a measure of the resistance of a particular structure to loads imposed by commonly occurring wind velocities. Obviously, wind of a given velocity will not have the same effect on two bridges that are identical in all respects except for the stiffening member, which is a truss in the one case and a girder in the other. Furthermore, the writer believes that vertical forces resulting from wind action are not uniform per square foot of floor area and that the total vertical wind force is not proportional to width of roadway on two structures that are identical except for width.

The writer suggests that an attempt be made to modify K so as to include some measure of the effect of wind action. Perhaps such a modification can be obtained from a study of the lift graph for the bridge cross section. In this way it may be possible to classify K-values for existing bridges in the order of their observed behavior. The writer has taken a step in this direction by means of the following simple device.

In Part 4, the author states that the limiting value of K per cable should be 300 for bridges with stiffening girders and 120 for bridges with stiffening trusses. By dividing all the author's values of K per cable for girder-stiffened bridges by 300/120 or 2.5, the writer has obtained a rearrangement of the order of stiffness of the bridges in Table 2 (see Table 10).

TABLE 10.—SUGGESTED REARRANGEMENT OF TABLE 2, WITH MODIFIED VALUES OF K

Year	Bridge	K per cable
1903	Williamsburg	1,082
1926	Philadelphia	735
1909	Manhattan	618
1936	Triborough	547
1936	Transbay	324
1930	Mid-Hudson	262
1931	Maumee River[a]	259
1931	St. Johns	252
1931	George Washington	244
1929	Detroit	212
1931	Waldo-Hancock	212
1929	Mount Hope	201
1929	Grand Mère	188
1939	Wabash River[a]	131
1937	Golden Gate	116
1938	Vancouver	112
1939	Bronx-Whitestone[a]	56.5
1938	Thousand Islands[a]	56.5
1938	Thousand Islands[a]	51
1939	Deer Isle[a]	27.6
1940	Tacoma[a]	24.8

[a] Denotes construction with solid-web stiffening girders.

Undoubtedly this is not the correct order; nor are the values truly representative, but this arrangement does appear to conform more nearly to the observed behavior of these bridges and indicates that very satisfactory results might be obtained from a more scientific modification of K per cable.

HARDY CROSS,[63] M. AM. SOC. C. E.[63a]—The author of this paper deserves thanks for effective condensation and clear presentation. The evidence presented is chiefly analytical; this is useful, but it is hoped that, in the closure, it will be supplemented by evidence from the author's extended practice.

Structural design is based on evidence from several sources; few problems illustrate this better than that of the stability of suspension bridges. Analysis furnishes evidence, but analysis is of necessity based on assumptions. It is important to note that model experiments are merely short cuts in analysis and are also based on assumptions. Experiments seriously attempt to avoid assumptions, but they rarely do so because of differences of useful scale. Full-scale experiments are scarcely practicable for suspension bridges, although it may be possible to make them with airplanes. Experience presents results of full-scale experiment; but the experiment is not planned and the results are fragmentary and often cannot be dependably correlated. Common sense—intuitive qualitative perception of relations—also furnishes evidence; but this evidence alone is not always dependable.

Evidence from no one of these sources is inherently more dependable than that from another; all may mislead—and often do so—and all sources supply valuable evidence. Because engineers are not too wise in the ways of nature, they are not often able to reconcile what seem to be conflicting data from the various sources; the current problem illustrates the point clearly.

Suspension bridges may begin to move in a wind and these movements can build up. "Autoresonance" from wind action is indicated and is generally accepted as an explanation. If autoresonance occurs, the strains in the structure will increase to failure unless the driving force dies out or reverses, or unless damping, either frictional or internal, absorbs the energy as fast as it is supplied, or unless the vibration characteristics of the structure change, as would occur with slack ties.

The first of these phenomena may occur because the wind dies out or because it is gusty. The author thinks that the lift relations change as the vibration builds up; this may be the explanation, or an explanation. The hypothesis is an interesting contribution to speculation and it deserves investigation even though this may be difficult.

Much information is not available on friction damping. Perhaps engineers try too hard to eliminate it in structures; but to absorb by friction the energy of a vibrating suspension bridge is itself a major problem. Even less is known of internal damping, but the writer has not yet seen data to indicate that it is an important factor in this problem.

Changeable vibration characteristics seem to be nature's solution of the resonance problem and perhaps the late John Roebling, M. Am. Soc. C. E., saw their value intuitively. The writer does not know how far this field of thought has been explored but he has reason to think that the author of the paper has studied it and hopes that he will comment on it.

The author feels sure that it is better to eliminate the cause of instability than to provide a cure. Whether this is true depends on the certainty of the

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[63] Prof., Civ. Eng., Yale Univ., New Haven, Conn.

[63a] Received by the Secretary July 21, 1944.

diagnosis of cause and the dependability of the recommended cure; good engineering depends on evaluation of insurance risks against probable occurrence and possible consequence. If the engineer can find out when—for what proportions and spans—these bridges are unstable, he may not care so much about why; certainly this is the case in many engineering problems. Usually the designer makes some observations on "when"; then he speculates as to "why"; and finally he revises and successively revises to approach dependability. The author has here contributed to the "why"; it is to be hoped that in his closure he will contribute data from his broad experience as to the "when."

FREDERICK C. SMITH[64] AND HERMANN FRIEDLAENDER,[65] ASSOC. MEMBERS, AM. SOC. C. E., AND GEORGE S. VINCENT,[66] M. AM. SOC. C. E.[66a]—In association with F. B. Farquharson, M. Am. Soc. C. E., the writers have been working on the development of equations which will express adequately the behavior of suspension bridges under vibration. Model studies of the Tacoma Narrows Bridge, one of the structures analyzed by the author, have been undertaken by the University of Washington, at Seattle, and the Washington (State) Toll Bridge Authority with the cooperation of the Public Roads Administration. These studies also involve certain supplemental tests recommended by the Advisory Committee on the Investigation of Long-Span Suspension Bridges.[67] As a part of the supplemental tests the original model was modified to represent a bridge with unloaded backstays.

The author's equations and those developed by the writers, in common with most engineering formulas, are not precise. The engineer starts with certain assumptions, makes a series of approximations of varying degree, and attempts to develop equations that will predict with fair precision the performance of the designed structure. The test of the equations is their reliability for such prediction. For this purpose, complex formulas are not justified except for limited comparative study if simpler formulas give precision equivalent to that of the observations. In the case of suspension bridge vibration due to wind, perhaps the best data for comparison with the results of equations are the observations on the Tacoma Narrows Bridge and its fifty-scale full model in the wind tunnel. The model duplicated with rather startling accuracy the behavior of the bridge in all movements covered by field observations, and provided, in addition, the opportunity to observe movements which had not been adequately observed in the field. The supplemental tests provided observations, not elsewhere available, for checking certain of the equations. It is proposed to discuss the basic assumptions made and the resulting equations, to compare the author's equations with some of the writers' equations, and to test both against observations made on the model.

[64] Associate Prof., Civ. Eng., University of Washington, Seattle, Wash.

[65] With U. S. Army; formerly, Engr., Webster-Brinckley Co., Seattle, Wash.

[66] Highway Bridge Engr., Public Roads Administration, Structural Research Laboratory, Univ. of Washington, Seattle, Wash.

[66a] Received by the Secretary April 24, 1944.

[67] *Civil Engineering*, October, 1943, p. 503.

Assumptions.—The basic assumptions by Messrs. Steinman and Rannie[3] agree in that:

(1) The suspenders are inextensible under inertial loads;
(2) The cable stresses due to inertial load are small compared to dead-load stresses;
(3) The tower stiffness against horizontal forces applied at the top is negligible;
(4) The moment of inertia of the truss, hinged at the tower, is uniform throughout the length of the bridge; and
(5) For bridges with suspended side spans, the cable is inextensible under inertial loads.

They differ in that:

(*a*) Mr. Steinman assumes that the deflection curves due to oscillation are sine curves, whereas Mr. Rannie develops the equations for the deflection curves;
(*b*) Mr. Steinman takes truss stiffness into account in all cases, whereas Mr. Rannie neglects truss stiffness in the case of the Tacoma Bridge, although he does develop more general equations which do include the effect of truss stiffness; and
(*c*) Furthermore, Mr. Steinman develops an equation for bridges with unloaded backstays in which he takes into account the extensibility of the cable under inertial loads, a bridge condition that Mr. Rannie did not consider.

In their studies, the writers also found it necessary to take cable extensibility into account in developing equations for the condition with unloaded backstays. Furthermore, the fact was recognized that the backstays carry their own weight and, therefore, are not straight. They are actually loaded equilibrium polygons subject to elastic distortion and, accordingly, were treated as loaded side spans but with unit loading different from that on the main span and correspondingly modified sag (the cable weight on the Tacoma Bridge was about 25% of the total weight).

Equations.—The author simplifies the derivation of many expressions by the use of approximations which, for the most part, appear to be permissible. However there are apparently certain errors in Eqs. 11*c* and 14. Consider first Eq. 11*c*. If, as is suggested by the author, the integration of Eqs. 2 and 4 is extended over the three spans the following equation is obtained:

$$K = \frac{\left[\frac{\pi^2 n^2}{l^2} + \frac{2\,(a_1)^2\,l_1}{a^2\,l}\frac{\pi^2\,(n_1)^2}{(l_1)^2}\right] H + \left[\frac{\pi^4 n^4}{l^4} + \frac{2\,(a_1)^2\,l_1}{a^2\,l}\frac{\pi^4\,(n_1)^4}{(l_1)^4}\right] E\,I}{1 + \frac{2\,(a_1)^2\,l_1}{a^2\,l}} \quad \text{..(95)}$$

If the term, $\frac{2\,(a_1)^2\,l_1}{a^2\,l}$ were equal to q of Eq. 11*c*, Eq. 95 would be identical with

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Eq. 11*b*, but $\frac{2\,(a_1)^2\,l_1}{a^2\,l}$ is approximately equal to $\frac{(n_1)^2}{2\,n^2}\frac{l}{l_1}$ $(\sec^4 \gamma_1)^2$ and not $\frac{(n_1)^2}{2\,n^2}\frac{l}{l_1}$ $\sec^4 \gamma_1$, which the author gives as the value for q. It seems to the writers that $(\sec^4 \gamma_1)^2$ is more nearly correct for the secant term than is $\sec^4 \gamma_1$ for the following reasons:

After developing Eq. 9 (in the development of which the author assumes no tower-top movement), Mr. Steinman states: "The resulting value of Δl_c or Δl will be the algebraic sum of the component values." The writers understand this to mean that, assuming that the tower tops do move, Δl caused by the multiple segment wave represented by the equation,

$$\eta = a \sin \pi\, n \frac{x}{l} (\sin 2\,\pi\, N\, t) \qquad (96a)$$

will be the same as Δl caused by a single segment wave represented by the equation,

$$\eta = \frac{a}{n} \sin \pi \frac{x}{l} (\sin 2\,\pi\, N\, t) \qquad (96b)$$

An approximate formula[68] for the change in length of the span, Δl, when there is a small change in the sag, Δf, is given by the formula,

$$\Delta l = \left\{ \frac{\frac{16\,f}{l}\left[5 - 24\left(\frac{f}{l}\right)^2\right]}{15 - 40\left(\frac{f}{l}\right)^2 + 288\left(\frac{f}{l}\right)^4} \right\} \Delta f \qquad (97)$$

An approximate formula[69] for the length of the side span cable commonly used is

$$L_1 = l_1 \left(\sec \gamma_1 + \frac{8}{3} \frac{(f_1)^2}{(l_1)^2 \sec^3 \gamma_1} \right) \qquad (98)$$

where l_1 is length of side span and γ_1 is the angle the chord of the side span makes with the horizontal.

If this expression is differentiated first in respect to f_1 and then in respect to l_1 and the resulting expressions equaled, the following expression is derived

$$\Delta l_1 = \frac{\frac{16}{3}\frac{f_1}{l_1}}{\sec^4 \gamma_1 - \frac{8}{3}\frac{(f_1)^2}{(l_1)^2}} \Delta f_1 \qquad (99)$$

Neglecting all terms but the first in both the numerator and denominator and substituting $\frac{a}{n}$ for Δf and $\frac{a_1}{n_1}$ for Δf_1 the following approximate formulas are

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[68] "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, John Wiley & Sons, Inc., New York, N. Y., 1917, p. 199.

[69] "Suspension Bridges," by D. B. Steinman in "Movable and Long-Span Steel Bridges," by George A. Hool and W. S. Kinne, McGraw-Hill Book Co., Inc., New York, N. Y., 1923, p. 292.

obtained:

$$\Delta l \approx \frac{16}{3}\frac{f}{l}\frac{a}{n} \qquad (100a)$$

and

$$\Delta l_1 \approx \frac{16}{3}\frac{f_1}{l_1}\frac{a_1}{n_1}\frac{1}{\sec^4 \gamma_1} \qquad (100b)$$

(For the Tacoma Bridge this approximation introduces an error of less than 2% in the value of Δl and less than 1% in Δl_1.) In Eqs. 100, Δl must be equal to $2\,\Delta l_1$; therefore,

$$a_1 = \frac{1}{2}\frac{f}{f_1}\frac{l_1}{l}\frac{n_1}{n}\, a \sec^4 \gamma_1 \qquad (101)$$

Substituting this value of a_1 in the expression, $\frac{2\,(a_1)^2\, l_1}{a^2\, l}$ of Eq. 95 and remembering that $\frac{f_1}{(l_1)^2} = \frac{f}{l^2}$ gives:

$$q = \frac{(n_1)^2\, l}{2\, n^2\, l_1}(\sec^4 \gamma_1)^2 \qquad (102)$$

A somewhat more precise expression for L_1 is[70]

$$L_1 = l_1 \left[1 + \frac{8}{3}\frac{(f_1)^2}{(l_1)^2} + \frac{1}{2}\tan^2 \gamma_1 \right] \qquad (103)$$

By a similar derivation this gives

$$\Delta l_1 = \frac{\frac{16}{3}\frac{f_1}{l_1}}{\frac{1}{2}(1 + \sec^2 \gamma_1)}\,\Delta f_1 \qquad (104)$$

and

$$q = \frac{(n_1)^2\, l}{8\, n^2\, l_1}(1 + \sec^2 \gamma_1)^2 \qquad (105)$$

These changes make very little difference in the value of K. In fact, for practical purposes, the secant term could be neglected entirely as its effect on the value of K is small.

If this correction is accepted, not only should Eq. 11*c* be modified as described, but Eq. 14 which is derived from the same assumptions should also be modified.

The fundamental equilibrium equation, as it pertains to suspension bridges, may be stated as:

$$w + \Delta w = -(H + \Delta H)\left(\frac{d^2y}{dx^2} + \frac{d^2\eta}{dx^2}\right) + E\,I\left(\frac{d^4\eta}{dx^4}\right) \qquad (106)$$

The quantity Δw, being the inertial load per foot per cable, may be written:

$$\Delta w = \frac{w\,a}{g} = -\frac{w}{g}\,\omega^2\,\eta_x \sin \omega\, t \qquad (107)$$

[70] "The Theory and Practice of Modern Planned Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, John Wiley & Sons, Inc., New York, N. Y., 1917, p. 195.

Eq. 106 recognizes that the curve of the deflected cable is proportional to the moment diagram of the loading carried by the cable, and therefore the load on the cable is proportional to the second derivative of the cable curve. It recognizes also that the load carried by the deflected girder is proportional to the fourth derivative of the deflection curve of the girder. Eq. 107 recognizes that the inertial load in vibration is proportional to the second derivative of the deflection with respect to time. The quantity ΔH, being the result of the inertial loads, also contains the factor, $\sin \omega t$, so that the fundamental equation can be written in the form:[71]

$$-\frac{E\,I}{H}\frac{d^4\eta}{dx^4} + \frac{d^2\eta}{dx^2} + \frac{w}{H\,g}\,\omega^2\,\eta = \frac{\Delta H\,w}{H^2} \qquad (108)$$

If, instead of the unknown frequency, ω, the dimensionless quantity

$$\mu = \sqrt{\frac{w}{H\,g}}\,\frac{l\,\omega}{2} \qquad (109)$$

is used, the following equation results:

$$-\frac{E\,I}{H}\frac{d^4\eta}{dx^4} + \frac{d^2\eta}{dx^2} + \left(\frac{2\,\mu}{l}\right)^2\eta = \frac{8\,f}{l^2}\frac{\Delta H}{H} \qquad (110)$$

Letting $\dfrac{H}{E\,I} = k^2\left(\dfrac{2}{l}\right)^2$:

$$\frac{d^4\eta}{dx^4} - \left(\frac{2\,k}{l}\right)^2\frac{d^2\eta}{dx^2} - \left(\frac{4\,\mu\,k}{l^2}\right)^2\eta = -\frac{8\,f}{l^2}\frac{\Delta H}{H}\left(\frac{2\,k}{l}\right)^2 \qquad (111)$$

Eq. 111 is a standard differential equation, the special solution of which is[72]

$$\eta = C_1 e^{2k_2\frac{x}{l}} + C_2 e^{-2k_2\frac{x}{l}} + C_3 e^{2i\mu_2\frac{x}{l}} + C_4 e^{-2i\mu_2\frac{x}{l}} + \frac{2f\Delta H}{H\mu_2} \qquad (112)$$

in which $i = \sqrt{-1}$

$$(k_2)^2 = k^2\left[\sqrt{\left(\frac{\mu}{k}\right)^2 + \frac{1}{4}} + \frac{1}{2}\right] \qquad (113a)$$

and

$$(\mu_2)^2 = k^2\left[\sqrt{\left(\frac{\mu}{k}\right)^2 + \frac{1}{4}} - \frac{1}{2}\right] \qquad (113b)$$

For the symmetric modes, η has the same value for $+x$ as it has for the corresponding $-x$. When this fact is used the first two quantities on the right side of Eq. 112 combine as a hyperbolic function and a single constant. The terms involving i may be written as trigonometric functions and there results the equation

$$\eta = Y\cosh\frac{2\,k_2\,x}{l} + A\cos\frac{2\,\mu_2\,x}{l} + \frac{2\,f\,\Delta H}{H\,\mu^2} \qquad (114)$$

[71] "The Failure of the Tacoma Narrows Bridge," a report to the Hon. John M. Carmody, Administrator of the Federal Works Agency, Washington, D. C., March 28, 1941, by a board of engineers consisting of O. H. Ammann, Theodor von Kármán, and Glenn B. Woodruff, Appendix VI, p. 15, Eq. 10.

[72] "Elements of the Differential and Integral Calculus," by W. A. Granville, P. F. Smith and W. R Longley, Revised Ed., Ginn & Co., 1934, p. 403.

The general relation between deflection and moment in the stiffening truss is

$$\frac{d^2\eta}{dx^2} = -\frac{M}{E\,I} \qquad (115)$$

Making use of this relationship, the following equation is obtained from Eq. 112:

$$\eta = \frac{2\,f\,\Delta H}{H\,\mu^2}\left[1 - \frac{(k_2)^2}{(\mu_2)^2+(k_2)^2}\,\frac{\cos\dfrac{2\,\mu_2\,x}{l}}{\cos\mu_2} - \frac{(\mu_2)^2}{(\mu_2)^2+(k_2)^2}\,\frac{\cosh\dfrac{2\,k_2\,x}{l}}{\cosh k_2}\right] \qquad (116a)$$

Eq. 116a holds between the limits of $x = \pm\,\dfrac{l}{2}$. For the side spans in which x varies between $\pm\left(\dfrac{l}{2} + l_1\right)$, the following equation may be derived:

$$\eta = \frac{2\,f\,\Delta H}{H\,\mu^2}\left[1 - \frac{(k_2)^2}{(\mu_2)^2 + (k_2)^2}\,\frac{\cos\mu_2\left(1+\alpha-\dfrac{2\,x}{l}\right)}{\cos\alpha\,\mu_2} - \frac{(\mu_2)^2}{(\mu_2)^2 + (k_2)^2}\,\frac{\cosh k_2\left(1+\alpha-\dfrac{2\,x}{l}\right)}{\cosh\alpha\,k_2}\right] \qquad (116b)$$

in which $\alpha = \dfrac{l_1}{l}$.

To this point the foregoing development follows that of Mr. Rannie quite closely, differing only in that a more exact term for truss stiffness is used by the writers. The writers' development differs further from that by Mr. Rannie in that cable extensibility is taken fully into account.

The extension of the entire cable from anchorage to anchorage, $L - L_0$, can be expressed as $\dfrac{\Delta H}{A\;E}\displaystyle\int_{-0.5\,l-l_1}^{0.5\,l+l_1}\frac{ds^2}{dx}$. The term, $\displaystyle\int\frac{ds^2}{dx}$, is often referred to simply as L_T, and was evaluated by Mr. Steinman in a previous paper.[73] The foregoing relationship may be expressed as

$$L - L_0 = \frac{\Delta H}{A\;E}\,L_T \qquad (117a)$$

With a fair degree of approximation the value of $L - L_0$ can be obtained from the geometry of the cable and stated as follows:

$$L - L_0 \approx \frac{8\,f}{l^2}\int_{-0.5\,l-l_1}^{0.5\,l+l_1}\eta\,dx \qquad (117b)$$

Substituting in Eq. 117b the expressions for η in Eqs. 116, integrating over the entire length of the bridge, and then equating Eqs. 117 results in the

[73] "A Generalized Deflection Theory for Suspension Bridges," by D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1145.

expression:

$$\tan \mu_2 + 2 \tan \alpha \mu_2 = (1 + 2 \alpha) \mu_2 - \left(\frac{\mu_2}{k_2}\right)^3 (\tanh k_2 + 2 \tanh \alpha k_2)$$
$$+ (1 + 2 \alpha) \frac{(\mu_2)^3}{(k_2)^2} - C (\mu_2)^3 \left[\frac{(\mu_2)^2 + (k_2)^2}{k^2}\right] \quad (118)$$

in which $C = \dfrac{L_T H_w l}{16 f^2 A E}$ and μ, μ_2, k, and k_2 have the values indicated in connection with Eqs. 112 and 113.

In the development of Eq. 117*b* the approximation is made that $\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{1.5} = 1$. If this approximation is not used the more rigorous solution gives the same equations as the foregoing except that $L_T = \displaystyle\int \frac{ds^2}{dx}$ is replaced by $\displaystyle\int \frac{ds^3}{dx^2}$, sometimes referred to as L_S. This solution satisfies the requirement that the sum of the kinetic energy, elastic energy, and energy of position must be constant for a given frequency and amplitude. The difference between L_S and L_T, however, is not significant.

If the dead load of the side span varies from that of the main span in the ratio of $w_1/w = \beta$, Eq. 116*b* becomes

$$\eta = 2 f \frac{\Delta H}{H \mu^2} \left[1 - \frac{(k_s)^2}{(\mu_s)^2 + (k_s)^2} \frac{\cos \mu_s \left(1 + \alpha - \dfrac{2x}{l}\right)}{\cos \alpha \mu_s}\right.$$
$$\left. - \frac{(\mu_s)^2}{(\mu_s)^2 + (k_s)^2} \frac{\cosh k_s \left(1 + \alpha - \dfrac{2x}{l}\right)}{\cosh \alpha k_s}\right] \quad (119)$$

in which

$$(k_s)^2 = k^2 \left[\sqrt{\beta \left(\frac{\mu}{k}\right)^2 + \frac{1}{4}} + \frac{1}{2}\right] \quad (120a)$$

and

$$(\mu_s)^2 = k^2 \left[\sqrt{\beta \left(\frac{\mu}{k}\right)^2 + \frac{1}{4}} - \frac{1}{2}\right] \quad (120b)$$

Similarly, Eq. 118 becomes

$$\tan \mu_2 + 2 \phi \beta \left(\frac{\mu_2}{\mu_s}\right)^3 \tan \alpha \mu_s = (1 + 2 \alpha) \mu_2$$
$$- \left(\frac{\mu_2}{k_2}\right)^3 \tanh k_2 - \frac{2 \phi}{\sqrt{\beta}} \left(\frac{\mu_s}{k_2}\right)^3 \tanh \alpha \sqrt{\beta} \frac{\mu_2}{\mu_s} k_2$$
$$+ (1 + 2 \alpha) \frac{(\mu_2)^3}{(k_2)^2} - C (\mu_2)^3 \left[\frac{(\mu_2)^2 + (k_2)^2}{k^2}\right] \quad (121)$$

in which $\phi = \dfrac{2 (\mu_2)^2 + k^2}{2 (\mu_s)^2 + k^2}$.

In Eq. 118, μ_2 must be determined by trial. This may be facilitated by plotting each side of the equation against μ_2 and noting the intersections of the curves that give the successive values of μ_2 which satisfy the equation.

Fig. 19 shows the general forms of these curves. Curves (1) show the left side of the equation, curve (2) shows the right side, and curve (3) shows the right side if the last term (which includes the effect of cable extension) is

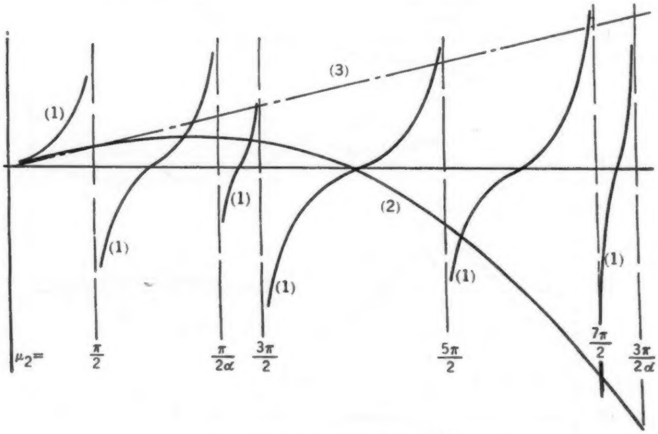

FIG. 19.—SOLUTION OF EQ. 118

omitted. It will be noted that cable extensibility may have a quite marked effect on the frequency of a lower mode but has relatively little effect on that of the higher modes. With or without this factor the intersections for higher values of μ_2 approach odd multiples of $\frac{\pi}{2}$ or of $\frac{\pi}{2\,\alpha}$. It should be noted, furthermore, that C is the only term in Eq. 118 that is affected by the dead load. The expression for C contains H_w and, therefore, is directly proportional to w.

Comparison with Observations.—Tests made on the model of the original Tacoma Bridge indicated that the frequency of oscillation of both the asymmetric and symmetric modes can be predicted with about the same degree of precision by either Mr. Steinman's, Mr. Rannie's, or the writers' equations. The author's equations have the advantage of simplicity, at least for the symmetric modes (n is odd).

When in the supplemental tests unloaded backstays were substituted for the loaded side spans, it was found that, for the asymmetric modes (but not for the symmetric ones), both the author's and the writers' equations had equal validity.

The symmetric modes with unloaded backstays (curves IV, V, and VI, Fig. 20), however, are greatly distorted as compared with the waves on the original bridge (curves I, II, and III, Fig. 20). No fundamental mode could be produced on the model, either by wind action or manually, but two distinct two-node (three-wave) movements developed readily under wind action. One had a very long middle wave and the other a very short one. A four-node

(five-wave) movement of nearly equal wave segments also developed readily. Eq. 15 gives values[73a] for frequency which usually miss the observed values by plus or minus 10% or more. Although the results of the writers' equations do not check observed values as closely as in the case of the original bridge, they

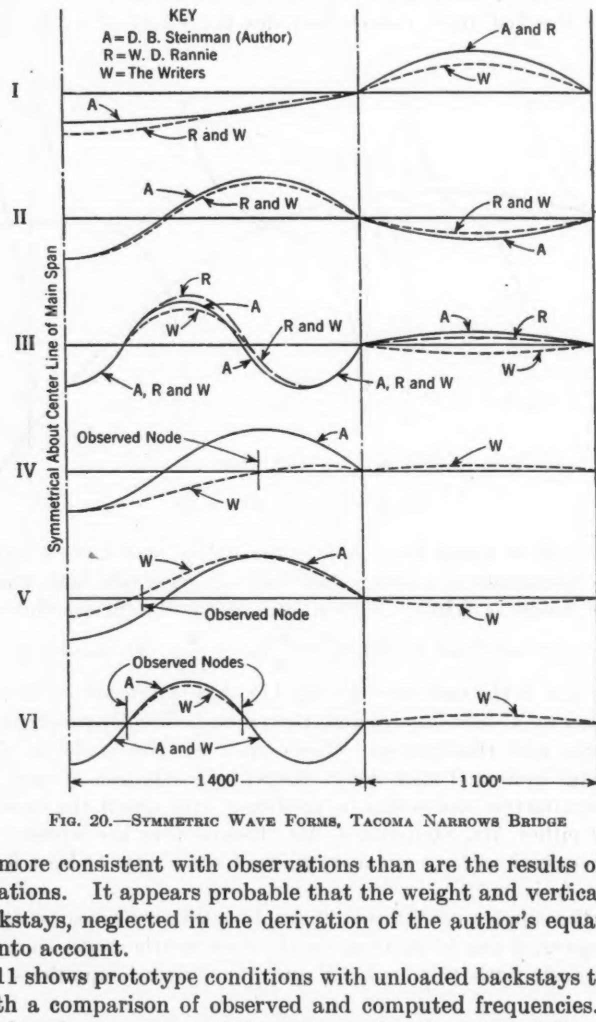

FIG. 20.—SYMMETRIC WAVE FORMS, TACOMA NARROWS BRIDGE

are much more consistent with observations than are the results of Mr. Steinman's equations. It appears probable that the weight and vertical movement of the backstays, neglected in the derivation of the author's equation, should be taken into account.

Table 11 shows prototype conditions with unloaded backstays tested on the model, with a comparison of observed and computed frequencies. The span lengths and main-span sag are the same as for the original Tacoma Bridge. The weight per foot of the main span is varied. The weight of the cable is constant throughout. Blanks in Table 11 indicate modes for which the equation or observation involved gives no value—that is, the equation does not indicate the mode or (in the "observed" column) the mode did not occur.

[73a] See corrections for *Transactions* on page 1084.

The wave forms (as distinct from the frequencies) of the asymmetric modes obtained from the different formulas are substantially similar, both for the original bridge and for the bridge with unloaded backstays.

TABLE 11.—FREQUENCIES (IN CYCLES PER MINUTE) WITH UNLOADED BACKSTAYS

Nodes	n	(a) 2,280 LB PER FT ONE TRUSS			(b) 2,850 LB PER FT ONE TRUSS			(c) 3,770 LB PER FT ONE TRUSS		
		Eq. 15	Eq. 121	Observed	Eq. 15	Eq. 121	Observed	Eq. 15	Eq. 121	Observed
0	1	13.5	....	....	12.3	....	....	10.8	....	....
2	3	12.7	11.5	12.2	12.8	11.2	11.8	12.4	10.6	11.2
2	3	....	15.0	16.0	....	14.8	15.5	....	14.3	14.7
4	5	20.8	22.9	24.5	20.9	23.1	23.0	20.4	22.2	22.4

However, for the symmetric modes (n is an odd number), the various formulas give wave forms that differ significantly, as can be seen by referring to the typical charts in Fig. 20. Curves I, II, and III are typical of the original bridge; and curves IV, V, and VI of the original bridge with unloaded backstays with the dead load per foot on the main span increased 32%.

It will be seen that all the equations are reasonably precise for the conditions in which the observed wave segments are about equal. The differences in amplitude, shown best in condition I, Fig. 20, arise principally from two sources:

(1) The curves in Fig. 20, calculated from Mr. Steinman's equations for the main span and side span, are assumed to be sine waves correlated by the required ratio of their amplitudes, $\frac{a}{a_1} = \frac{2\, l_1}{l\, n_1} \frac{n}{\sec^4 \gamma_1}$, whereas those by Mr. Rannie are not true sine curves but are represented by a modified trigonometric equation similar to Eq. 116a but with the hyperbolic term omitted. Because of Mr. Rannie's basic assumptions the negative and positive areas under his curves must balance. This difference is most evident in the form of the main-span wave (see Fig. 20, curve I).

(2) The curves in Fig. 20 calculated from the writer's equations differ from both of the others because the extension of the cable recognized in the writers' formulas permits a given downward deflection of the main span to take place without being fully balanced by the upward deflection of the side span. The effect of this factor is shown also in condition III, Fig. 20, in which the writers' side-span deflection is actually reversed from the direction required to balance the waves. A more striking example of this effect is found on the model of the proposed new design for the Tacoma Bridge. No vertical oscillations have been produced by wind action on this model but several

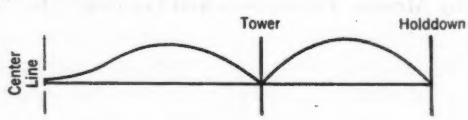

FIG. 21

modes have been produced manually. Among these is one of the form shown in Fig. 21. This is reproduced faithfully by the writers' Eqs. 116. Of course, this form of wave can occur only if the length of the cable changes during oscillation.

In cases where the observed lengths are quite unequal, as in conditions IV and V (unloaded backstays), Fig. 20, the writers' equations give very satisfactory results. It is obvious that Eq. 15, based upon wave segments of equal length, cannot represent these conditions adequately.

Eqs. 21*a* and 21*b* properly relate torsional rigidity to vertical rigidity when the torsional stiffness of the suspended structure is neglected. Investigations to date seem to indicate that the torsional stiffness of the suspended structure as usually designed may be neglected without appreciable error. However, the writers' studies indicate that the effect of torsional stiffness does become significant if top and bottom lateral systems are provided.

The author presents, in simple form, equations which adequately predict a large proportion of the movements of a suspension bridge. However, the writers believe that the equations they have derived following Mr. Rannie's basic method have some advantages over those of the paper because, as has been shown by tests on the model, not only can these equations be used to predict performance with results equal in validity to those obtained from the author's equations in those cases in which the author's equations apply, but can also be used to make reliable predictions in those cases in which the author's equations would lead to erroneous conclusions. However, it seems reasonable to anticipate that the author will be able to modify his equations to give them more general and rigorous application.

Corrections for *Transactions:* In November, 1943, *Proceedings*, on page 1365, Eq. 15, change "$\frac{1}{n}$" to "$\frac{32}{\pi^3}\frac{1}{n^4}$"; on page 1367, in Table 2, change $K = 12.88$ for Manhattan Bridge to $K = 25.76$; on page 1371, in Eq. 24, delete the first minus sign (the second minus sign is correct); on page 1380, in Eq. 47, change "0.024" to "0.045"; in June, 1944, *Proceedings*, on page 1009, in the six subcaptions of Fig. 18, the expression "$V = \frac{1.52}{S}$," etc., should read "$V = 1.52$ ft per sec," etc.; on page 1011, in Table 8, the cut with the eleven diagrams should be rotated 180°; on page 1012, line 20, change "Messrs. Prandtl and Tietjens[47]" to "section 4"; and, on page 1012, line 21, change "shown in tests by Messrs. Theodorsen and Garrick[46]" to "developed in section 2."

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

MILITARY AIRFIELDS
A SYMPOSIUM

Discussion

By D. O. Murray, Thomas B. Pringle, John R. Haswell, W. J. Turnbull and William H. Jervis, Ralph R. Proctor, D. M. Burmister, Ralph Hansen, H. V. Pittman, G. R. Schneider, and H. M. Williams

D. O. Murray,[20] Assoc. M. Am. Soc. C. E.[20a]—The designing engineer will find, in this Symposium, considerable valuable information, presented in convenient form. He should bear in mind, however, that differences in hydrological features of widely separated regions will necessarily modify some of the standards proposed.

For example, Australia lies closer to the equator than the United States and, although snow falls on the tablelands and mountains of the southeastern part of Australia, trouble from frost is rare. For the most part, airdromes are so located that loss of subgrade support due to moisture-content increase is not a probability. Some airdromes have been constructed on poor subgrades. One site, abandoned before the war by the civil authorities, was later used because of the exigencies of war.

In Australia, particular care is taken to avoid conditions that will assist capillary moisture to rise under the surface. This precaution is most essential because of the thin bituminous surfaces typical of Australian airfields. Difficulty in procuring supplies has been a sufficient cause in itself for this procedure. Also, during past years, the various road authorities have paid particular attention to the construction of thin bituminous coats. In prewar days, under favorable circumstances, shire engineers were able to obtain as much as a year's service from a surface that had received only a heavy primer. The

Note.—This Symposium was published in January, 1944, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1944, by James B. Newman, Jr.; March, 1944, by Thomas E. Stanton; April, 1944, by W. E. Howland, and David S. Jenkins; May, 1944, by William E. Rudolph, Raymond L. Irwin, G. G. Greulich, Hibbert Hill, Jacob Feld, and Robert Horonjeff.; and June, 1944, by Clarence S. Jarvis, D. D. McGuire, E. D. Parmer, A. Casagrande, and Harry E. Cotton.

[20] A Prin. Engr., Headquarters Base Section, U. S. A. S. O. S., South West Pacific Area.

[20a] Received by the Secretary May 18, 1944.

same treatment has now been given to a few airdromes, generally those in Class III.

Because of the confidence of designers in Australia that there will be no trouble from moisture in the subgrades, the opinion is gaining favor that the combined thickness of pavement and base can be 80% of the values given by Colonel Stratton in Fig. 3.

Surface treatment requires the use of the heaviest primer that the surface will take. If necessary, trial lengths with different grades of primer may be laid down to determine the heaviest that can be used satisfactorily. The rate of application of primer is limited to a maximum of about 0.25 U. S. gal per sq yd. The binder is usually bitumen fluxed to 85–100 penetration, the rate of application being about 0.3 U. S. gal per sq yd. The surface is then covered with screenings not exceeding a 0.4-in. gage. Only a sufficient quantity is spread so that there will be a minimum of loose material. Some engineers favor the use of tar instead of bitumen for surfacing areas upon which gas or oil may be spilled because of the greater stability of tar in the presence of these oils.

The rainfall intensity curves for rainfall on the eastern coast of Australia conform closely with the curves given by Mr. Hathaway in Fig. 15. Table 12(*a*), containing data presented by G. B. H. Sutherland (49),[20b] indicates the close conformity with the curves shown in Fig. 15 (compare with Table 12(*b*)).

TABLE 12.—COMPARISON OF RAINFALL INTENSITIES

Probability (years)	Curve No. (Fig. 15)	Length of Duration of Storm in Minutes							
		10	20	30	40	50	60	120	240
(*a*) Data Presented by G. B. H. Sutherland									
Maximum recorded	....	10.2	8.3	7.0	6.0	5.3	4.8	3.15	2.05
10	....	6.8	5.3	4.3	3.6	3.15	2.8	1.65	0.95
30	....	6.2	4.8	3.8	3.2	2.75	2.45	1.4	0.75
3	....	5.3	4.0	3.2	2.6	2.3	2.0	1.15	0.6
(*b*) Data Presented by Gail A. Hathaway (See Fig. 15)									
....	2.8	6.6	4.8	4.0	3.7	3.1	2.8	1.7	1.0
....	2.4	6.0	4.5	3.7	3.1	2.8	2.4	1.4	0.9
....	2.0	5.2	3.9	3.1	2.7	2.15	2.0	1.1	0.7

A common practice in municipal drainage design is to adopt the formula,

$$i = \frac{K}{\sqrt{t}} \qquad (13)$$

in which i = intensity of rainfall in inches per hour; t = time of concentration of rainfall (that is, the time taken by the rain water to reach the point under consideration from the extremity of the catchment area); and K = a constant so chosen as to give a curve corresponding to the desired probability.

[20b] Numerals in parentheses, thus: (49), refer to corresponding items in the Bibliography at the end of the Symposium, and at the end of discussion in this issue.

It is a worth-while practice to calculate the value of K for heavy storms or to check with Fig. 15 so as to develop some idea of the relative intensities of various storms. In this way, a better appreciation of the appropriate number for the curve corresponding to a specific storm will be developed. The formula gives low results for storms of short duration and high results for storms of long duration.

Frequently, to drain an airdrome, the neighboring region will have to be serviced by the airdrome drainage system. If the area involves more than about 1,000 acres, the use of time contours will lead to economy in the design of the main drainage ditches. The total area will have to be subdivided according to its separate small catchment areas as determined by roads, taxiways, or natural or other features. Each of these subdivisions will have its own features which will control the design of its drainage. The places reached by the drainage water from each subdivision are marked on the map for equal lapses of time. Usually lapses of 10 or 20 min are suitable—the larger the areas the longer should be the time interval. In this way, it will often be found that drainage from one subdivision has passed before drainage from another subdivision arrives.

The time contours are not constant. They vary with the quantity of water. The quantity of water will govern the depth of flow, and the velocity of the flow will vary with the hydraulic radius of the ditch. The trial computations may be made with either greater or less intensities of design storms. The effects of the time taken for the storm to travel across the full catchment area in various directions may also be considered. Useful information will be obtained as to the possibility of an economical design for a higher design storm.

The use of time contours to allow for ponding is imperative and simple. The total discharge may simply be the addition of the various discharges from the contributing parts of the catchment area.

The need for further research is clearly indicated. However, the latter part of the Symposium dealing with the design of drainage facilities makes available an excellent tool for a diligent application of principles of hydraulics to these drainage problems. Thus, the designer can visit the works he has designed and find the system functioning reasonably well in conformity with his design.

THOMAS B. PRINGLE,[21] M. AM. SOC. C. E.[21a]—During 1941–1942 and 1943 the Corps of Engineers constructed many million square yards of airfield pavements. Now that the major construction program is complete, it becomes increasingly more evident that the problem of developing and maintaining the airfield system will be a tremendous task and that to complete this work will require careful planning. Colonel Stratton has emphasized the importance of the development of airfield pavement design to carry the loads in use since the beginning of World War II and has called attention to the importance of traffic and its effect on pavement service. He has also stated that it is too early to forecast the ultimate service behavior of airfields of recent construction.

[21] Senior Engr., Office, Chf. of Engrs., Washington, D. C.

[21a] Received by the Secretary May 31, 1944.

However, it is not too early to lay the groundwork and begin planning the airfield development of the future. This planning will require the collection of factual data which will be the working tools of the planner or forecaster.

The average lives of various pavement surface types can be estimated by carefully compiling data on every airfield in the United States and assembling these data into orderly statistical form, showing for each surface type the square yardage constructed each year and the yardage remaining at the end of the year. The various state highway departments in cooperation with the Public Roads Administration have made studies, known as "Road Life Studies," which have given very reliable data on average pavement lives of various surface types for the nation as a whole, for regions, and even for individual states (50). Although there is no reason to believe that the pavement life of airfields will be the same as that of highways, the procedure for estimating the life will be identical and the statistical records that must be compiled before any comprehensive study can be made will be similar. The method of design must be substantiated by the statistical records and known service behavior of pavements. All these data can be collected only through the orderly compilation of existing information, such as pavement condition, maintenance costs, and reconstruction. These statistics, when studied with traffic records from the time of construction to the death or retirement of the pavement, give factual data for the future development of airfields and for setting up maintenance and reconstruction programs, and they form a basis for handling many complex problems of airfield administration. The actual number of load repetitions is far more significant in the case of airfields than in the case of highways, since the highways must be designed for an unlimited number of load repetitions, whereas on airfields (even those used for capacity operation) the number of repetitions is much less than on heavy traffic highways. From the standpoint of economy it will be necessary that airfield designs be predicated on anticipated life, estimated traffic, and average maintenance costs. Anticipated life can be estimated with reasonable accuracy from the collection of necessary data concerning pavements and the maintenance of reliable records of service, installations, retirements, and inventories. Retirement data and survivor curves, similar to those developed by life insurance companies, will be the basis of determining the probable life of the pavement. This information, if used to guide the expert judgment of men who know pavements and their maintenance problems, should produce a reasonable answer. Purely statistical data will not give the correct solution to the individual problem. However, these data can be of inestimable value to the experienced paving engineer in determining construction and maintenance programs.

Data on traffic volume and character will be required. These data should include the number and distribution by weight of airplanes using the field; and, for planning purposes of peacetime, airfields should predict the distribution for the hours of the day. These data will show with reasonable accuracy what facilities are required to handle present and future traffic requirements and

will help establish priorities for improvement and predict maintenance requirements for the various pavements at an airfield.

About 1915 highway departments thought in terms of new construction and the day when their system would be complete. These systems have been completed to a considerable extent, and their maintenance is as big or an even bigger item than the original construction. The same problems of increased maintenance will apply to airfields as years pass, together with the problems of reconstruction due to obsolescence and deterioration.

It is vitally necessary that an effective planning organization be established to collect the necessary data on service behavior of pavements, including traffic records by type and character, which should contain not only design weights of airplanes but data on actual pavement loading and the maintenance costs for various pavement designs and surface types. The present and future needs of each installation must be studied carefully in order that development can proceed according to the most feasible and economical plan that will utilize existing facilities to the greatest extent and require the least new construction.

Airfield planning and development as discussed herein are mainly problems that will come with peace and the development of the airplane for army and civilian peacetime needs. However, it is important that these questions are not completely overlooked, even during the stress of wartime. The Corps of Engineers has undertaken a study known as an "Evaluation Survey" which will form a nucleus of data that will be invaluable not only in formulating and proving pavement designs but also in planning efficient layouts and in guiding maintenance programs, and especially in planning airfield development for the future. The results of these evaluations must necessarily be held confidential until after the war emergency. Since the evaluation survey is being conducted only at airports of military occupancy, it is, therefore, not considered desirable to disseminate this information until after it ceases to be of military importance.

Colonel Stratton has most ably discussed the design method employed by the Corps of Engineers for airfield pavement construction. The method of design as now employed by the department was not in use during the early construction period and knowledge of the effect of the heavy wheel loads to which pavements are now subjected was unknown from actual experience. Unfortunately, some of the pavement in the runways, taxiways, and aprons of the hundreds of airfields constructed by the Corps of Engineers and other agencies were not built according to the designs discussed in Colonel Stratton's paper.

When the tremendous construction program was almost complete, it became obvious that many airfields had been constructed whose actual carrying capacity was unknown. For the Army Air Forces to set up a training program which would utilize all airfields most efficiently and economically and insure the least interruption, due to failure of pavements under heavy loads, it was necessary to determine the safe carrying capacity of each field. The increasing weights of airplanes and the necessity for designating fields to be used by certain type of aircraft made it essential to obtain a record of the evaluation of each airfield based on the carrying capacities of the pavements as actually constructed.

The field work investigation and preparation of reports are being done under the supervision and direction of the U. S. Engineer Department. For the purpose of this evaluation two kinds of operation are considered—namely, capacity and limited:

1. Capacity Operation.—Capacity operation is defined as the maximum traffic that can possibly operate on an airfield for a period of approximately 20 years. Daily operation may be assumed as varying from 100 wheel loadings for very heavy airplanes to 1,500 wheel loadings for very light airplanes.

2. Limited Operation.—Limited operation is defined as a few wheel loadings a day for a period of approximately 20 years (about 10% capacity operation) or as the maximum traffic that can possibly operate for a period of 2 to 4 years. However, the use of a pavement rated for limited operation by the maximum traffic that can possibly be operated for a period of 2 to 4 years may require greater yearly maintenance than would an airfield rated for capacity operation.

The over-all evaluation as made is for the airfield as a whole and the runway system as a whole. These evaluations are termed field and runway evaluation:

(*a*) Field Evaluation.—Each airfield is given an over-all field evaluation based on the carrying capacity of the pavements or sections of pavements. The field evaluation is considered as the gross plane weight that may be operated at the field for capacity operation or limited operation without any hindrance due to excessive maintenance or reconstruction of any major pavement.

(*b*) Runway Evaluation.—Runways are evaluated separately from the field as a whole. The mechanics of evaluation are the same for the runway pavements as for field evaluation. Runway evaluation specifies the gross weight of plane that may be operated at limited and capacity operation on any runway without hindrance due to excessive maintenance or reconstruction except possibly at the end of runways.

The evaluation of each pavement is studied carefully, using all available information, and is determined either by actual tests or by estimation of the carrying capacity from design and construction data or other information. This leads to a unit evaluation of each and every pavement.

The evaluation is based on the design criteria presented in Colonel Stratton's paper. Instead of making tests of soil and construction materials, determining the results expected, and thereby determining the design requirements, the evaluation tests the in-place values and from the in-place values assigns the carrying capacity of the pavements. The principles of evaluation are the same as those for design except that they are applied in reverse order. For the evaluation of flexible pavements, the in-place CBR values of subgrade and base courses are determined. This eliminates certain factors that must be considered in design, such as construction control and the possibility that the materials in the prototype will not conform with design values. Similar procedure applies to rigid pavements. The actual *k*-value or modulus of soil reaction and the flexural strength of concrete in place are determined. Consideration is given to possible frost action in connection with the evaluation of

both flexible and rigid pavements and load carrying capacity is reduced because of probable frost action where justified.

The evaluation will be the beginning, and the basis, of service behavior data on airfield pavements. All investigations will be conducted under uniform procedure. The in-place or as-built pavements can be watched through their life, and from the known service behavior a definite relation can be established between theoretical design, as now advocated by the Corps of Engineers and described by Colonel Stratton, and the actual pavement performance. The present empirical curves developed from the results obtained from traffic tests, covering the entire United States, can be verified and corrected, if necessary, from the record of actual service behavior of many pavements constructed under all conditions with a variety of construction materials.

JOHN R. HASWELL,[22] M. AM. SOC. C. E.[22a]—Under the heading, "Drainage of Airfields: Subsurface Drainage Systems," Colonel Stratton aptly states that under certain conditions, with a properly designed surface drainage system, no subsurface drainage is necessary. However, the ground-water table is not fixed and a site that may appear satisfactory in a dry year, or even over a cycle of several years of low precipitation, may later develop excess moisture. Therefore, a single line of drains on each side of a runway pavement somewhat as shown in Fig. 8(*b*) would be a valuable precaution in many instances. There will be some seepage through the pavement that should be considered and provisions should be made for its removal. In heavy soils a "perched" water table may develop in the upper few feet of soil when the real ground-water table is somewhat deeper.

The second type of subsurface system likened to "a farm tile drainage system" prompts this discussion. For many years agricultural engineers have been working away from the old costly uniform grid and endeavoring to put the most drains where the wettest condition is encountered in the subsoil. On a reasonably flat field one would not look for seepage to be intercepted by cross drains, but a slope may occur in the subsoil strata or springs may come up almost vertically through a tough clay. Gravel layers or lenses may be tapped and may serve as laterals in some favored places.

The general subject of planning and constructing agricultural drainage systems in the United States is briefly covered in "Drainage in the Humid Regions" (51) and many state agricultural colleges have publications covering local land drainage probems.

The maintenance, where possible, of the old agricultural drainage, as part of the airfield development, has been treated by H. H. Nicholson (52). After quoting the Ninth Report of the Select Committee on National Expenditures dealing with Air Services, Mr. Nicholson writes:

> "It is to be noted that soil and drainage are mentioned last of the factors to be considered in choosing a site, but there can be no doubt that these two points exert a telling influence on the operational efficiency of an aerodrome.

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[22a] Received by the Secretary May 31, 1944.

"Some of the special difficulties and possible sources of trouble in this sphere of drainage work can easily be envisaged. The need for an even surface of considerable extent means that pre-existing ditches must be piped and filled in; ridges and furrows must be obliterated; excrescences must be removed and hollows filled in. Existing land drains may or may not be broken in the process, especially where buildings are erected or runways laid down. Whatever is done by way of such operations, surplus water will continue to move along the tracks which it has followed in the past, and wherever the drains are broken or the underground drift of the water is interrupted, there will trouble arise, unless adequate measures are taken to forestall it."

The necessity for placing "natural earth backfill" as shown in Fig. 8(*b*), or "impervious bedding" as shown in Fig. 37, under the tile drain may be questionable. There are trenching machines of the digging wheel type, capable of making a satisfactorily graded ditch and leaving the bottom slightly curved to receive the tile. If the trench is dug in a haphazard manner as to grade, there is ample chance for enough variation in depth of fill, in tamping, and in kinds of soil, to cause sufficient differences in settling to ruin the grade. For most airfields, the placing of drain tile on the undisturbed subsoil, properly prepared to receive it, would seem far safer than the placing of drain tile on an uncertain backfill.

The use of "impervious bedding" would result in the loss of at least one half the diameter of the drain tile in the effective depth of the drains as related to the soil below the pervious fill. Surely some water may be expected to seep into the subsoil from the base course. The movement of such water is downward in the soil to about the tile level and then horizontal to the drain, rising up in the open joints and entering almost entirely at the under side. It is to be expected that in some cases more water will enter the drains shown in Fig. 8(*b*) through the bottom of the open joints than through the perforations on the upper side of the tile. The same practice persists in some highway standards where the engineer seems to consider the soil mass as so much impervious material and not as a structure with pores and various water-holding capacities.

W. J. Turnbull,[23] M. Am. Soc. C. E., and William H. Jervis,[24] Assoc. M. Am. Soc. C. E.[24a]—The problems encountered at the beginning of the war construction program undertaken by the U. S. Engineer Department are covered in this Symposium. Colonel Stratton and Mr. Hathaway are to be congratulated for their contributions. Rigid adherence to schedules set up for completion of the various airfields to fit in with military necessity made these problems much more critical than they would have been in normal times. The solutions described in the Symposium were remarkable, especially considering the speed with which they were made.

Since 1942 the California Bearing Ratio (CBR) as a tool for the design of flexible pavements has been the object of much discussion by many engineers

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[24a] Received by the Secretary June 1, 1944.

connected with the design of airfields. The discussion of the CBR test has been about equally divided between favorable and critical comment. Those engineers who support the use of the CBR test as a design tool recognize the fact that it is comparatively new with the Engineer Department. However, these engineers feel that it is better than any previous method and further that at the present time there is no alternate procedure which is as good. The engineers who are critical of the test think that it is something new and untried and that the background of previous experience is a better criterion for designing and building military airfields.

The CBR test measures the shear strength of a soil and is not a classification test. Since the strength of any soil varies widely under different moisture and density conditions and often varies widely between the undisturbed and remolded states, any shear test, whether a CBR, triaxial, an unconfined compression, or a direct shear test, must be divided into two parts—the preparation of sample for test and the actual test. The secret of proper performance of any shear test depends largely on the proper preparation of the specimen so as to reproduce faithfully conditions in the field. If this is not done, the test results will be misleading and may lead to either underdesign or overdesign. With this in mind, the Engineer Department, through the Mississippi River Commission, U. S. Waterways Experiment Station, in Vicksburg, Miss., has been conducting an extensive investigation of the CBR test technique involved both in preparation of the specimen and in performance of the test. This investigation has been supplemented by studies from numerous other laboratories in the Engineer Department. As a result of these comprehensive studies, the CBR test technique has been developed to such an extent that reasonably accurate and uniform results can be obtained by the ordinary testing laboratory.

Much of the adverse criticism originally directed against the CBR test as a whole was due, not to difficulties entailed by performance of the test, but to lack of knowledge of the effects on the test of certain variables introduced in preparation of test specimens. At the beginning of the investigation it was thought that the variables with the most effect on the CBR would be density, water content, and soil type. Later in the investigation it was learned that other variables existed, the principal one being water content at the time the sample is molded for test. The indications have been that this water content is extremely critical for the soaked CBR values determined on all materials except cohesionless soils and high swelling clays.

Several changes in the test technique as a result of the Engineer Department investigation have occurred since Colonel Stratton prepared his paper. As previously mentioned, many variables involved in the CBR test now seem evident and may be controlled and adjusted to such an extent that the CBR test can be used practically for design purposes. As with any other testing procedure, refinements and variations will be made as further studies in the laboratory and the field become available.

In connection with the development and continuation of the use of the CBR test, the laboratory design test and actual field behavior should be correlated. In other words, it is not known whether the structure of the soil specimen produced in the laboratory is similar to the structure produced in the field

by the field compaction equipment generally in use. For a complete and intelligent use of the CBR test, it will be necessary to correlate laboratory and field results. This may be accomplished by either of the following two methods:

(1) Observations and laboratory and field investigations of airports which have been constructed; and
(2) A closely controlled test of field versus laboratory compaction supplemented by actual field observations at existing airports.

Either method should be equally good with the exception that the first one will take considerably longer and its success will depend to great extent on the available field records of water content and density at the time of construction.

As of 1944, a method of conducting field in-place CBR tests has been developed which appears quite satisfactory and which agrees reasonably well with laboratory tests on remolded and undisturbed samples when moisture, density, and soil structure effects are considered. The field in-place test has many advantages because it can be used directly for evaluation purposes without the necessity of considering the many variables which exist in the test on remolded samples for use in design. The main criterion of the field in-place test is that the engineer, using or making the test, must have knowledge of the condition of the subgrade or base-course material with respect to the probable ultimate condition of moisture and density which will exist over the period of the life of the airfield.

Under the heading, "Base Courses," Colonel Stratton states that "Base-course materials sometimes are stabilized with commercial admixtures such as portland cement, cut-back asphalts, emulsified asphalts, and tars." The stabilization of base courses by means of various commercial admixtures should be approached cautiously since it has been proved in several instances that essentially stable base-course materials have been weakened by the use of admixtures. The CBR test is an excellent tool for detecting any increase or reduction of strength in the natural material when admixed with any so-called stabilizing material.

Because improper drainage often may have been responsible for the failure of military airfields, the necessity for surface and subsurface drainage systems at airports cannot be overemphasized, birdbaths on the surface of runways, taxiways, and aprons, as a result of the combination of low transverse grades and consolidation of the base courses and subgrades, are believed to be extremely detrimental, since, if any cracks exist in the surface pavement, water will enter and a localized weakened condition in the base course and subgrade will result. Nonplastic base courses, compacted to a density equal to that obtained at optimum moisture by the AASHO Method, as modified by the Engineer Department, to a great extent, will eliminate detrimental consolidation.

The construction of military airfields has been a huge program and, due to the war emergency, has been completed in record time. In spite of the difficulties inherent in so hurried a program, the design methods which were adopted

should in general prove themselves adequate. The CBR test, even though admittedly a new and unperfected tool, has contributed materially to the success of the military airfield program.

RALPH R. PROCTOR,[25] M. AM. SOC. C. E.[25a]—The description, by Colonel Stratton, of the failure of highway design and construction methods to meet the demands for increased subgrade loadings required for military landing fields is most interesting and has prompted this discussion.

Relationship Between Soil Loadings in Earth-Fill Dams and Landing Fields.—More use should be made of earth-fill dam design and construction methods in landing field construction; the soils in the foundation and lower part of a high earth-fill dam are subjected to greater pressures than any mentioned by Colonel Stratton and must withstand these pressures (about 1 lb per sq in. for each foot of height of the dam), without undue settlement or distortion. This condition of loading requires soil stability, secured from the coefficient of friction and cohesion, under greater loadings than those contemplated by Colonel Stratton.

Uncertainty of Plate Bearing Test Results.—Plate bearing tests involve elements of uncertainty. Three separate soil conditions may be involved; the soil may be saturated at the start of the test, it may become saturated during the test, or it may not become saturated at all during the test. It is believed that no formula for evaluating the relationship of tests under these conditions can be developed for the three following reasons:

(*a*) A fully saturated soil has a modulus of elasticity determined by the soil and water during the time the water is being forced out of the soil by the applied load;

(*b*) An unsaturated soil is permanently deformed by consolidation, even under loadings of 5 lb per sq in., and develops some elasticity at the end of the consolidation period, but consolidates further upon application of a greater loading; and

(*c*) A soil that becomes saturated by consolidation passes through both condition (*a*) and condition (*b*) with an additional influence of compressed air in the voids of the soil when under loading.

These unpredictable variations in soil characteristics (conditions (*a*), (*b*), and (*c*)) led the writer to the conclusion that the best way to avoid such complications in earth-fill dam construction is to compact the soils used to dry weights and moisture contents that would permit full soil consolidation without involving saturation, with consequent reduction of the effective coefficient of friction from internal pressure.

Prediction of Soil Consolidation by Use of the Indicated Saturated Penetration Resistance.—Fig. 40 shows the mean measured soil consolidation, under various loadings, that was secured from 859 laboratory tests of compacted soils of practically all soil types. The solid lines represent the range of the tests and the dotted lines represent interpolated values. The consolidation of broken

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[25a] Received by the Secretary June 5, 1944.

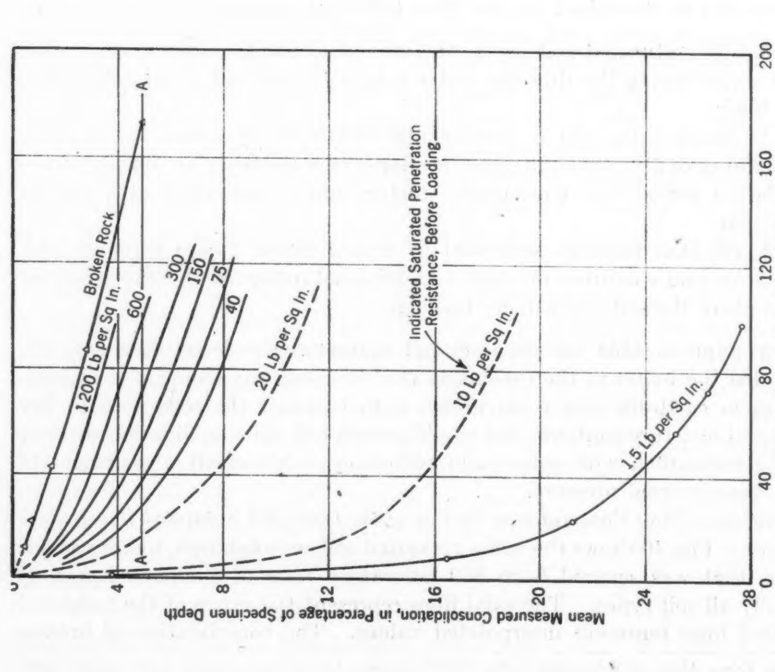

FIG. 40.—MEAN MEASURED CONSOLIDATION OF 859 SOILS WHEN PLACED AT VARIOUS INDICATED SATURATED PENETRATION RESISTANCES AND CONSOLIDATED UNDER VARIOUS LOADINGS

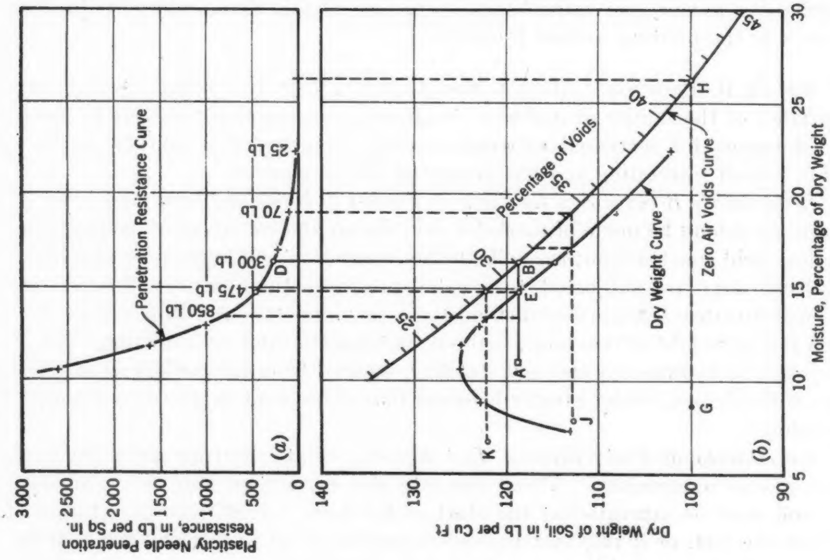

FIG. 41.—TYPICAL COMPACTION-PENETRATION CURVES

granite, from 1.5 in. to 2.5 in. in size, when hand placed in a layer 14 in. in diameter by 7 in. deep, is also shown; the compressive strength of the rock was 14,950 lb per sq in. It will be noted that the amount of soil consolidation to be expected from a given loading is shown in terms of the "indicated saturated penetration resistance" of the soil. Fig. 41 shows the method of determining this value. Point A shows an assumed moisture content and dry weight of the soil whose characteristics are shown by the dry weight and penetration resistance curves. Point B shows the moisture content at which the soil would become saturated at the dry weight shown at point A. Point D shows the indicated saturated penetration resistance of the soil at a dry weight represented by line AE. The indicated saturated penetration resistance is seen thus to be dependent on the relative dry weight of the soil.

Prevention of Internal Pressure Caused by Soil Consolidation.—This method of evaluating the relative density of a soil as compacted or as found in place has been used by the writer for the last 12 years. Fig. 40 has been in use (with revisions from time to time as more test results became available) since 1936 for predicting the consolidation of soils in earth-fill dams in order to provide room for soil consolidation, by proper control of the compacted soil dry weight and moisture content, without the consolidation causing the soil to become saturated. If the soil represented by Fig. 41 is compacted to the dry weight and moisture content represented by point E, subsequent consolidation, without any change in moisture content, will be represented by a vertical line in Fig. 41 from point E. When the dry weight of the soil represented by point E is increased by 3.5 lb per cu ft, complete saturation is reached. Fig. 40 indicates a required depth of fill of about 30 ft to cause this consolidation. However, this consolidation also involves compression of entrapped air in the soil. Therefore, for a 30-ft depth of fill loading, or 30 lb per sq in., it is better to compact this soil, in the case of the dry weight shown at point E, at a moisture content not higher than 13.5%, thus allowing for the aforementioned consolidation to take place with a resulting soil dry weight and moisture content that falls on the dry weight curve. If a soil becomes saturated by consolidation, part of the applied load is carried by water pressure, reducing the effective coefficient of friction of the soil. The writer's opinion is that no appreciable internal pressure will occur in the soil at this and lesser moisture contents when the consolidated dry weight and moisture content are not to the right of the right-hand side of the soil dry weight curve.

Variations in Density of Natural Soils.—Points G, J, and K of Fig. 41 show the variations in soil density of a particular soil encountered during the foundation excavation for a dam. The comparative consolidation to be expected from these soils can be found by noting the indicated saturated penetration resistance of each and comparing the consolidations shown in Fig. 40 at these resistances for some particular soil loading. Point G is a commonly found relative soil density and consequent saturated penetration resistance for soils one to two feet below the original ground surface. This indicates the need for extreme care in using natural soils for subgrade purposes.

Miscellaneous Notes on Soil Testing Methods.—The modified CBR test described by Colonel Stratton has one serious defect—that is, the establishment

of an "optimum" soil density as that presumably secured by dropping a 10-lb, tamper eighteen inches. The writer has not found the use of an "optimum" soil density, thus secured, to be reliable as a standard for soil compaction. However, this should be a considerable improvement over the standard CBR method, as a study of the foregoing discussion of Figs. 40 and 41 shows that the consolidation secured from tests after compressing a soil under 2,000 lb per sq in. would have no connection with the consolidation when the soil was compacted to lesser dry weights. A better method for performing this test would be to compact and test the soil at about three different densities, representing the range of variation in moisture content in a well-controlled compacted fill; that is, at penetration resistances of 500, 1,500, and 4,000 lb per sq in. penetration resistance. The lower value is selected in order that the effect of compacting while the soil is too wet may be determined. There is one important point in experimental soil compaction that is overlooked generally. Whenever soil is compacted the soil particles break down; the percentage of fines is increased, evidently by erosion of the larger sized particles. The experimental compaction method should not alter the soil characteristics in this respect to a greater extent than occurs during the actual compaction by the rollers. Repeated compaction of soil samples can change their characteristics enough to make the tests worthless. Another rather common practice in preparing compaction-penetration resistance curves should be avoided; that is, an entire soil sample should never be dried in an oven and then tested by adding various amounts of water to secure predetermined moisture contents. These curves are best prepared by dividing the soil sample in two parts at about 800 lb per sq in. penetration resistance and then progressively drying and compacting to secure the left half of the curves, and progressively wetting and compacting to secure the right half of the curves.

If a plate bearing test is made of this soil at the density and moisture content shown at point G, the resistance to penetration by the plate should increase as the penetrating load is increased, in accordance with the soil consolidation curves of Fig. 40, until the applied pressure becomes great enough to start actual penetration, at which time the deformation curve turns in the opposite direction, following a path similar to that shown in Fig. 4. However, if this soil is tested similarly when in a nearly saturated condition, most of the test load is carried almost immediately by water pressure and a resistance curve similar to Fig. 4 is secured. Fig. 5 indicates that the bearing plate deformation remains constant for a given unit pressure loading for plates larger than 30 in. This indicates that the laws of soil consolidation, rather than of penetration, should apply or that, during the tests that established this ratio, there was not sufficient depth of soil below the plates to determine the true relationship of load to deformation. The writer would expect to secure results similar to those of Fig. 5 by testing a 4-ft thick layer of compacted soil placed on a hard surface.

Fig. 40 shows the soil consolidation that may be expected from a confined soil; it is believed that the results shown in Fig. 40 more nearly portray the action to be expected in a subgrade from either compacted or natural soils than do the results of penetration tests, the plate bearing test is essentially a

large penetration needle test. The results secured in Fig. 40 occurred over a period of about two weeks; however, an average of 75% of the consolidation occurred within a period of 5 min. The load repetition test merely determines the time required for a given load to consolidate the soil below the pavement sufficiently to cause failure by excessive slab deflection. For instance, if the duration of an aeroplane wheel loading is 0.1 sec, 75% of the soil consolidation shown in Fig. 40 should occur after 3,000 trips, if it is not necessary to force water from the soil to permit this consolidation.

Both Fig. 5 and Fig. 40 show an increased rate of subgrade resistance with increased deformation; this appears to indicate a serious error in any method of computing slab thickness that is based on the assumption of constant subgrade reaction.

Application of Laboratory Methods to Construction.—It appears that the use of the determined indicated saturated penetration resistances of compacted or natural soils, assembled from compaction-penetration curves made from soil samples taken from frequent field soil density tests as the subgrade is being constructed, should serve as a better guide to the bearing power of a subgrade than plate bearing tests. This is particularly true if the relationship between the indicated saturated penetration resistance of soils and bearing power of soils for slabs is determined by trial loading of test slabs. This testing procedure during actual construction may be illustrated best in connection with Fig. 42, which shows typical soil compaction results secured during one month's work on an earth-fill dam project involving 7,000,000 cu yd of compacted earth fill; 374,000 cu yd were compacted during this month, equivalent to two 7,000-ft runways, 200 ft wide, with 3 ft of compacted fill below the subgrade. A constant check of the addition of water to the soil, both at the dam and borrow pits, was maintained by use of the compaction cylinder and needle. A total of 238 compacted fill density and penetration resistance measurements was made and the results compared with results secured by immediate hand compaction of the removed soils. The mean of these results is shown in Fig. 42 as: Mean cylinder needle, 2,198; mean fill needle, 1,880; mean cylinder, 122.7; and mean fill, 120.7. The minimum compacted penetration resistance was 700 lb per sq in.; the maximum was 3,900 lb per sq in. From the 238 soil density measurements, 69 representative soil samples were selected for preparation of compaction-penetration resistance curves; 25 of the 69 specimens selected for test were taken from the area showing the lowest soil density secured for each day's work. The determination of the indicated saturated penetration resistance of the fill was made from these curves, 28 samples being selected from the 69 samples for which compaction-penetration resistance curves were prepared for consolidation-percolation tests. A corresponding number (28) of plate bearing tests to the consolidation tests would place one every 500 ft along the runways. The soil density and compaction-penetration resistance tests (69) would be spaced apart 200 ft and the compacted density hand-compaction comparison tests (238) would be spaced apart 60 ft. As all of these measurements and tests have been found necessary to control soil compaction in dams, they should be required for similar compaction control in subgrades and should also yield the data for determining pavement thickness as they are obviously

much more representative of actual soil conditions than plate bearing tests ever could be. The variations found in actual measured consolidations of apparently similar samples of the same soil show that the mean of a large number of soil tests is more to be relied upon than that of a few tests. A soil

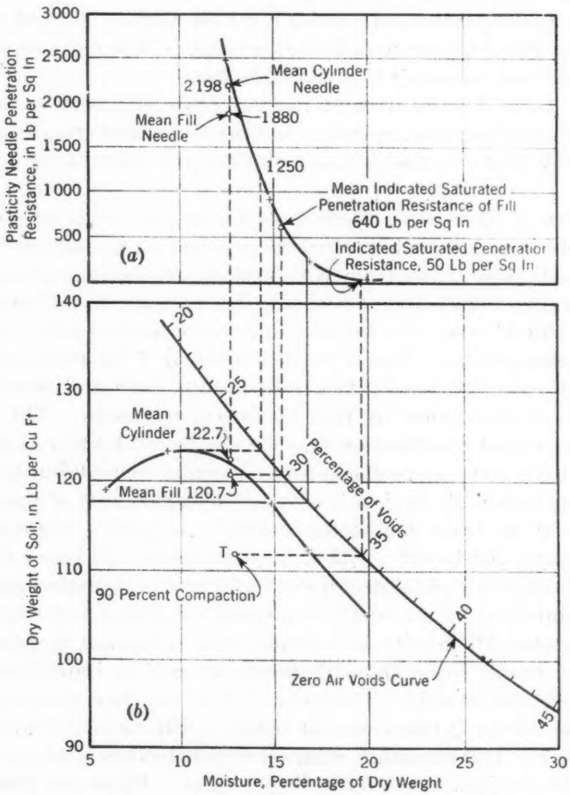

FIG. 42.—A REPRESENTATIVE COMPACTION-PENETRATION RESISTANCE CURVE FOR 374,000 CU YD COMPACTED EARTH FILL, SHOWING THE MEAN VALUES SECURED FROM 238 COMPACTED FILL DENSITY TESTS AND HAND-COMPACTION TESTS

density and compaction-penetration resistance test costs six man-hours. A consolidation test costs six additional man-hours. It is obvious that dozens of these tests could be made for the cost of a plate bearing test; furthermore, no heavy equipment need be provided, with consequent high hourly charges for its use. Inspection of the rates of consolidation of soil samples indicates that at least 24 hr should be allowed for each added increment of soil loading for a plate bearing test and, if five increments are added, a total time of six days would be required for each test, involving at least $1,500 of equipment rental and being equivalent in cost to about 100 consolidation tests. The aforementioned 24-hr increment assumption applies to soils that do not become

saturated by consolidation; several weeks might be required for an accurate plate bearing test on dense, clayey soils.

Control of Expansion of Compacted Soils When Saturated.—The foregoing condition of compaction (see Fig. 42) has not, in fourteen years of experience of compacting soils by this method, resulted in a dry enough soil to permit subsequent expansion from the absorption of water. Conditions represented by point A of Fig. 41 may possibly permit such expansion; those represented by points G, J, and K of Fig. 41 are almost certain to permit expansion of lightly loaded clay soils.

Reduced Bearing Power from 90% Compaction.—It is interesting to note the comparison of the results shown with the "90% compaction" standard. The relative consolidation under a given loading to be expected from the mean indicated saturated penetration resistance secured of 640 lb per sq in. may be compared with the 50 lb per sq in. indicated for the 90% compaction point T, Fig. 42. Incidentally, the degree of compaction shown in Fig. 42 is higher than is usually secured by the many variations of hand-compaction methods now in use. It has been the writer's practice for years to require hand compaction that secures a maximum dry weight of soil at about 2,500 lb per sq in. penetration resistance. All of the various "modifications" of the originally published methods for hand compaction (53), such as dropping various weight tampers from different heights, mechanical tampers, etc., were tried and discarded as too cumbersome and not worth while several years ago. No use is made of the actual peak dry weight; the measure of soil compaction used is the indicated saturated penetration resistance. Referring to Fig. 40, the consolidation under a loading of 10 lb per sq in. would be 2.9% for the 90% compaction and 1.3% for the mean compacted fill density shown. Also, referring to line AA of Fig. 40, it will be seen that a loading of 172 lb per sq in. is required to secure a consolidation of 4.9% for the broken rock, 110 lb per sq in. is required for the mean of the compacted fill, and 32 lb per sq in. is required for the 90% compaction. This indicates that, if the loading required to cause a given deflection of a bearing plate is to be used as an index of the bearing power of this soil, the bearing powers of the mean compacted fill (dry weight, 120.7 lb per cu ft) and the 90% compaction (dry weight, 112 lb per cu ft) will be 61% and 19%, respectively, of that indicated for the broken rock. Assuming the broken rock bearing power to correspond to the k of 800 lb per sq in. per in. of Fig. 1, the corresponding values of k for the mean compacted fill and 90% compaction will be represented in Fig. 1 by 490 and 152 lb per sq in. per in., respectively. Applying these values along the 75,000-lb wheel loading curve of Fig. 1(a) the following concrete thicknesses are secured: Broken rock, 5.5 in.; mean compacted fill, 7.2 in.; and 90% compaction, 9.7 in. It is thus seen that, for whatever wheel loading the 75,000-lb curve might represent under these assumptions, the mean compacted fill would require 2.5 in. less concrete than the 90% compaction when used in a landing field subgrade.

Bearing Power of Natural Soils.—The results of plate bearing tests on natural soils cannot be used safely because the process of preparing the subgrade and placing the concrete is almost certain to disturb or remold such soils enough to affect materially their bearing power in their natural state. Con-

crete, cement, or aggregate trucks in particular are almost certain to have this effect. This remolding of natural soils can also occur after the pavement is placed, particularly in the case of bituminous pavements or where concrete pavements become cracked or broken. It is the writer's opinion that the values for soil consolidation secured from Fig. 40 by use of the indicated rather than the actual saturated penetration resistance are the more reliable for natural soils.

Preparation of Compacted Fill Subgrade for Paving.—There are two practical points of construction that appear to have been overlooked in subgrade construction. One is that the soil should be compacted to at least a foot above the finished subgrade and then trimmed carefully to grade immediately ahead of the placing of the pavement. This is probably the most important point contained in this discussion. It has been found repeatedly in dam construction that soil density tests must be taken from a greater depth than 10 in. from the surface of compacted fills in order to secure a true check of the density of the compacted fill; samples taken closer to the surface show lower soil density in most cases. In pouring the concrete, placing machines have a tendency to leave large areas of rock pockets next to the subgrade. A half inch of mortar brushed on the freshly trimmed subgrade, from which all loose soil has been removed, just ahead of the pavement placing will increase greatly the contact area between concrete or bituminous pavement and subgrade. One landing field came to the writer's attention at which the sheepsfoot fill was of fair order but the loose material remaining after the sheepsfoot rolling was finished was compacted by a smooth-drum roller. The indicated saturated penetration resistance of this part of the fill was 50 lb per sq in. The actual penetration resistance at the time of construction was much higher; the subgrade appeared to be firm and hard. After the first good rain an aeroplane with a tire pressure of 60 lb per sq in. penetrated the 4-in. oiled macadam surfacing and the subgrade until the sheepsfoot compacted fill was reached, just as had been predicted from the soil density test made at the time of construction. It is almost certain that no trouble would have been experienced if the sheepsfoot fill had been trimmed as described.

The use of at least 3 ft of compacted fill subgrade below the trimmed surface, unless the natural soil has an indicated saturated penetration resistance of at least 400 lb per sq in., will eliminate much of the uncertainty of the bearing power of subgrades as the subgrade will at least be uniform in bearing power if the work is done properly. The cost of this work should not be more than for 1.5 in. of extra concrete pavement thickness.

Design and Use of Sheepsfoot Roller.—All of the elements of roller design found necessary to secure soil compaction results comparable to Fig. 42 may be combined in one machine. The materials used successfully for compacted fills in earth-fill dams include decomposed granite and diorite, sandstones and shales soft enough to be dug with a rooter, glacial moraines, alluvial soils, volcanic ash, glacial clay, and heavy California adobes. The roller consists of three drums, universally mounted abreast, 5 ft long outside dimension, by 42 in. inside diameter and fitted with six circumferential rows of thirteen teeth each, arranged so that no more than two teeth may bear on a plane surface at one time, when directly below the axis of rotation of the drum. The teeth are

8.5 in. long and are fitted with removable shoes having areas of 6 sq in., 7 sq in., 8 sq in., and 10 sq in. The area used is dependent on the material, the choice of area being made to secure a final penetration of 3 in. to 5 in. after sixteen roller trips have been made. The weight, including frame, on each drum should be 10,500 lb, with provision for adding water in the drums to increase the weight to 13,000 lb per roller drum, or 39,000 lb for the entire unit. If required, water and sand can be placed in the roller drums, increasing the weight per drum to 15,500 lb. This roller, at 39,000 lb total weight, can be handled economically by the larger tractors, with a drawbar pull of 13,000 lb at 240 ft per min, up to 6,500 ft above sea level. It may be necessary to use two-drum rollers having a loaded weight of 20,000 lb each to compact some heavy clay soils; this type of roller has also been used successfully to compact decomposed granitic rocks. The aforementioned tractor should haul the three-drum roller, loaded to 20,000 lb per drum, at a speed of 176 ft per min. However, there may be some advantages to the higher speed of travel attained by the use of only two drums of this weight as the compacting load is applied more suddenly. The correct soil moisture content for use with such rollers will be that wherein a penetration resistance in the compaction cylinder from 2,000 lb per sq in. to 4,000 lb per sq in. is secured, the maximum moisture content should be that of an indicated saturated penetration resistance of 400 lb per sq in., with average results of about 600 lb per sq in. indicated saturated penetration resistance.

All of the dam construction experience of the writer indicates that the correct use of the type of roller described will secure substantially similar bearing power from all materials that can be compacted. Actually the only problem in a subgrade is to determine the proper slab thickness for a given loading after the soil is compacted by the roller available for use. All that technical supervision can do is to regulate the soil moisture content so the roller will get the maximum compaction it is capable of producing and to make sure the correct number of roller trips are made over all parts of the fill.

Emergency Construction Procedure.—For hasty or emergency construction the use of the three-drum roller described previously will automatically create high soil densities with no other precaution than controlling the soil moisture so the 7-sq-in. tooth penetration with 13,000-lb drums will be between 3 in. and 5 in. and making certain that sixteen roller trips are made over all parts of the fill. A similar roller, with 10,500-lb drums, was used to secure the results shown by Fig. 42. This method applies only to soils that need addition of water. Where most of the soils are too wet for use some of the previously mentioned decomposed sedimentary or igneous rocks, glacial moraines, and even alluvial soils may be found in place while so dense that, when saturated, the moisture content is low enough to permit compaction of the type described. Where excessive moisture cannot be avoided the use of larger shoes and 10,500-lb roller drums to prevent complete penetration of the roller teeth will secure the maximum soil density possible. The method of testing outlined will determine the indicated saturated penetration resistance of the resulting fill and, in itself, furnish data for determining the pavement thickness or will show which areas of the runways are representative of the fill in order that tests of any other

type may be made at locations known to be representative of actual compacted fill conditions.

Concluding Remarks.—The foregoing discussion describes methods of soil compaction that should be especially applicable to hasty or emergency construction; that is, the work of preparing the subgrade could start without delay and the construction forces, by use of the methods described herein, could secure the best compacted subgrade possible with the equipment and materials at hand, after which the type and thickness of paving could be selected to fit the constructed subgrade. The success of emergency or deliberate construction depends on the functioning of the engineering force in charge; poor engineering will result in poor compaction. The engineering procedure for securing this type of compaction control, while exacting, is simple and does not involve complicated test methods; personnel for the control of dam construction have been assembled and trained for several different projects without difficulty. A carefully planned series of tests to determine the required thickness of pavement, landing field or highway, in terms of the indicated saturated penetration resistance of the subgrade would be most useful in that, at least for emergency construction, the data for determining pavement thickness could then be secured from the records of the engineering inspection of the compaction of the subgrade soils.

It appears from this discussion that subgrade soils should be compacted to relative densities represented by "Mean Cylinder 122.7" of Fig. 42 if possible, rather than holding the soil dry weight to a lower value as was done in connection with dam construction in this case to provide for soil consolidation without appreciable internal pressure. It appears that the close approach to saturation of such soils, with consequent internal pressure, will do no harm but, rather, the necessity of forcing water from the soils will slow the consolidation and result in a longer trip-life for the pavement if it is subjected to wheel loads that, if static, would break the pavement. The same reasoning indicates that subgrades compacted to lesser dry weights, because of the presence of too much water in the soil to allow full compaction, should be also compacted to the dry weights and moisture contents represented by the right-hand half of the dry weight curve, as close to saturation as possible. In the case of the slab being designed to withstand full soil consolidation under loading a longer period will elapse, more load trips, before the full slab deflection is reached, allowing the concrete to attain greater strength before being stressed fully.

D. M. Burmister,[26] Assoc. M. Am. Soc. C. E.[26a]—A most important and timely contribution to the design and construction of airports is given by this Symposium. The Corps of Engineers, U. S. Army, should be credited with developing the principles and construction practices for the unprecedented program of wartime airport construction. There are a few comments the writer wishes to make on Colonel Stratton's paper.

Subgrade Support.—It is evident from the design curves of Fig. 1 and Fig. 3 that subgrade support is of somewhat less importance for rigid concrete

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[26a] Received by the Secretary June 22, 1944.

pavements than for flexible bituminous pavements. With the small deflections permissible in concrete slabs to keep the tensile stresses within safe limits, only a minor increase in subgrade support can be developed by deflection of the slab directly under the load. In other words, a rigid concrete pavement has a very considerable load-spreading effect on the pressures transmitted to the subgrade, the load being distributed more uniformly over an area, possibly as large as 10 ft or more in diameter, at correspondingly greatly reduced intensities. On the other hand, the subgrade support is far more important for flexible bituminous pavements, because the deflections are greater and the load-spreading effect is smaller. The greater concentration of stress on the subgrade and the greater curvature of the pavement under the load tend to cause greater deformations in the subgrade, which, under the action of repeated loadings, tend to accumulate to the point of failure of the pavement, unless kept to sufficiently small values. Whether this limitation can be expressed in terms of a maximum allowable settlement under a single load application for different types of soil conditions is an open question. The performance of existing airfields under traffic conditions should clarify this subject considerably. It is hoped that Colonel Stratton will be able to present some factual data on settlement or deflection limitations for design purposes.

Base Courses.—Colonel Stratton has emphasized that extremely thick concrete pavements can be avoided by the use of base courses to improve the subgrade support. Although the improvement of pavement support was not always evidenced by plate bearing tests, traffic tests and service performance have demonstrated the value of base courses. The importance of the restraints exerted by a concrete pavement on the supporting capacity of a base course has been investigated (54). The effect may well increase the effectiveness of the base course by 50% to 100%. The restraint effects are due (*a*) to the surcharge effect of the weight of the concrete slab; (*b*) to its rigidity and marked load-spreading effect, which prevents any tendency for upheaval resulting from the deflection of the slab; and (*c*) to the shearing restraint at the interface between the slab and the base course, which prevents lateral displacements in the base course.

In the case of flexible bituminous pavements, the base course furnishes the primary support, and the bituminous layer serves as a wearing course. In Fig. 3, which gives the design curves of required thickness of base course for flexible pavements, there is no distinction made as to the quality of the base course. It is evident, however, from the minimum values of $C\,B\,R = 50\%$ for light plane loads to 80% for the heaviest plane loads that there is a very considerable range of quality and strength. The supporting capacity depends on the kind of material and the grading, the construction procedures of placing and rolling, and the degree of compaction achieved. When performance under actual traffic conditions has been observed in sufficient detail, it may be possible to establish minimum standards of good construction practice for design and construction purposes. The type of base-course construction can then be selected with due regard for economic and practical considerations, local use of materials, and the thickness required to provide adequate support and

length of service. Minimum standards of base-course construction are suggested in Table 13 as a basis for discussion (54).

TABLE 13.—MINIMUM STANDARDS FOR BASE-COURSE CONSTRUCTION

Type	Quality	Degree of compaction	Relative Density (%) From:	Relative Density (%) To:	Tentative thickness factor[a]
		(a) Crushed Stone Bases			
B-1	Best	Maximum	90	100	0.50
B-2	Good	Good	70	90	0.60
		(b) Gravel Bases			
B-3	Well graded	Maximum	90	100	0.75
B-4	Run of bank	Good	70	90	1.00

[a] Relative factors based on run-of-bank gravel as 100%.

Experience and performance records will indicate what the relative thickness factors should be. Relative density should be a more suitable measurement than percentage of compaction for the degree of compaction achieved in the field. This is illustrated in Fig. 43. Relative density is based on the loose,

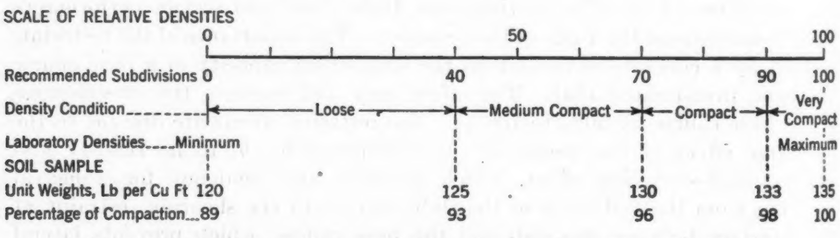

FIG. 43.—COMPARISON OF RELATIVE DENSITY AND DEGREE OF FIELD COMPACTION

dry state as 0% and on the maximum compacted state as 100% as convenient references (laboratory), which give a clear picture of conditions with respect to degree of compactness and also supporting capacity. For example, 95% compaction is only 70% relative density for the material shown, and can be considered only fair to good, being on the border line between medium compact and compact. With the development of better rolling equipment and compacting procedures, it may be possible to obtain 95% to 100% relative density, and ultimately even 110% or more, based on laboratory densities.

Good compaction is the most essential factor. The close packing and interlocking of the larger particles, whatever the grading may be, give high strength and resistance to deformations. To achieve this condition practically, especially in the upper layer directly beneath the pavement, where the stress concentrations are greatest, the methods used so successfully in the construction of "water-bound macadam" pavements may be used, omitting, however, the

topping of fines with water. The principles are simple: A single-sized coarse layer (1¼ in. to 2½ in. in size) 4 in. thick is rolled to maximum compactness. This achieves the close packing and interlocking of the large particles. Choke or filler stone (½ in. to 1 in. in size) is spread and worked into the interstices of the base and rolled until the base is filled. This choke stone provides the proper bedding, so that the large particles cannot rock or shift. An almost ideal condition is achieved for the top course of the base. Where the heaviest wheel loads are anticipated, the best bases so far constructed should be used to avoid excessive thicknesses.

The service and performance records under traffic conditions of the airports constructed by the Corps of Engineers would be of inestimable value for future construction, when correlated with the design and construction procedures given in the Symposium.

Ralph Hansen,[27] Assoc. M. Am. Soc. C. E.[27a]—A valuable contribution to the design of airfield pavements is presented in this excellent paper by Colonel Stratton. As the author states (see heading, "Synopsis"), "* * * the design and construction procedures have been verified by accelerated traffic tests utilizing heavy, rubber-tired earth-moving equipment." The following is a description of one of the traffic tests on flexible pavements, initiated by the Chief of Engineers, the results of which confirm the design curves tentatively established for wheel loadings in excess of 12,000 lb.

Barksdale Field, near Shreveport, La., was selected for this accelerated traffic test as it was believed the subgrade at this site would be representative of plastic clay subgrades which might be encountered in the construction of flexible pavements in the southern part of the United States. The specific purpose of the Barksdale Field test was to determine the following for a 20,000-lb and a 50,000-lb wheel load on flexible pavements with a 3-in. asphaltic concrete wearing course:

1. The thickness of pavement and base necessary to prevent detrimental shear deformation in the subgrade;
2. The effect of the quality of base materials on the total specified thickness of pavement and base;
3. The quality of base materials required directly under the pavement;
4. The thickness of a soil cement base needed to prevent detrimental shear deformation in the subgrade; and
5. The adequacy of the compaction requirements as adopted by the Office of the Chief of Engineers to prevent detrimental consolidation of the base materials and subgrade.

Extensive tests on the subgrade and materials locally available for base-course construction were conducted to determine the CBR, compaction requirements, and other properties prior to design and construction of the test pavements. The subgrade was stripped level, compacted, and a test section consisting of two test tracks, one for the 20,000-lb wheel load and the other for the

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[27a] Received by the Secretary June 29, 1944.

50,000-lb wheel load, was constructed. Each test track comprised the following four different types of bases: (*a*) Layered base construction consisting of several layers of different materials with a high CBR material directly under the pavement; (*b*) layered base construction similar to the first type but with a lower CBR material directly under the pavement; (*c*) a high CBR material for

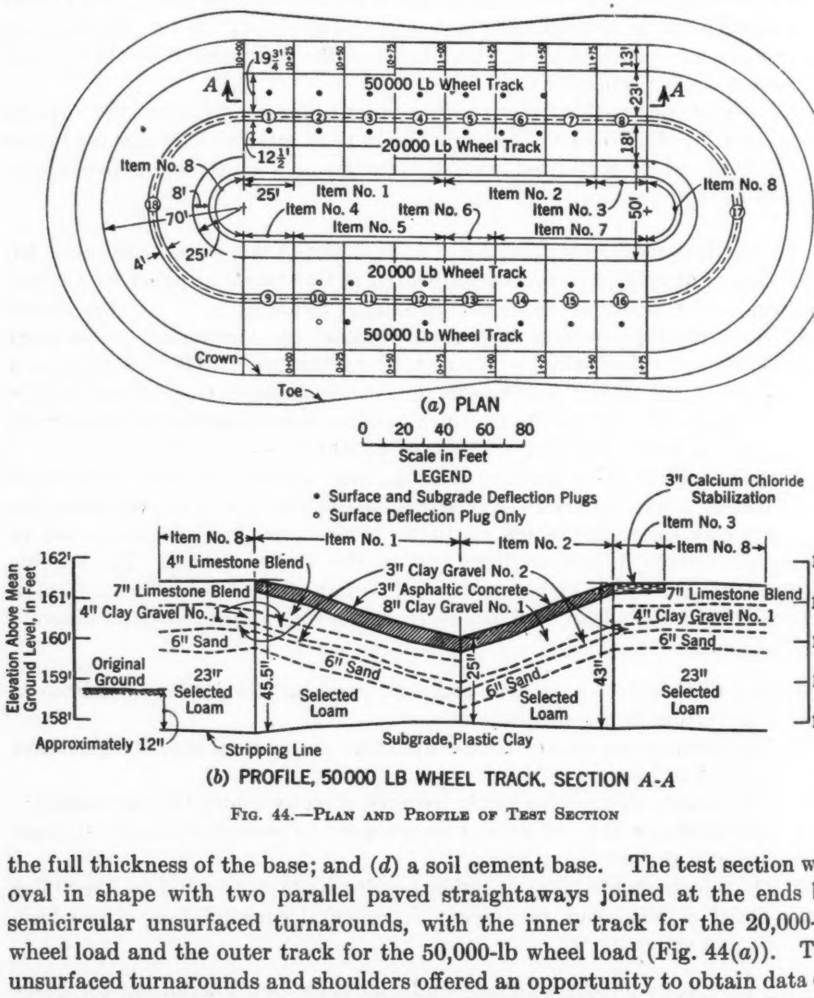

FIG. 44.—PLAN AND PROFILE OF TEST SECTION

the full thickness of the base; and (*d*) a soil cement base. The test section was oval in shape with two parallel paved straightaways joined at the ends by semicircular unsurfaced turnarounds, with the inner track for the 20,000-lb wheel load and the outer track for the 50,000-lb wheel load (Fig. 44(*a*)). The unsurfaced turnarounds and shoulders offered an opportunity to obtain data on the effect of repetition of heavy wheel loads on unsurfaced bases. Deflection plugs were installed in both tracks to measure the deflection, by means of electrical apparatus, of the pavement, base, and subgrade under the action of moving and static wheel loads and plate bearing loads. In Fig. 45 some typical oscillograms are shown for deflections under moving wheel loads at item 1, section 3, Fig. 44(*a*). The curves are recorded on a 60-cycle time base. The T

gage measures the deflection of the surface of the pavement and the B gage measures the compression of the base materials. The difference between the two readings is the deflection in the subgrade. Hydrostatic and earth pressure cells were installed in both the 20,000-lb and 50,000-lb tracks. Loaded earth-

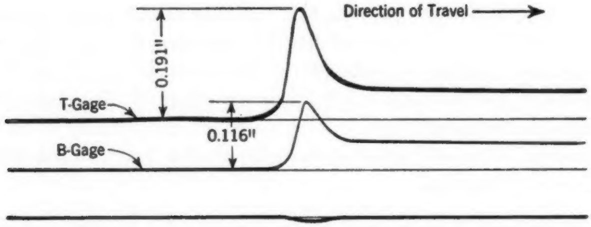

FIG. 45.—TYPICAL OSCILLOGRAMS FOR DEFLECTIONS UNDER MOVING WHEEL LOADS, 50,000-LB TRACK

moving equipment with tire contact areas approximately comparable to those of planes with similar wheel loads were used to produce the specified wheel loads. The loaded equipment was pulled around each test track by traction units, with one rear wheel of the loaded equipment traveling successively in three adjacent tracking paths, each path equal in width to the width of the tire imprint. Traffic was continued until 5,000 trips were made in each tracking path. Traffic was halted at various intervals for test measurements, maintenance and repair to the tracks, and observations. After completion of the traffic test, trenches were excavated in each test track to determine the lateral displacement and consolidation of the base materials and subgrade.

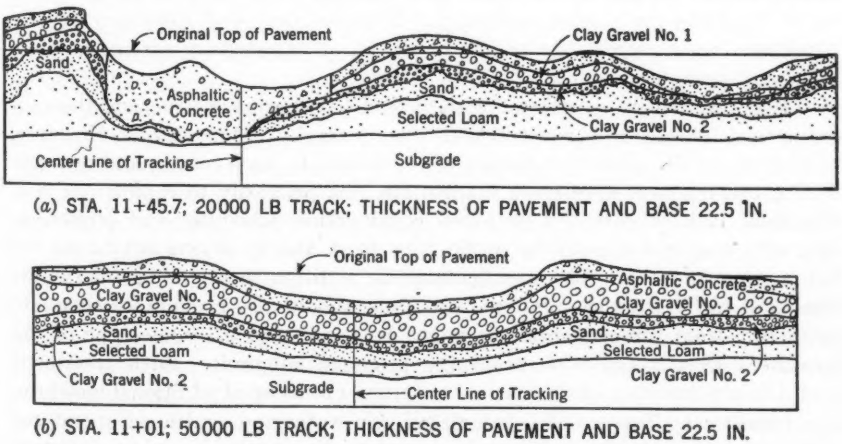

FIG. 46.—TYPICAL CROSS SECTIONS, VERTICAL FACES, AFTER TRACKING

In Fig. 46 are included typical cross sections (item 2, Fig. 44(*a*)) of the lateral displacement of the base materials and subgrade, vertical deformation of the pavement, and curvature of the 3-in. asphaltic concrete pavement resulting from repetitions of wheel loads. Extensive field and laboratory tests were made

on the asphaltic concrete pavement, all base materials, and the subgrade. After they have been assembled and analyzed and after correlation with the data from other tests, these data will be used in improving the design technique for airfield pavements.

H. V. PITTMAN,[28] M. AM. SOC. C. E.[28a]—Some of the more serious problems encountered in the construction of flexible pavement with clay-gravel base during late fall and winter are included in this discussion of Colonel Stratton's paper which deals particularly with "Construction of Concrete Pavements" and "Drainage of Airfields."

Construction of Concrete Pavements.—As stated by Colonel Stratton (see heading, "Concrete Paving"),

> "A well-prepared rigid pavement design may be vitiated by carelessness or improper construction, and emphasis is placed on the quality of subgrade and base-course construction; protection of the prepared subgrade, base courses, and the concrete during the construction; the selection of proper equipment for construction; construction sequences; and adequate curing of the concrete."

These requirements are considered about the minimum essentials necessary to obtain satisfactory results. Experience has shown that, due to the exigencies of the war necessitating tight construction schedules which at times appeared almost impossible of execution within the time limit specified, the final results are considerably less than these minimum essentials or those which would obtain under more normal conditions. Therefore, the ultimate service behavior of some of the wartime airfields is certain to reflect some faults in construction and it should be evaluated accordingly. However, even under the very adverse conditions encountered, the results obtained, as attested by service use, are remarkably good.

From the writer's experience with airfield pavement construction, the subgrade and base-course construction was generally satisfactory. However, provisions for the proper protection of the subgrade, base course, and concrete during construction were often inadequate, due primarily to rapidity of construction, lack of carefully prepared construction schedules and sequences, etc., with resultant damage by traffic, rain, frost, and by drying out during the hot summer months. The specifications, in addition to providing for the exclusion of traffic from the prepared subgrade, should also provide for adequate protection from the elements. It might be possible to use some type of soil treatment that would waterproof the prepared subgrade. Such treatment would insure retention of the original moisture, exclusion of additional moisture, and permit easy disposal of accumulated rain water and continued operations with a minimum of delay. The spanning of one lane with a covering consisting of some type of substantial but lightweight housing of the low-trussed type is deemed worthy of consideration. Some type of protection similar to the foregoing is very much needed so that a minimum of from 800 to 1,000 lin ft of

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[28a] Received by the Secretary June 30, 1944.

prepared subgrade can be adequately protected from the elements and kept in a state of readiness for pavement operations at all times.

More adequate protection and curing of concrete during construction should be provided, particularly during the hot summer months. Specifications for airfield pavement usually provide for the use of any one of five methods of curing concrete—namely, impervious membrane, cotton mat, ponding, wetted earth, and waterproof paper. Of these methods which serve the dual purpose of protecting and curing the concrete, contractors invariably select and use the impervious membrane curing method because of the ease in handling and applying the membrane and because of the resultant low cost. This type of curing does not adequately protect the concrete from the high summer temperatures which are known to cause severe warping of the pavement. The other methods specified, particularly the cotton mat method, would, if used, provide adequate protection and curing. However, contractors will continue to use the cheapest and most convenient method permitted by the specifications even though it is less effective.

Alternate and Successive Lane Construction.—Colonel Stratton's statements (see heading, "Construction of Concrete Pavements") that "* * * there is need for improvement in placement methods" and that

> "Alternate lane construction of multi-strip airfield pavements is conducive to serious disturbance to the subgrade by the operation of construction equipment and by ponding of rain water on the intervening subgrade between completed strips"

are in accord with observations of the writer. In order to meet design requirements fully the alternate lane method of construction should be eliminated, particularly where concrete is placed directly on a prepared subgrade. Experience has shown also that superior and satisfactory results can be obtained by using the successive lane method of construction of airfield runways progressively from the center line to shoulders by following a carefully prepared construction schedule, based on a proper order of work and operations sequence. This method results in improved drainage conditions on the shoulders and excludes traffic from the prepared subgrade. In addition, where all work of grading, subgrade preparation, and paving is properly coordinated, a minimum of time is lost in backtracking and in waiting until the concrete pavement has aged sufficiently to accommodate the necessary construction equipment.

Drainage of Airfields.—Referring to the section on "Drainage of Airfields," a serious drainage problem encountered during construction should be mentioned—namely, the design location of the pavement below ground surface at the shoulders, a problem not normally encountered in highway construction. Such location or depression of the pavement, in the writer's experience, has proved particularly troublesome during rainy weather and winter construction, has resulted in serious delays to progress and has necessitated excessive ditching at shoulders, pumping out excess rain water, and, in some cases, removal, replacement, and reworking of the subgrade. Whenever possible the pavement grade should be raised so that the base of pavement will correspond to ground elevation at shoulders after stripping, thus affording free surface drainage of the

prepared subgrade from center line to shoulders and thence to the drainage disposal system. This procedure would increase the amount of shoulder fill where subgrade is in fill, and would reduce the amount of shoulder excavation where subgrade is in cut. In cases where the stripped natural ground would coincide with the base of elevated pavement, the excavation would be reduced to a minimum as the material in such cases requires only scarifying and compacting, and the operations of excavating, wasting, or hauling to fill to accommodate a depressed pavement are eliminated.

The elevation of pavement grades would also be advantageous at those fields where the terrain is extremely flat and where the drainage system slopes for disposal of surface and subsurface water to border ditches and beyond to natural drainage systems are at a very minimum. The combination of depressed pavements with successive lane construction should eliminate many of the difficulties, delays, and expenses in the construction of some airfields. Later designs employed in some districts have provided grades which preclude most drainage difficulties encountered earlier in the program.

The location of the water table in rice-growing areas should be given more serious consideration than usual—particularly in late fall, winter, or early spring. In such areas, the cultivated topsoil is invariably underlain with a so-called clay hardpan—a dense, impervious material the existence of which determines the suitability of such land for rice growing. In many cases this hardpan occurs near the surface with a correspondingly high water table and very wet topsoil. In such cases, it is necessary to waste the otherwise suitable topsoil and to substitute borrow material having suitable moisture content for the compacted subgrade. This procedure often results in delay and additional cost while more soils investigations are made and in the necessity for longer hauls. In some cases it was necessary to use the only borrow available which was less desirable than that originally specified. In selecting a site for an airfield in a rice-growing area, it is very important to determine if the site contains sufficient suitable borrow for completion of the compacted subgrade in case seasonal conditions prohibit the normal use of the designated topsoil. Also, this determination of availability of suitable borrow should be made in advance of final selection of the site and, in any case, well in advance of construction.

Problems and Difficulties of Winter Construction.—The construction of concrete pavements can be successfully prosecuted during winter, even in regions of rather severe climatic conditions, without appreciable increase in cost or letdown in design requirements. The most serious problem is that encountered in the preparation and protection of a satisfactory subgrade—a problem common to all types of pavement. On the other hand, the construction of flexible pavement with a stabilized clay-gravel base under winter conditions in areas of frequent rainfall and freezing temperatures, such as prevail in latitudes of Little Rock, Ark., and Memphis, Tenn., is difficult, involves additional time for completion and an appreciable increase in costs, and, more often than otherwise, results in a pavement considerably below the usefulness contemplated by design requirements.

Among the most serious problems encountered are the following:

(*a*) Exposure to damage of large areas of prepared subgrade by all the vagaries of the weather and by construction equipment in delivering, spreading, and compacting the base course.

(*b*) Exposure of large areas of partly or completely prepared base course to like damage by weather and traffic while being constructed in two layers of equal thickness. Often the initial layer is subjected to damage by rain and frost action and requires reworking and compaction before the succeeding or final layer can be placed. When this condition occurs, excess moisture penetrates the subgrade, necessitating complete reworking and compaction of both the subgrade and the initial base course. This same condition may obtain during or after placement of the second or final base course and, at times, after application of the prime coat. Once moisture, appreciably in excess of the specified optimum, enters the base-course material, its removal is a difficult problem due especially to low temperatures and a very low rate of evaporation. Also, the clay-gravel base course is very susceptible to damage by freezing which destroys its stability, necessitating reworking of all areas so affected.

(*c*) Exposure of large areas of primed surface to the elements necessitated by the required minimum curing period of 48 hr and in order to provide sufficiently large primed surfaces for the economical prosecution of paving operations. Such exposure often results in serious damage to the base course, thereby requiring reworking, compaction, and repriming. Also, exposed primed areas hold excess surface water which must be dissipated before application of the wearing surface, resulting in additional delay.

TABLE 14.—Number of Days Suitable for Work (Sundays Excluded) for Pavements with Flexible Base, and for Concrete Pavements

Month	Flexible Base				Concrete			
	Average, 1937–1942		Actual Days per Month		Average, 1937–1942		Actual Days per Month	
	Days per month	Cumulative days	Minimum	Maximum	Days per month	Cumulative days	Minimum	Maximum
January	7	7	0	14	16	16	6	25
February	5	12	0	8	18	34	10	22
March	15	27	11	19	23	57	20	25
April	14	41	10	17	21	78	18	24
May	20	61	16	24	24	102	23	26
June	18	79	15	21	23	125	22	25
July	20	99	15	25	24	149	22	27
August	20	119	13	24	24	173	22	26
September	21	140	16	24	23	196	21	26
October	22	162	15	26	25	221	22	26
November	13	175	10	18	22	243	20	25
December	11	186	6	16	22	265	20	24

Table 14 shows, in summary, a comparison between the days suitable for work (Sundays excluded) on flexible base and concrete pavement, based on climatological data for Little Rock and vicinity for 1935 to 1942. Table 14 indicates that much time is lost in construction of flexible pavements with a

clay-gravel base during the winter months and during periods of heavy rainfall, whereas the experience on concrete shows relatively short periods of lost time occurring during cold weather and following heavy rainfall. Table 14 should afford a good comparison of the effects of weather on construction progress for the two types of pavement.

Winter construction of flexible pavement with a clay-gravel base should be limited to areas of more favorable climatic conditions than exist in the vicinity of Little Rock and Memphis, if results comparable to those of concrete are to be obtained.

G. R. Schneider,[29] Assoc. M. Am. Soc. C. E.[29a]—The rapid advances in airfield pavement and drainage design procedures are emphasized in this Symposium, and the authors are to be commended for the thorough manner in which the information has been presented. However, some of the unusual problems encountered in the field during the recent construction program in the United States are not mentioned, and it may not be out of place to note some of them in order to complete the record.

Many elements affected the location of the sites selected, and it was sometimes necessary to construct the fields in areas where the conditions were unsuitable from an engineering standpoint. When military airfields are to be constructed for training many thousands of fliers in a short time, they must be located far enough apart to permit construction of auxiliary or satellite fields from which fliers can train without interference with each other and with fliers from other fields. Obviously, a training field should not be located on an important airway. It should be placed where there is a reasonable amount of safe flying weather during the year, where air currents will not cause an undue hazard, and where it will not be subject to overflow by flood waters. Railroad and power line connections must be reasonably close because of critical labor and materials, and an adequate water supply must be readily obtainable. Nearness and accessibility of population centers must be considered because of the labor supply for construction and maintenance of the field and because of the social and recreational benefits. A location where the field would be useful after the war was preferred to one which would have little or no utility in peacetime. It was desirable to avoid hilly land because of the large amount of grading necessary, but flat land where little grading was involved made drainage difficult. Cultivated land was undesirable because the time and energy expended in working the soil to make it suitable for farming rendered it unsuitable as a pavement foundation. On the other hand, if the land was not cleared, the operations of clearing and grubbing, in addition to being time consuming and expensive, disturbed the original soil structure and often resulted in a poor foundation. It was usually desirable to have an all-over landing field in which case the soil had to be suitable for the growth of sod. The aforementioned limitations on site location meant that sites were selected which, in many cases, involved difficult foundation, drainage, and construction problems.

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[29a] Received by the Secretary July 17, 1944.

A further difficulty was the urgent need for speed in selection, planning, and construction. It was not possible to spend the desired amount of time on preparation of plans; and the time of construction could not be chosen so as to conform with favorable weather conditions. Because of the magnitude of the construction program, all sources of concrete aggregate and base materials that could be considered at all suitable were used, and, in some cases, materials that were not as good as might be desired had to be utilized. The railroads did an excellent job in hauling enormous quantities of concrete aggregates and base materials, and the Office of Defense Transportation distributed the available railroad cars and expedited their movement very efficiently. However, the pavement design was influenced in some instances by consideration of the difference in railroad haul requirements of the construction materials.

Airfields in the northern part of Louisiana and the eastern part of Arkansas were usually located in the flat alluvial areas where relief is slight and drainage problems are serious. In fact, some of the sites were selected by studying maps of areas that had been overflowed by major floods and then examining the narrow strips of relatively high land between the streams that had not been overflowed for the most suitable site. The soils generally are relatively impervious sandy clay silts, in some instances underlain by sand and in other instances by clay pan. Some of the latter areas were planted in rice or had been used for rice cultivation. Generally, these subgrade soils fall in the Public Roads Administration classification A-4-6. In their natural state they are loose and in the summer they dry out until they are quite hard, but they rapidly become saturated in the presence of water. When wet, the original structure of the subgrade soils is quickly broken down by the passage of construction equipment and their shear strength is largely lost.

Three important considerations in designing a pavement to be constructed on a subgrade of this type under any except the most favorable weather conditions are the depth of the clay pan below the finished subgrade, the limitation imposed by the subgrade on the wheel load of the construction equipment, and the selection of a type of pavement that can be constructed with a minimum of working of the subgrade. If the impervious clay pan is near the surface of the ground, water will collect readily and will saturate the subgrade making construction impossible unless the subgrade soil is removed to the clay pan and replaced with more stable material. Even though favorable weather conditions permit construction, there is the possibility of pavement failure during a wet season with field operations at or near capacity load. Consideration should also be given to the use of runway gutters and inlets, as shown by Fig. 8(*c*), to remove water from the pavement. Otherwise, the turf along the edge of the pavement will act as a dam in accumulating water which saturates and seriously weakens the subgrade at the edge of the pavement. This type of runway gutter is particularly desirable along taxiways if the field is to be used by planes in which the pilot's field of view is restricted so that he cannot see the edge of the pavement.

Subsurface drainage is generally ineffective with subsoils of this type, but if a subsurface drainage system is used special care must be taken in the design and construction of the filter surrounding the pipe to prevent the soil from

being washed into the drains. It is essential, too, that the pipe used in the storm drainage system have absolutely tight joints or soil will be washed into the pipes, undermining the fill over the pipe and resulting in sinkholes.

This Symposium provides a sound rational basis for the design of airfields. However, so many factors are involved in the selection, design, and construction of an airfield that the engineer should study carefully all the local conditions that will affect the cost or utility of the field, in addition to familiarizing himself with the features described in the Symposium.

H. M. WILLIAMS,[30] ESQ.[30a]—A most acceptable method of airfield drainage design, which has proved useful during the present war emergency construction program, is presented in the paper by Mr. Hathaway. The suggested methodology, however, does not provide for the determination of runoff from lengths

TABLE 15.—DETERMINATION OF SYN

No.	Location of watershed (1)	Area (acres) (2)	Date of storm (3)	Length (Ft): Overland (4)	Length (Ft): Channel (5)	n: Overland (6)	n: Channel (7)	S (%): Overland (8)	S (%): Channel (9)	Adjusted L (Ft): Overland (10)	Adjusted L (Ft): Channel (11)
1	Zanesville, Ohio	3.57	6-21-37	400	....	0.40	....	14.0	....	110	....
2			6- 6-37	400	....	0.40	....	14.0	....	110	....
3			8-10-37	400	....	0.40	....	14.0	....	110	....
4	Hastings, Nebr.	3.40	6- 5-41	250	300	0.40	0.10	3.5	5.3	140	30
5			8- 7-42	250	300	0.40	0.10	3.5	5.3	140	30
6	Hastings, Nebr.	3.74	6-20-42	300	100	0.40	0.10	4.0	5.0	170	20
7			10- 2-41	300	100	0.40	0.10	4.0	5.0	170	20
8			6- 2-41	300	100	0.40	0.10	4.0	5.0	170	20
9			6 -8-41	300	100	0.40	0.10	4.0	5.0	170	20
10			8-11-39	300	100	0.40	0.10	4.0	5.0	170	20
11	Hastings, Nebr.	0.69	5-17-40	310	....	0.40	....	5.8	....	130	....
12			10- 2-41	310	....	0.40	....	5.8	....	130	....
13	Hastings, Nebr.	0.69	6- 8-40	310	....	0.40	....	5.5	....	130	....
14			9-24-40	310	....	0.40	....	5.5	....	130	....
15			6- 5-41	310	....	0.40	....	5.5	....	130	....
16	Hastings, Nebr.	0.69	6- 8-40	310	....	0.40	....	8.0	....	110	....
17			5-17-40	310	....	0.40	....	8.0	....	110	....
18	Edwardsville, Ill.	49.95	3-15-38	500	1,500	0.40	0.06	2.0	2.0	360	160
19			3-22-38	500	1,500	0.40	0.06	2.0	2.0	360	160
20			5-27-38	500	1,500	0.40	0.04	2.0	2.0	360	110
21			6-21-42	500	1,500	0.40	0.04	2.0	2.0	360	110
22			2- 9-39	500	1,500	0.40	0.06	2.0	2.0	360	160
23			5-17-43	500	1,500	0.40	0.04	2.0	2.0	360	110
24			5-17-43	500	1,500	0.40	0.04	2.0	2.0	360	110
25			5-17-43	500	1,500	0.40	0.06	2.0	2.0	360	160
26			9- 2-41	500	1,500	0.40	0.06	2.0	2.0	360	160
27	Albuquerque, N. Mex.	40.5	8- 2-39	200	2,400	0.15	0.06	0.9	5.0	75	180
28			8-20-39	200	2,400	0.15	0.06	0.9	5.0	75	180
29	Colorado Springs, Colo.	35.6	8-10-38	400	1,500	0.40	0.10	7.5	3.1	150	200
30	Colorado Springs, Colo.	35.4	7-21-40	900	1,500	0.40	0.10	6.0	3.1	380	200
31	Vega, Tex.	95.9	5-30-38	900	2,700	0.20	0.04	2.0	1.5	320	200
32			5-30-38	900	2,700	0.20	0.04	2.0	1.5	320	200
33			6-21-39	900	2,700	0.20	0.04	2.0	1.5	320	200
34			7-26-39	900	2,700	0.20	0.04	2.0	1.5	320	200

[30] Office, Chf. of Engrs., War Dept., Washington, D. C.

[30a] Received by the Secretary July 17, 1944.

of flow greater than 600 ft. The selection of such a limiting value is basically correct since the phenomenon of sheet flow as expressed in the basic equations is normally not experienced on typical airfield covers for lengths greater than 600 ft. In fact, it may be limited to much shorter distances before channelization occurs. However, in many types of airfield layouts, other than that indicated in Fig. 26, the engineer is frequently confronted with conditions under which it is entirely feasible and desirable to permit considerably longer lengths of flow than are covered by Mr. Hathaway.

In an effort to provide such an extension to the design criteria, consideration has been given to the development of a synthetic hydrograph procedure following, in general, the methodology suggested by Franklin F. Snyder, Assoc. M. Am. Soc. C. E. (29). For the necessary data on which to base the required hydrograph constants, recourse has been made to the data of the U. S.

Synthetic Unit Hydrograph Constants

Total adjusted L (ft) (12)	t_r (min) (13)	T_p observed (min) (14)	t_p estimated (min) (15)	t_d estimated (min) (16)	Q_{pR} observed (cu ft per sec) (17)	Volume of runoff (in.) (18)	q_u (cu ft per sec) (19)	Q_{pR} reproduced (cu ft per sec) (20)	T_p reproduced (min) (21)	No.
110	11.0	14.0	7	1	7.4	0.66	4.52	9.2	14	1
110	7.5	9.0	4	1	3.3	0.21	7.02	3.5	11	2
110	3.0	5.0	4	1	8.0	0.43	5.98	8.8	8	3
170	6.0	13.5	9	2	2.8	0.19	4.67	2.7	12	4
170	3.0	10.5	9	2	3.4	0.21	4.75	3.2	10	5
190	16.0	21.0	10	2	3.4	0.47	2.81	4.8	20	6
190	11.0	16.0	9	2	1.6	0.17	3.27	2.1	16	7
190	12.5	20.0	12	2	1.0	0.12	2.71	1.4	17	8
190	10.0	17.5	11	2	0.8	0.10	2.48	1.3	15	9
190	5.0	11.5	9	2	3.0	0.30	2.71	4.7	12	10
130	29.0	29.0	8	1	0.21	0.16	4.53	0.23	31	11
130	13.0	16.5	7	1	0.23	0.11	4.65	0.26	16	12
130	6.0	12.0	8	1	0.44	0.21	3.34	0.65	11	13
130	10.0	14.5	8	1	0.35	0.18	3.66	0.48	14	14
130	5.0	11.0	8	1	0.34	0.17	3.08	0.57	13	15
110	10.0	12.0	5	1	0.50	0.19	6.23	0.54	13	16
110	11.0	14.0	6	1	0.46	0.20	5.19	0.54	14	17
520	17.0	24.0	13	2	24.6	0.32	2.04	30.6	26	18
520	18.0	30.0	17	2	16.3	0.18	2.21	16.8	27	19
470	11.0	17.0	10	2	87.0	0.75	2.89	84.0	21	20
470	16.0	21.0	10	2	89.0	0.66	3.92	67.0	25	21
520	10.0	25.0	19	2	18.0	0.20	2.48	21.8	22	22
470	7.0	16.0	11	2	76.0	0.74	2.43	89.3	19	23
470	10.0	16.0	10	2	62.0	0.65	2.14	75.0	20	24
520	40.0	46.0	17	2	20.7	0.39	1.66	27.7	44	25
520	5.0	19.0	18	2	21.9	0.19	2.39	22.1	19	26
255	5.0	11.0	8	1	17.6	0.14	3.35	19.7	13	27
255	8.0	13.0	8	1	12.1	0.12	2.77	15.5	15	28
350	10.0	15.0	9	1	83.3	0.66	4.43	59.9	19	29
580	10.0	22.0	16	2	37.5	0.39	2.88	25.6	27	30
520	13.0	20.0	12	2	136.0	0.73	2.40	144.0	24	31
520	13.0	21.0	13	2	96.0	0.66	1.83	131.0	24	32
520	63.0	67.0	20	2	62.0	0.60	2.36	53.9	67	33
520	23.0	32.0	15	2	64.0	0.50	1.91	48.8	31	34

Soil Conservation Service for small, turfed watersheds having characteristics which, in so far as possible, were similar to those of typical airfield areas. The data for the various watersheds which have been studied are assembled in Table 15.

To provide a common basis of comparison for these various watersheds, use was made of the relationships expressed in Fig. 17 by means of which an effective length for $n = 0.40$ and $S = 1\%$ was determined for each area. In these determinations the effective length is subdivided into effective channel length and effective overland flow length. The length of flow in the channel was measured along the proposed collecting channel or swale for that section in which appreciable depth of flow may reasonably be expected to occur during the design storm. Length of overland flow is the average distance from the end of the effective channel to the edge of the drainage area measured in the direction of flow as indicated on the proposed grading plans.

From Mr. Snyder's concept of the unit hydrograph, the following equations may be obtained:

$$t_p = T_p - K_q (t_r - t_d) - \frac{t_r}{2} \qquad \text{(14a)}$$

and

$$q_u = \frac{37.5}{t_p} = \frac{Q_{pR}}{R\,A}\left[1 + \frac{K_q (t_r - t_d)}{t_p}\right] \qquad \text{(14b)}$$

Also,

$$Q_{pR} = \frac{37.5\,R\,A}{t_p + K_q (t_r - t_d)} \qquad \text{(15)}$$

In Eqs. 14 and 15, in addition to the notation of the Symposium:

K_t and K_q = factors which vary as functions of $\frac{t_r}{t_d}$;
Q_{pR} = peak rate of discharge for given area for given rainfall excess;
q_u = peak rate of discharge of unit graph, in cubic feet per second per acre;
R = rainfall excess, in inches;
T_p = time, in minutes, from beginning of rainfall excess to peak discharge;
t_p = lag, in minutes;
t_r = length of rainfall excess, in minutes; and
t_d = unit of duration of rainfall excess, in minutes $\left(\text{assumed equal to } \frac{t_p}{5.5}\right)$.

(This notation has, of necessity, been varied from that presented in Mr. Snyder's paper (29).)

Because of the possible wide range in duration of design rainfall, when using the procedure of constant rainfall intensity for critical durations, it was found necessary to change the assumption by Mr. Snyder that $K_q = K_t = \frac{1}{2}$. The addition of the dimensionless unit hydrograph, discussed subsequently, to

determine hydrographs for durations of rainfall in multiples of t_d indicated that K_q and K_t varied as functions of $\frac{t_r}{t_d}$. This relation is shown in Fig. 47.

Values of t_p and q_u shown in Table 15 were obtained for the various runoff occurrences by means of Eqs. 14, and, in turn, were plotted against the effective

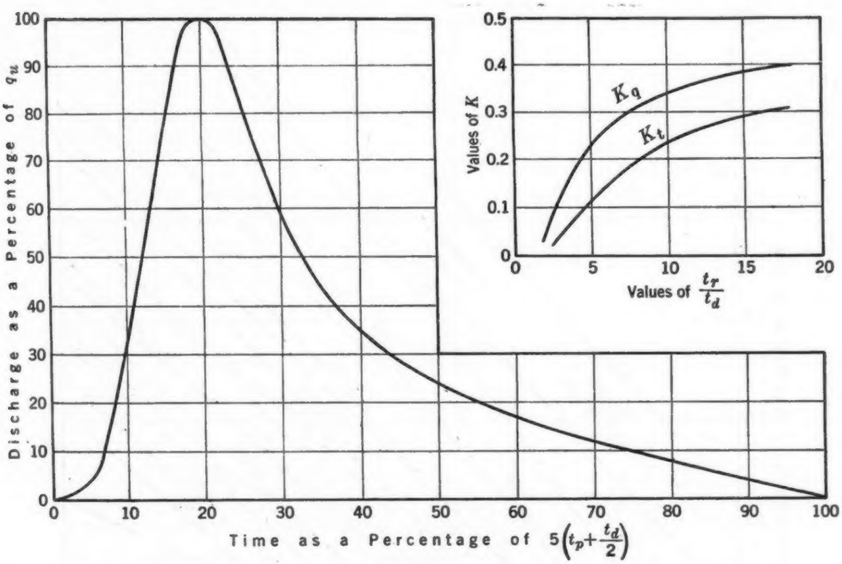

FIG. 47.—DIMENSIONLESS UNIT HYDROGRAPH AND RELATION OF K_q AND K_t TO $\frac{t_r}{t_d}$

length of the corresponding drainage areas. From these data, the following relations were determined:

$$t_p = 0.466\ (L\ L_{CA})^{0.3} \qquad (16)$$

or, if L_{CA} is assumed to be $\frac{1}{2}L$, then:

$$t_p = 0.378\ L^{0.6} \qquad (17a)$$

and

$$q_u = \frac{37.5}{t_p} \qquad (17b)$$

A dimensionless unit hydrograph was adopted (Fig. 47), the peak of which could be determined for any drainage area by Eqs. 17. The length of the recession was assumed to be four times the rising side of the hydrograph or a total hydrograph base length of $5\left(t_p + \frac{t_d}{2}\right)$. With these instruments, several of the hydrographs were reproduced with a resulting degree of agreement which was considered good.

Runoff data obtained by the foregoing method have been incorporated in a reproduction of Fig. 23 for a comparison of results of the two methods, as shown

in Fig. 48. For lower values of supply, the synthetic unit hydrograph produces a higher rate of peak discharge, and, also, for peak discharges reduced by any appreciable amount of ponding and with the more common supply values of between 1.0 and 2.0, the required drain-inlet capacity does not vary greatly for either method. However, because of the inherent differences in the basic

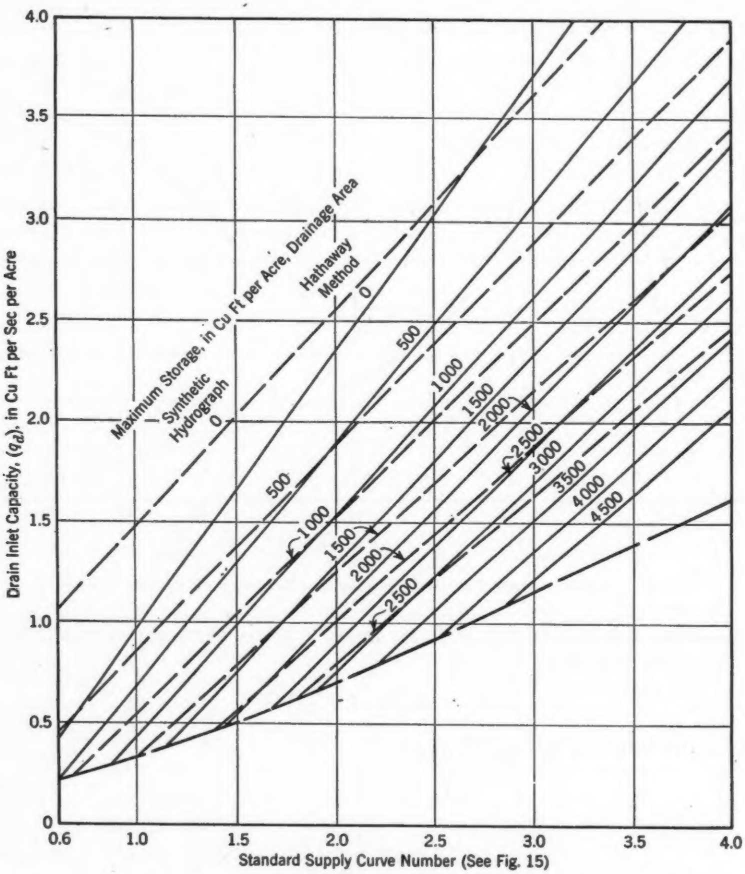

FIG. 48.—COMPARISON OF REQUIRED DRAIN-INLET CAPACITY BY MR. HATHAWAY'S METHOD AND BY SYNTHETIC UNIT HYDROGRAPH (L = 400 Ft, S = 1%, and n = 0.40)

principles of the unit hydrograph and the theory of overland sheet flow, it would be impossible to achieve complete agreement in the results obtained by both methods for a given area subject to a given rainfall intensity.

For a further comparison of the two methods, the runoff from an assumed turfed airfield area 400 ft wide and 2,000 ft long was computed for a supply of 1-in. volume occurring in 20 min. Runoff from the area was assumed to be collected in a shallow swale with 1.5% transverse slope and 0.5% longitudinal slope along the long axis of the area. To conform with the limits of overland

flow suggested by Mr. Hathaway, the area was subdivided into twenty subareas, 200 ft square. The hydrographs for the subareas, determined by Mr. Hathaway's method, were routed through the storage in the swale. It was observed that the resulting hydrograph for the total area was very similar to the synthetic hydrograph, both having a peak of 23 cu ft per sec, but the synthetic hydrograph lagged 6 min. Needless to say, the effort required to route the hydrographs for the subareas through storage greatly exceeds that required to develop the synthetic hydrograph.

The records on which to base a unit hydrograph study of this nature are somewhat limited, but additional records are becoming available rapidly to amend these preliminary investigations. However, the study presented herein indicates a feasible method of extending the design criteria presented by Mr. Hathaway to the determination of runoff from larger airfield drainage areas.

Acknowledgment is made to Mr. Snyder for his helpful comments in connection with these studies and to the U. S. Soil Conservation Service for field data on small watersheds.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

GEOLOGY IN HIGHWAY ENGINEERING

Discussion

By Karl Terzaghi, D. P. Krynine, Robert F. Legget, and Irving B. Crosby

Karl Terzaghi,[19] M. Am. Soc. C. E.[19a]—This review of the relation between geology and highway engineering deserves the attention not only of highway engineers, but of every engineer who deals with rock and soils. The influence of geologic factors on the success or failure of engineering operations is so manifold and decisive that it cannot be overemphasized. Hence, the writer agrees with Mr. Huntting on this subject. However, the manner in which soil mechanics has been treated, or rather ignored, calls for some comments regarding the competency of the geologist to express an opinion on the design of structures and regarding the circumstances which brought soil mechanics into existence.

In his college days the writer enjoyed the benefits of a specialized training in geology and petrology under the guidance of prominent experts in these fields. His first professional papers were devoted to purely geological subjects and throughout his subsequent professional career as a civil engineer he never ceased to draw upon whatever geology and experienced geologists had to contribute toward the solution of his problems which, in his early days, were primarily connected with hydroelectric power developments. However, as his engineering experience increased, the writer became painfully aware of the existence of a large group of engineering problems in the region between the domains of geology and of applied mechanics. These were the problems involving the mechanical effects of loads and seepage pressures on soils. Geology furnished valuable information regarding the origin, sequence, and shape of the individual soil strata, and applied mechanics made it possible to compute the stresses in the soil produced by given systems of forces. However, there was no method of evaluating the corresponding deformations and of

Note.—This paper by Marshall T. Huntting was published in December, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by W. W. Crosby; March, 1944, by Carl B. Brown; April, 1944, by F. H. Kellogg, Berlen C. Moneymaker, E. F. Bean, A. T. Bleck, Lyman W. Wood, Philip Keene, and Jacob Feld; May, 1944, by H. E. Marshall, K. B. Woods, and D. J. Belcher; and June, 1944, by R. Woodward Moore, George E. Ekblaw, and Allen S. Cary and Ade E. Jaskar.

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[19a] Received by the Secretary May 26, 1944.

defining the stress conditions for failure. The empirical rules used for bridging the gap were sadly inadequate. Since this fact was continuously impressed on the writer by observation, he became more and more obsessed by the desire to remedy the intolerable situation.

His first impression was that the no man's land between geology and engineering could most successfully be invaded from the geological viewpoint because he thought that definite relationships might exist between the geological character and the petrographic character of the soil strata in the field and their reaction on the forces which act on them during and after construction. To discover the existence of such relations, it is necessary to proceed in a purely empirical manner, by correlating construction experience with whatever can be ascertained by methods of field geology at the sites of construction. After the writer had spent several years collecting data of this type at random, he found the process too slow, and he decided to take advantage of what at that time was the boldest and most promising experiment in engineering geology ever made—namely, the construction of dams, open cuts, and tunnels, under extremely different geological conditions, by the U. S. Reclamation Service. During 1912 he visited practically every one of the larger construction camps of the Reclamation Service. With the active cooperation of the late F. H. Newell, M. Am. Soc. C. E., Director of the Reclamation Service, who was keenly interested in the writer's quest, geological data pertaining to the observed sites were secured. After the voluminous material was assembled, the writer tried to extract from it whatever definite relations may exist between the geological character and strength and bearing capacity of the different strata. The results of his labors were so utterly disappointing that he did not even consider it worth while to publish them. They can be condensed into a single sentence. There are no definite relations, because, for a given geologic origin and petrographic character of a stratum, the strength of the material, rock, or soil depends on conditions that are not, and cannot be, recognized by a purely geological investigation. For almost every geologically well-defined stratum the possible scattering of its physical properties from the statistical average is so important that one cannot even make an acceptable guess based on precedent.

At that stage of his investigation—a stage of bitter disappointment and disillusionment—the writer became interested in what is now known as soil mechanics. During the following 25 years he spent most of his time on observation and experimentation in this particular field. He aimed to develop suitable experimental methods for investigating the physical properties of soils and to explain the behavior of soils under the influence of load and seepage pressure on the basis of the laws of mechanics of solids and fluids. Between 1926 and 1929, the writer spent much time and labor on advocating the purely empirical procedure of correlating soil constants and soil profiles with the observed performance of the road surface in the field. Since the relative merits of the different methods of soil mechanics are already evident by their success or failure in connection with design and construction, their legitimate place among the tools of the practising engineer is no longer a matter of personal opinion.

Soil mechanics, without the assistance of geology and general statistics, can be used only in those rare instances in which a structure must be built in, or above, fairly uniform and homogeneous soil strata. In every other instance reasonably accurate values for the soil constants and information on the vital details of the structure of the subsoil can be obtained only by combining the results of soil tests with the results of investigations on the geological origin of the subsoil. However, the results of these investigations merely represent the prerequisite for solving the engineering problem. The solution itself requires an evaluation of the mechanical effect of the forces acting on the soil during and after construction. This major item in the solution is far beyond the domain of the geologist. As a matter of fact, many geologists know so little about the mental equipment necessary for this decisive part of the design that they consider themselves competent to make positive recommendations. Since this fact is often ignored even by engineers, its practical implications deserve serious consideration.

Although a competent geologist knows more about rocks than any engineer, no engineer would expect him to prepare an adequate design for a stone arch bridge, because the design of such a bridge requires a thorough training in mathematics and mechanics. Nevertheless, it is not uncommon for engineers to consider seriously the opinion of a geologist regarding the safety of a dam foundation, and this is a serious mistake. Few dam foundations can be designed on a purely mathematical basis, because generally the foundation conditions are too complex to permit an accurate forecast of the mechanical effects of load and percolating water. However, precisely because of this complexity, the design of such a foundation requires a far more thorough training in the mechanics of solids and fluids than does the design of a stone arch bridge, because the capacity for evaluating correctly the mechanical implications of various geological details without any computation does not develop in the human mind until a stock of subconscious knowledge has been accumulated as a result of the habitual application of the laws of mechanics to the solution of a great variety of problems. Since the geologist has neither the training nor the opportunity to acquire such a background, his forecast of a mechanical effect can be very misleading. The following example should serve to illustrate this fact.

In 1915, the National Academy of Sciences in Washington, D. C., appointed a committee of distinguished scientists to report on the causes of slides on the Panama Canal. Since one of the members (a prominent geologist) had previously acquired some knowledge of mechanics, he presented his findings in mathematical terms, involving an elaborate theory of the relation between the compressive strength of the rocks and the stability of the slopes. The basis of his theory was "that the tension T, acting along an arc δs of the surface of sliding is equivalent to a normal pressure $T\ \delta\psi$." To justify his statement the geologist referred to a classical textbook on mechanics. Since the writer was not aware of the propriety of applying this principle to the mechanics of solids, he consulted the reference and found that the statement refers not to solids, but to the relation between the surface tension and the normal pressure at the boundary between gases and liquids.

This incident demonstrates that an acquaintance with the literature of mathematics and mechanics, a necessary tool to the engineer, is not sufficient to establish competency as an engineer. Professional training has recognized the need and wisdom of following the acquisition of scientific knowledge by an apprenticeship for the purpose of developing the art of the profession. Competency for the practice of a profession is measured by both education and experience. Neither one alone is sufficient. The experiences necessary to become a competent geologist are not likely to be the same as those necessary to become an engineer. Hence, it would be dangerous for the engineer to accept an opinion of a geologist on a question of engineering. In science, a serious error in computation or judgment merely constitutes a blunder, but in civil engineering it may lead to a catastrophe.

Several geologists have published instructive descriptions of landslide preventive measures. However, if the reader of these discussions is familiar with the subject, he will notice that almost all these measures were developed, practised, and described by civil engineers prior to 1880. The geologists merely discovered that successful preventive means had already been used by civil engineers at a time when the landslides had not yet aroused the interest of the members of their own profession. As a matter of fact, there are exceptions to this rule. About 1935 a German engineering geologist proposed preventing slides on the slopes of a deep cut in stiff, fissured clay in northern Germany by grouting the fissures in the clay with portland cement. This procedure was entirely new. The writer saw the cut after the grouting was done, the cut was made, and the slopes had failed by sliding.

Under the heading, "Investigation of Proposed Tunnel Sites," Mr. Huntting discusses the influence of geological factors on tunneling. Essentially the same information was published in 1874 by F. Ržiha (61)[19b] and since that time no new material has been added.

Mr. Huntting implies (under the heading, "Frost-Heave Problems") that the danger of frost heave increases with increasing clay content of the soil. This and several other statements indicate that he does not pay as much attention to the laws of physics as these laws deserve. However, it would be preposterous to blame him, because he is a geologist and not an engineer. His deficiencies in physics are more than compensated by his broad vision in the field of geology, and merely indicate that the field of competency of every professional man, the geologist included, has certain limitations. If the geologist is careful enough not to trespass, his potential usefulness in connection with civil engineering cannot be overestimated.

Because of the obvious value of reliable geological information, large construction organizations, such as the Tennessee Valley Authority or state highway departments, are inconceivable without a staff of competent geologists. However, a vast amount of construction work is still being handled by very small staffs or by individual engineers. The success or failure of some of these jobs may depend to a large extent on a correct interpretation of the geological conditions. Thus, one may ask whether or not the investigation of

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[19b] Numerals in parentheses, thus: (61), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

the geology of a given site requires the services of a professional geologist. The answer to this question depends quite obviously on the nature of the job and a reliable answer can be expected only from an engineer who is thoroughly familiar with the elements of physical geology and with the influence of geological factors on the mechanical effects of loads and seepage pressures. Any other engineer might ignore the existence of geological complications on the job or overestimate the soundness of his own judgment in connection with problems in the domain of the professional geologist. Therefore, an elementary training in geology should be considered an essential prerequisite to the practice of highway, tunnel, or foundation engineering. In recent years most of the universities in the United States and abroad established courses in soil mechanics. This fact alone makes it necessary to teach engineering geology. The reason is obvious. Soil mechanics, as taught and practised at present, is likely to create the dangerous illusion that the processes in natural soil strata are always, or at least usually, as clear and simple as those in the cylindrical specimens subject to investigation in the laboratory. This idea, however, is due not to an inherent defect of soil mechanics, but to the incurable tendency of the human mind toward unwarranted generalization. Soil mechanics is not a substitute for engineering geology but a supplement to it. Sometime in the future when its novelty has worn off and its extravagances have been forgotten, soil mechanics may even turn into a useful tool in connection with purely geological research. Pending this future development, a course in engineering geology is needed as an antidote, and Mr. Huntting deserves the gratitude of his readers for his apt presentation of the manifold practical values of purely geologic information.

D. P. Krynine,[20] M. Am. Soc. C. E.[20a]—The predominant industry in the United States is agriculture, the construction business ranking second. Of the two billion dollars spent on public construction in 1938, about 45% was invested in roads and highways. The nation makes a considerable effort, financially, to keep its highways in tune with requirements of its life. The user wants a highway located where he needs it; he wants good riding conditions; and he wants low taxes. Hence, the major problem of the highway engineer is by no means a technical one. First, the highway engineer thinks of the purely human part of his problem, including human relations and economics, and for this purpose good planning is necessary. Again, this planning is not primarily of a technical nature. When finally the time comes for technical planning, the problem is again broken into two parts—what to build and on what to build. The former part of the question, of course, is more important than the latter. When a proper design of the superstructure and a proper choice of pavements (including materials) has been made, according to the needs of traffic, the question of the subgrade and of the foundation arises, and here the advice of the soil mechanician and of the geologist is necessary. Essentially, the highway engineer asks the soil mechanician and

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[20a] Received by the Secretary June 5, 1944.

the geologist to advise him whether the soil, near the surface or remote from it, will support a given structure safely.

Soil Mechanics, Soil Science, and Geology.—The engineering study of the properties of earth masses began with the early construction of fortifications in France, perhaps in the sixteenth century. The term "geology" in its present sense was probably first used sometime in the seventeenth century. Thus, the engineering study of earths (later termed "soil mechanics") and geology are approximately of the same age—about three hundred years old. Only recently did these fields begin to approach each other, in scope and content, to the mutual benefit of each. In this connection the purpose of this paper as defined in the "Synopsis" is timely and appropriate.

Near the beginning of the twentieth century, a group of geologists and agriculturalists started a new branch of knowledge which they named "Soil Science," and this particular branch of human knowledge, concerning the agricultural study of topsoils and their properties, has become an independent one. Highway engineers and soil mechanicians can find many interesting ideas in the field of agricultural soil science, such as the capillary moisture in soils, which is exceedingly important to those interested in the structural properties of soils (62). Geologists, particularly hydrogeologists, are more interested in free or gravitational water.

Thus, whereas soil scientists study topsoils mostly for agricultural purposes, geologists are interested in the earth as a whole, including the mantle covering the solid rock—with the exception, perhaps, of topsoils. Highway engineers and soil mechanicians are interested in the upper part of the mantle (about 300 ft thick), including any rock within that range of depth. At greater depths, stresses caused by the heaviest engineering structures are insignificant, and, in a practical sense, there may be no measurable strain from additional loads. Topsoils must be considered when simple earth roads are being built; but neither geologists nor soil scientists can advise the highway engineer effectively about topsoils.

Engineering and Geological Points of View.—There is considerable difference between the engineering and geological approach to the study of that part of the earth mantle which is important to engineers. Three differences are basic: (*a*) Relative scale of outlook, (*b*) relative appreciation of the strength of a material, and (*c*) relative concept of the time element.

The geologist tends to characterize a locality in broad terms whereas the engineer needs data concerning a particular point. In other words, the geologist thinks on a wider scale than does the engineer. Even excellent geological maps cannot be used for engineering purposes directly (except for preliminary studies) without supplementary borings and other soil investigations. An experienced boring contractor or foreman should make the borings rather than an average geologist or an average soil mechanician. Of the latter two, the person best qualified to do so must supervise the borings, interpret the results, and conduct the laboratory and field tests. If there is a geologist who is familiar with the test technique and who in addition understands the function of engineering structures, he may well be entrusted with the tests. In a general case, however, a well-trained soil mechanician is

preferable because of his specialized and intimate acquaintance with the strength of soils.

The third difference between a geologist and an engineer is in the relative concepts of time. The geologist's unit for measuring time is large—the geological "age." Ancient and medieval engineers also tried sometimes to build for eternity. The modern engineer, however, has a feeling that in a relatively much shorter time every structure built by man must outlive its usefulness and be replaced. Hence, as a rule he builds for a limited time only, generally for the period of amortization (dams with silting reservoirs, apartment houses, etc.). Especially is this true of highway pavements, for which there is a certain maximum service life (63).

Right Type of Geologist.—The impression gained by a reader of this paper is that a geologist can perform a great number of useful tasks in highway engineering. Granted that this may be so, the question still remains as to the choice of a proper geologist. An experienced Swiss engineering geologist, M. Lugeon (64), has stated (writer's translation):

> "A geologist who works with the engineers must remain a naturalist and must use the methods of natural science only. This is not only to avoid assuming the rôle of an engineer with subsequent conflicts, but mostly because geologists have been trained in schools of naturalists * * *. A geologist who has only studied the history of the earth and of those who have lived on it is of no use to the engineer * * *; but, if this man was also preoccupied in studying the mechanics of the formation of earths and seas, if he tried to know the creative and destructive agencies of the mantle, then practical men will take him from the ranks and use him."

The chief engineer of a highway project is usually a man with wide general interests and technical experience. He will accept no statements which in his opinion are too abstract and cannot be used immediately in practice. Thus, the geologists must know something about the behavior of structures and pavements to speak the highway engineer's language. In addition, considerable previous practice and a certain degree of reorientation will be needed.

Through intimate contact, the writer has come to look upon geologists as exceedingly fine scholars and useful people. In all frankness, however, it should be admitted that in everyday highway practice the services of a geologist, as such, are not essential. Excellent results are attained by engineers with some construction practice and school training in soil mechanics and elements of geology. However, in exceptional cases (for instance, if there are important structures such as tunnels or large dams or if a considerable number of landslides occur), it would be a serious mistake not to seek the advice of an able and competent geologist.

True Value of Soil Investigations.—As a matter of course, whoever is in charge of the preliminary soil investigations must furnish correct results and must report a true picture of both surface and subsurface soil conditions. Nevertheless, the accuracy of soil investigations is not a guaranty of the safety of a given structure. An excellent proof of this statement is furnished by Mr. Huntting himself (see heading, "Competency of Materials for Bridge Foundations") when he mentions an $80,000 bridge that failed due to poor

design although presumably soil conditions were fairly well known. Structures stand successfully either because they were adequately designed and built by skilful engineers or purely because they have been lucky enough to escape critical conditions. The writer disagrees with the statement that the San Francisco-Oakland Bridge "stands as a monument to the worth of careful preliminary foundation investigations." If that bridge is to be considered as a monument, it is a monument to the ability of its designers and builders. During the period of most active railroad construction, many structures were built practically without soil investigations. If many of them still stand, can they be considered as monuments to the absence of soil investigations?

Uncertainties.—Superficially, one would conclude that in any investigation of the physical properties of a pavement subgrade an extremely meticulous study of all hard and soft spots should be made. On the contrary, Francis M. Baron, Assoc. M. Am. Soc. C. E., has shown (65) that a hard spot corresponding to a 25% reduction in computed deflections results in the rather small reduction of about 18% in the computed critical stresses.

In some cases, therefore, the study of the pavement subgrade may be simplified and standardized considerably if in a given locality there are only a few types of soils to deal with in the subgrade. In such cases simple auger borings, to a depth of, say, 4 ft, are made, and the thickness of the pavement is determined from practical observations of the behavior of existing pavement slabs on similar soils.

Frost-Heave Problems.—Under the heading, "Frost-Heave Problems," Mr. Huntting states that "The clay acts as the medium through which water rises from the water table by capillarity until it reaches the frost line where it freezes * * *." As a matter of fact, from the point of view of frost heaves, clays are less dangerous than silts since the latter possess both sufficiently great height of capillary rise and fair permeability. Clays, as a rule, lack permeability (66).

Balancing Cuts and Fills.—A field in which geologists could learn something from the engineering profession is in the approach to determining the degree of compaction of the earth mantle. In glacial zones, such as the Southern New England Zone, excavated earth (not rock) when placed in a fill shrinks —perhaps an average of 15%. This means that, in nature, glacial soils are not compacted enough although they have been compressed by huge masses of ice. Furthermore, the degree of compactness is not uniform throughout a given region.

The designing engineer must know how many cubic yards of fill can be made from 100 cu yd of excavation at a given place. Although some information on this subject can be obtained by conducting small excavating and filling experiments, a better approach is needed. In this connection, it is regrettable that (see heading, "Prediction of Character of Material to Be Excavated") Mr. Huntting did not mention anything about the influence on the excavated material, especially on its volume changes, of the following two factors: (*a*) Depth of the water table at the borrow pit; and (*b*) season of excavation. The writer recalls a case in which 100 cu yd of a material containing a certain percentage of fine particles, excavated in the neighborhood of a lake, pond, or

river, furnished less than 100 cu yd of fill in summer and more than 100 cu yd of fill in early spring, due, of course, to additional water pumped from the water table during the frosts.

Conclusion.—Various specialists play a useful rôle in highway work—surveyors, designers, construction and maintenance engineers, mechanics, contractors of all kinds, chemical engineers and asphalt technologists, mathematicians and stress analysts, economists, statisticians and accountants, weather men, soil mechanicians, and geologists—and each of them has his place in the entire picture.

ROBERT F. LEGGET,[21] M. AM. SOC. C. E.[21a]—There will be little quarrel with most of the general statements formulated by Mr. Huntting. Many of them present, with renewed emphasis, well-known facts which may often be overlooked by engineers in practice. The examples presented in the paper are all interesting and useful to supplement the general propositions.

In this paper Mr. Huntting has covered an extremely wide field. It must be admitted that highway engineering itself covers many phases of engineering activity. At the same time, it must also be observed that the term "highway engineering" is used generally to denote engineering work specifically related to the construction of highways. Mr. Huntting goes far beyond this special field and really covers much of the general practice of civil engineering. In view of this, his statement (see heading, "Introduction") that "* * * no treatment of the subject as a whole is available" might have been qualified in some way. The writer may perhaps be forgiven for mentioning a book, "Geology and Engineering (12)," in which six separate chapters are devoted to six of the main topics discussed by Mr. Huntting, most of the remaining topics being dealt with in other parts of the book, although not to the extent of entire chapters.

Again, Mr. Huntting appears to suggest that the geologist, by himself, can undertake many of the tasks discussed in this paper. Therefore, the following countersuggestions (see heading, "Investigation and Prediction of Subsurface Water Conditions and Location of Underground Water Supplies") are satisfactory: "The geologist's prediction is not infallible, * * *" and (see heading, "Competency of Materials for Bridge Foundations") "A purely geological exploration of a proposed bridge site is seldom productive of enough detailed and precise data to be sufficient in itself, * * *." It is hoped, therefore, that Mr. Huntting will be willing to apply such qualifications to most if not all the problems discussed in his paper. In such work, the cooperation of geologist and engineer is most desirable. The problems, however, are essentially engineering problems in the solution of which the engineer calls to his aid the information which he can obtain through the detailed scientific study of the geologist. Doubtless this is what Mr. Huntting intended to suggest in his paper but certain parts may cause different impressions in the minds of readers not privileged to observe such cooperation as he suggests.

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[21a] Received by the Secretary June 16, 1944.

A particular instance of possible overemphasis on geology is the "Bibliography" of the paper which lists a number of publications quite definitely in the field of soil mechanics (essentially an engineering study), for which geology provides only a most necessary background. It is of interest perhaps to recall what Karl Terzaghi, M. Am. Soc. C. E., called "* * * the striking contrast between what we expected when digging into the earth or loading it, and what really happened * * *." Professor Terzaghi then stated (67):

> "I * * * hoped to discover the philosopher's stone by accumulating and co-ordinating geological information * * *. It took me two years of strenuous work to discover that geological information must be supplemented by numerical data which can only be obtained by physical tests carried out in a laboratory."

These comments are intended only to emphasize Mr. Huntting's arguments as noted, to the effect that geology is but one means of assisting engineering work.

The date 1888 (see heading, "Introduction") presumably refers to the publication of a paper in the *Proceedings* of the Geologists' Association (Great Britain) (68). Quite a number of publications before this date are available which include references to geology and highway engineering, most notably the writings of John Loudon McAdam; but, despite this long history, it is indeed significant that so little has been published with regard to this application of geology.

The author makes a reference (see heading, "Location of Suitable Road-Surfacing Material Pits and Quarries") to test pits, the use of which is described as haphazard. It is suggested that a very important function of geology in relation to highway engineering is guidance in the proper selection of test pits as a more useful means of exploring subsurface conditions. Test pits are sometimes placed in a haphazard manner, but this does not seem to justify the rather critical comment on them in the paper.

With reference to the surveys of road materials by the state highway departments, the Geological Survey of Canada has made investigations of this type for a considerable period, some of these publications dating back to 1916 (69).

The writer would appreciate Mr. Huntting's explanation of the term "traprock" (see heading, "Suitability of Various Earth Materials for Surfacing, Concrete Construction, and Other Highway Uses"), since its use does not appear to be generally accepted in geological circles as a specific name. It is frequently used in engineering circles but seems to be one of those very general terms, the use of which is unfortunate and, therefore, to be avoided in all carefully prepared publications. It is interesting to note (see heading, "Subgrade Treatment and Classification") the suggestion that the fundamental geological classification of soils (with reference to their origin) should be a "supplementary" classification. Surely this classification is of prime importance and should be utilized before other classifications since so many subsidiary features depend on it.

Finally, and with respect, the writer would ask what is meant by the term "within limits" in the suggestion (see heading, "Prediction of Character of

Material to be Excavated") that "Within limits, the depth and shape of such deposits are reliably predictable by competent geologists." Determination of the extent of deposits of soil is always a most difficult matter. Here indeed geology can be of assistance in guiding the necessary subsurface exploration work.

These critical comments all refer to matters of detail only. They do not question Mr. Huntting's main thesis with which the writer is in entire agreement. It is indeed encouraging to find such a paper as this in the *Proceedings* of the Society, its publication being in itself evidence of the growing appreciation of geology by the engineering profession. The paper should do much to develop this appreciation.

IRVING B. CROSBY,[22] AFFILIATE AM. SOC. C. E.[22a]—Geology in highway engineering is one phase of engineering geology, that is, the application of geology to the problems of civil engineering. It appears to have become fashionable to consider engineering geology as something new, but for more than half a century geology has been applied increasingly to a great variety of civil engineering problems on many types of projects. The use of geology in highway engineering was among the first of these applications. As early as 1889 an article was written in the United States about geology and highways (70). By 1918 considerable literature was available on the geology of road materials.

Although the geology of highway engineering had an early start more attention was given to other phases of engineering geology, particularly to the geology of dams. This was probably due to the fact that these problems are generally more critical and the need for geologic investigation more pronounced. Dam projects, however, involved many cases of the application of geology to road construction both in the relocation of highways about reservoirs and in the building of access roads.

Engineering geology has developed into a separate profession which requires competence in many of the specialized fields of geology and also experience in allied sciences and an understanding of engineering. The breadth of the field makes cooperation with other sciences and techniques essential to the successful practise of engineering geology. To meet the many demands of this profession successfully, the practitioner must give his full time and energy to its study. It cannot be practiced successfully by one whose primary interest is in some other field of geology or engineering.

The twelve problems listed by the author do not cover all the possible applications of geology in highway engineering, but they do include a wide range of problems in which geology is useful. Some of them involve other sciences or techniques, as is frequently the case in engineering geology, but all of them involve geology and experienced engineering geologists can contribute to their solution. The pertinency of geological study of frost-heave problems (Problem 5) may be challenged by some, but the basic investigations

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[22a] Received by the Secretary August 21, 1944.

which gave the first true understanding of the cause of this phenomenon were made by a geologist (15) (38). An engineering geologist can locate areas where frost heaving is likely to occur, and if these areas cannot be avoided in laying out a road provisions can be taken by the engineers to minimize frost heaving.

The foundations of bridge piers (Problem 7) present problems which may be simple and easily handled by an engineer or which may be complex and may require geologic investigation. An experienced engineer should recognize the cases where geologic cooperation is needed. Buried, former river channels, which were missed by drilling, have been found by geologic study, and the detection of buried channels may be an important problem in connection with the location of bridge piers. The physiographic interpretation of aerial photographs is often helpful in this connection.

Problem 8, the location of bridges and highways with respect to possible changes of stream channels, is most important with aggrading streams. These changes may occur, due to local conditions, in areas which are in the youthful stage of erosion and where the general tendency is downward erosion. Therefore, recognition of the true regimen of the particular part of the stream in question is important. The behavior of meandering streams is not yet thoroughly understood but progress recently has been made on this problem by the Sub-Committee on Landforms of the Committee on the Dynamics of Streams of the American Geophysical Union. This committee, composed of engineers and geologists, has published several reports (71) and one of its members has published the results of field and laboratory investigations on meanders (72). Application, by an engineering geologist trained in physiography, of the known principles of stream meandering to this problem can make possible a selection of locations which will obviate or minimize future trouble from shifting channels.

Ground-water problems are handled by both professions, but the occurrence and movement of ground water depend upon the character and structure of geologic formations, and geologic investigation is desirable except in the simpler problems. A special problem of relatively rare occurrence is to detect places where highway excavation and drainage lowers the ground-water level and injures ground-water supplies.

Engineering geology is a valuable tool which, when used by an experienced, competent man, can aid in the solution of these and many other problems of engineering. It is no panacea, however, and it has its limitations. Some problems can be solved by surface, geological observation alone, many others require subsurface exploration by drilling, test pitting, tunneling, or geophysics, and for some difficult problems it is economically not feasible to obtain an accurate solution. In this latter case, however, the engineering geologist can, and should, show the limits of the conditions that can occur and explain to the engineer the type of difficulties which he must prepare to overcome. When extensive subsurface exploration is necessary some think that a geologist is not needed. On the contrary, however, an engineering geologist, by means of his understanding of the geologic forces that have acted on the formations, can lay out the investigation to greatly reduce the amount of exploration necessary; and then he can obtain the maximum information from it by correct

interpretation. This requires understanding on the part of the engineer; he should allow sufficient exploration to enable the geologist to reach an intelligent and useful conclusion. On the other hand, there have been cases in which a geologist has demanded an inordinate amount of exploration to cover up his lack of experience. There is a happy medium which, unfortunately, is not always attained.

Obtaining competent engineering geologists to solve the problems of highway engineering or the other problems of engineering geology constitutes a problem with many difficulties. Engineering geology apparently appeals to many geologists as an attractive, easy, profitable field, and many geologists without experience in engineering have attempted to conduct investigations in it to the great detriment of the profession of engineering geology. At the other extreme are those who believe that, after a few courses in geology, engineers become competent to handle problems in engineering geology. This is just as dangerous as the other practice mentioned. Some understanding of geology by the engineer is essential, however, and the better he understands it, the less the danger of his overlooking some problem in which geologic aid is necessary. The most important requirements for competence in engineering geology are a thorough training in geology and a very broad experience in its application to many types of engineering projects on all types of geologic formations. A man experienced on a few types of problems in a limited area may make disastrous mistakes when he encounters different problems or different conditions. The apparent failures of engineering geology which are sometimes charged against it are often the failures of individuals who were not competent to handle the problems.

The early work in engineering geology was done by consultants who specialized in such work. Later some of the larger governmental organizations developed geological staffs composed largely of young geologists with little or no previous experience in engineering geology who were trained on the job at the expense of the job. A few state geological surveys have engineering geologists who are used in connection with highway work as has been explained by the author.

The use of consultants on special assignments is well adapted to the problems of dams where the work is concentrated and usually of considerable size. The more widely scattered problems of a highway department may not appear to be so well suited for investigation by a consultant, but when the work is well organized and especially when a permanent geologist or staff has collected data and done much routine work the more important problems can be handled, quickly and efficiently, by an experienced consultant.

The wide variety of problems in highway engineering may appear to require a large number of specialists but such is not the case. It is true that there will be problems in engineering geology, structural geology, sedimentation, physiography, ground-water hydrology, economic geology, petrology, materials testing, soil mechanics, and geophysics, but a competent engineering geologist must be trained and experienced in all but the last three and should know

something about them. When particular problems in petrology arise, for example, they can usually be handled by a specialist in a university. A state highway department may have sufficient geophysical work to justify the full-time employment of a geophysicist; then, too, it may be more economical and satisfactory to engage outside consultants for that work. A department with an experienced engineering geologist and a specialist in soil mechanics and testing with assistants in proportion to the amount of work and arrangements for geophysical investigations would be competent to meet the wide variety of problems. On problems of special importance an experienced consultant in engineering geology should be used to obtain the advantage of broader experience. Some departments may handle their work most efficiently by depending entirely upon consultants for investigations in engineering geology.

Mr. Huntting has done a service both for engineers and geologists by describing many of the problems in which geology can be useful in highway engineering. He has outlined briefly the way in which geology is related to some of the problems; he has explained its value as a tool for the engineer, and discussed some of its limitations.

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(70) **"Roads and Material for Their Construction in the Black Prairie Region of Texas,"** by Robert T. Hill, Texas Univ., December, 1889.

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DISCUSSIONS

COEFFICIENTS FOR VELOCITY DISTRIBUTION IN OPEN-CHANNEL FLOW

Discussion

BY E. R. VAN DRIEST, AND HUNTER ROUSE AND JOHN S. MCNOWN

E. R. VAN DRIEST,[11] ASSOC. M. AM. SOC. C. E.[11a]—The theorems of momentum and energy have proved to be very important tools of the hydraulic engineer and therefore their proper application should be understood completely by those who would use them. To be sure, some problems lend themselves more readily to solution by the momentum theorem, whereas others are solved more easily through the use of the energy theorem. Although this paper does not elaborate on the application of these principles, it does attempt to clarify some of the basic facts underlying them.

It is true that the standard textbooks derive the Bernoulli equation by either the application of the Newtonian law to a particle of fluid or the conservation of energy law to a mass; however, in the use of the Newtonian law the acceleration is written as the derivative $u \frac{du}{dL}$ rather than in the restricted form entered in Eq. 2. In most texts the acceleration equation is integrated directly, whereas in certain treatises[9] the line integral is formed along the streamline thus indicating that the work done on the fluid particle is equal to the change in kinetic energy. In this interpretation the left-hand member of Eq. 3*a* should read $\int_A^B F\, dL$.

The author's development of Eq. 7 for the case of gradually varied flow is not a strict application of the momentum theorem as such. The theorem states that in steady flow the sum of all external forces acting on a fluid mass is equal to the flow of momentum through the fixed surface bounding that mass at the instant in question. This theorem is expressed by the mathe-

NOTE.—This paper by William S. Eisenlohr, Jr., was published in January, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1944, by A. A. Kalinske, and Edward H. Taylor.

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[11a] Received by the Secretary June 2, 1944.

[9] "Fundamentals of Hydro- and Aero-Mechanics," by L. Prandtl and O. Tietjens, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, p. 112.

matical relation obtained from the Newtonian law for the L-direction,

$$\sum F_L = \int^A \rho\, u^2\, dA \quad \text{.....................(23)}$$

If this theorem is applied to the mass of Fig. 1:

$$\int^{A_A} p_A\, dA - \int^{A_B} p_B\, dA + (\text{weight})_L - F_{r,m}$$
$$= \frac{\gamma}{g}\int^{A_B} u^2{}_B\, dA - \frac{\gamma}{g}\int^{A_A} u^2{}_A\, dA \quad \text{..............(24)}$$

Rewriting Eq. 24 in terms of the mean velocities across sections A and B, and abbreviating the left-hand member,

$$\sum F_L = \frac{\gamma}{g}\left(C_{m,B}\, V^2{}_B\, A_B - C_{m,A}\, V^2{}_A\, A_A\right) \quad \text{............(25)}$$

It can readily be shown that Eq. 25 can be approximated by Eq. 7 when V_B approaches V_A and the difference between C_B and C_A approaches zero more rapidly than the difference between V_B and V_A. In other words, since the values of C depend only upon the shapes of the velocity distribution curves, as long as the shapes of the curves become similar faster than the velocities (and this is undoubtedly a good assumption for gradually varied flow), the two equations become sensibly the same.

Whereas the momentum theorem has been seen to be applied to the case of gradually varied open-channel flow, the energy theorem is most often used because the energy loss can be evaluated by means of a known formula. Reverting to the general energy theorem—namely, that the work done on a system by external forces is equal to the change in energy of that system—the author has not very carefully derived an energy relation for flow between two sections of an open channel. Reconsidering the case, if an elemental tube of the fluid mass is allowed to move over an interval of time Δt and the work done and energy change over this interval are obtained, then division by Δt will yield the time rate of change of work and energy in the element and summation throughout the entire mass will give the total rate of change of work and energy in the mass at the position considered. The quantity $\frac{p}{\gamma} + Z$ still remains constant over each section so that, upon division by the weight discharge $\gamma\, Q$, the following equation results

$$\left(\frac{p_A}{\gamma} + Z_A\right) - \left(\frac{p_B}{\gamma} + Z_B\right) = \frac{I'}{\gamma\, Q} + \alpha'_B\, \frac{V^2{}_B}{2\, g} - \alpha'_A\, \frac{V^2{}_A}{2\, g} \quad \text{....(26)}$$

in which α' is the "mean-cube" coefficient defined by $\dfrac{\int u^3\, dA}{V^3\, A}$, and I' represents the rate of internal work done by friction. Although Eq. 26 concerns the energy transformation per second of a mass of fluid per pound of fluid flowing per second, it can be interpreted to state that the work done by the external

forces (pressure and gravity), per pound of fluid occupying the space of one pound of discharge as it moves from section A to section B, such that each element has a common displacement irrespective of the time required for each element to travel from A to B, is equal to the change in energy of that pound between sections A and B. Thus, if the internal energy transformation in the fluid volume for the given displacement could be computed or measured, Eq. 26 could be useful, especially for large distortions of the velocity profile.

With reference to the illuminating paper by Professor Bakhmeteff,[6] there is attainable an energy relation involving the "mean-square" factor instead of the "mean-cube" factor. The "mean-square" equation is derived readily by considering the work done on an element of fluid as it moves between two sections; thus:

$$\frac{dp}{dL}\,\beta\,dA\,dL + \gamma\,\beta\,dA\,\frac{dZ}{dL}\,dL - dF_r\,dL = \frac{\gamma}{g}\,\beta\,dA\,\frac{d\left(\frac{u^2}{2}\right)}{dL}\,dL \quad \text{....(27)}$$

in which β is the length of the element, and the differential lengths are inserted to indicate the idea of work. Hence, upon division by $\gamma\,\beta\,dA$ and integrating along the path of motion, there results per pound of fluid on each streamline:

$$\left(\frac{p_A}{\gamma} + Z_A\right) - \left(\frac{p_B}{\gamma} + Z_B\right) = i + \frac{u^2{}_B}{2\,g} - \frac{u^2{}_A}{2\,g} \quad \text{.........(28)}$$

in which i represents the internal work due to friction. Since the integration is between two cross sections of flow, the work and energy equation for the movement of a cross-sectional volume of length dL is given by

$$\int\left(\frac{p_A}{\gamma} + Z_A\right) dA_A\,dL\,\gamma - \int\left(\frac{p_B}{\gamma} + Z_B\right) dA_B\,dL\,\gamma$$
$$= \int i\,dA\,dL\,\gamma + \frac{\gamma}{2\,g}\int u^2{}_B\,dA_B\,dL - \frac{\gamma}{2\,g}\int u^2{}_A\,dA_A\,dL \quad \text{.....(29)}$$

Whence,

$$\left(\frac{p_A}{\gamma} + Z_A\right) - \left(\frac{p_B}{\gamma} + Z_B\right) = \frac{I}{A\,dL\,\gamma} + \alpha_B\,\frac{V^2{}_B}{2\,g} - \alpha_A\,\frac{V^2{}_A}{2\,g} \quad \text{....(30)}$$

in which α is the "mean-square" coefficient defined by $\frac{\int u^2\,dA}{V^2\,A}$, and I is the internal energy change for the block. It is to be noted that the work done and the energy changed is per pound of fluid which traverses a distance such that each element of it has a common displacement and occupies the cylindrical region that one pound would occupy at any section of the channel. It thus follows that the two equations, Eqs. 26 and 30, compare the energy transformations in fluid volumes for a given displacement, one containing the "mean-cube" coefficients and the other the "mean-square" coefficients; it would seem that

[6] "Coriolis and the Energy Principle in Hydraulics," by B. A. Bakhmeteff, *Theodore von Karman Anniversary Volume*, Contributions to Applied Mechanics and Related Subjects, 1941, p. 59.

either equation is applicable provided the amount of internal energy transformation is known.

Reverting to the purpose of this paper, that of demonstrating the proper use of the correction coefficients for velocity distribution, there seems to be no question but that the "mean-square" factor must be used in the case of the momentum theorem application, whereas in the case of energy, either coefficient may be used depending upon which of the foregoing equations is considered. However, in gradually varied open-channel flow it is apparent that neither factor should be favored in view of the fact that the energy losses are ordinarily computed from empirical equations in which V_B is equal to V_A and in which the coefficients do not enter into the problem at all. In such a method of solution it seems advisable to ignore the correction coefficient entirely; certainly this appears justified since α and α' are close to unity in ordinary cases and experimental evidence[12] is available.

In closing, the writer wishes to emphasize the necessity for mathematical precision in coping with problems of the type investigated in this paper. It is hoped that none of these remarks are taken as discouraging in any manner.

HUNTER ROUSE,[13] M. AM. SOC. C. E., AND JOHN S. MCNOWN,[14] JUN. AM. SOC. C. E.[14a]—Attention is again focused by this paper upon the century-old controversy concerning the correct interpretation of the energy and momentum principles in hydraulics. Some aspects of the problem apparently have never yet been presented with adequate substantiation, as witnessed by the disagreement in even the most recent literature on this subject. Perhaps the thought and discussion provoked by the author's analysis will serve to settle at least the principal points of argument which remain. The author's oversimplified approach, unfortunately, has resulted in several conclusions which seem to have fundamental significance but which are in no way justified by a more careful analysis. The writers therefore submit herewith a criticism of the author's presentation together with an original analysis which they believe will clarify the general problem in a simple yet rigorous manner.

The type of motion considered by the author is gradually varied steady flow with a velocity distribution which changes from section to section. The only sound approach to even this particular phase of the general problem is the application of the basic equation of mechanics,

$$F = m \frac{dv}{dt} \qquad (31)$$

to a differential volume of fluid, and the subsequent integration of this expression throughout a finite volume according to established principles of the calculus. The author, however, has sought to proceed from particular first integrals of this basic equation—assertedly the fundamental equations of energy and momentum—without fully acknowledging the restrictions which these

[12] Discussion by J. C. Stevens of "Back-Water and Drop-Down Curves for Uniform Channels," by Nagaho Mononobe, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 990.

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[14a] Received by the Secretary June 29, 1944.

integrals imposed upon the subsequent analysis. He has, in fact, tacitly attributed to the integrals certain pseudo-differential properties, thereby eliminating from his derivation the very characteristics which distinguish the momentum and energy principles. A typical instance is found under the heading, "Development of the Momentum Equation," which in itself would serve to invalidate the final conclusions of the paper. If the change in velocity along a stream filament is appreciable, it follows that the accompanying change in cross-sectional area must also be appreciable. The fallacy in the author's proof to the contrary lies in treating as a differential expression what is actually the integral of a differential expression over a finite length of stream tube; under such circumstances it is obviously no longer permissible to ignore small quantities as though they were still infinitesimals of higher order than the other terms.

The general energy equation of hydraulics is fundamentally a scalar relationship, in which the boundary forces exerted upon the fluid between two cross sections do not appear because they do no work; it must include, however, a term embodying the dissipation of energy due to fluid deformation. The general momentum equation, on the contrary, is a vector relationship; therefore, all external forces become significant, whereas the internal stresses which produce the energy dissipation yield a force resultant of zero. Although the basic equation of motion may be written as a differential equation of acceleration, impulse-momentum, or work-energy through proper organization of terms, the essential differences between the integral equations of momentum and energy make it impossible to convert either one into the other by a comparable reorganization of terms—unless the type of flow under consideration is so simplified that the distinguishing characteristics of the integral equations disappear. The latter is the case only if the flow is very nearly uniform and the velocity is essentially constant over every section; under these circumstances, the familiar equation of gradually varied flow may be derived upon either a momentum or an energy basis. Once the acceleration becomes appreciable, or the velocity distribution variable, the two methods of approach necessarily lead to quite different mathematical expressions which cannot be interchanged.

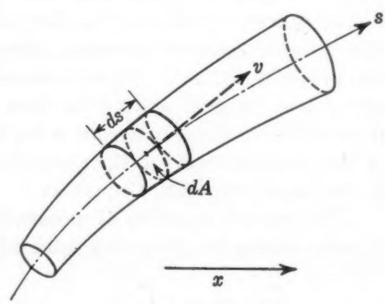

Fig. 4

As has already been pointed out, the basic equations which are sought can be obtained only from consideration of the dynamics of an infinitesimal fluid volume. For simplicity in analysis, the volume selected (see Fig. 4) is bounded by the streamlines forming the wall of a stream tube and by two normal sections a distance ds apart. The external forces acting upon this elementary free body are the attraction of the earth and the pressure and shear exerted by the surrounding fluid, which may be resolved in any direction x and equated to the product of the mass of the ele-

ment and the corresponding component of its acceleration:

$$\Sigma(dF)_x = dm\, a_x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32a)$$

The acceleration may be transformed as follows: $a_x = \dfrac{dv_x}{dt} = \dfrac{\partial v_x}{\partial s}\dfrac{ds}{dt} = v\,\dfrac{\partial v_x}{\partial s}$. Furthermore, dm may be written as $\rho\, ds\, dA$, so that

$$\Sigma(dF)_x = \rho\, v\, \frac{\partial v_x}{\partial s}\, ds\, dA \ldots\ldots\ldots\ldots\ldots\ldots\ldots (32b)$$

Eq. 32*b* expresses, in effect, the equality between the impulse per unit time and the accompanying rate at which the momentum of the fluid element is changed. It should therefore replace Eq. 2, which is neither a basic differential equation nor a correct integral thereof.

A very essential point—possibly the crux of the controversy—must now be noted. Before evaluating any double integral, the two principal variables must first be made independent. Since the quantity dA varies with s, the foregoing equation cannot be integrated correctly along the axis of the stream tube unless means are taken to express dA in terms of a variable which is independent of s. Fortunately, such means are immediately at hand in the fact that $v\, dA = dQ$, the elementary rate of flow through the stream tube, which is necessarily the same at all cross sections; thus,

$$\Sigma(dF)_x = \rho\, \frac{\partial v_x}{\partial s}\, ds\, dQ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (32c)$$

Integration in the s-direction then yields the momentum equation for a finite length of stream tube:

$$\int_{s1}^{s2} \Sigma(dF)_x = \rho\, (v_{x2} - v_{x1})\, dQ \ldots\ldots\ldots\ldots\ldots\ldots (32d)$$

Since the pressure forces between successive elements are equal and opposite, in the process of summation they cancel, leaving in the term at the left only the external forces of pressure, shear, and gravitational attraction acting upon the filament of fluid. Were it possible to express these forces simply, the foregoing first integral written for flow in the x-direction would yield the correct form of Eq. 5; notably absent is the factor 2 which appears in the denominators of the velocity terms of the author's expression due to his assumption of Eq. 2 as the basic differential equation.

The general equation of momentum for accelerated steady flow is obtained by completing the foregoing integral throughout the volume:

$$\int_{vol} \Sigma(dF)_x = \int_{Q} \rho\, (v_{x2} - v_{x1})\, dQ = \int_{A2} \rho\, v_{x2}\, v_2\, dA - \int_{A1} \rho\, v_{x1}\, v_1\, dA \ldots (32e)$$

Since internal shears as well as pressures cancel in the process of summation, the left side now represents the x-component of the resultant of all external forces (that is, the end pressures, the boundary pressures, the boundary shear, and the

fluid weight), which can be evaluated only through measurement or arbitrary assumption as to type of variation. The right side, which represents the difference in flux of the x-component of momentum past the two end sections, requires equally explicit knowledge as to the corresponding velocity distributions. As a matter of fact, only in case the flow at the two end sections is in essentially the same direction can the equation readily be applied to conditions in which the velocity varies across the flow, under which circumstances it reduces to the alternative forms,

$$\Sigma F = \rho\, Q\, (C_{m2}\, V_2 - C_{m1}\, V_1) = C_{m2}\, \rho\, (V_2)^2\, A_2 - C_{m1}\, \rho\, (V_1)^2\, A_1 \quad (33)$$

in which, as the author has stated,

$$C_m = \frac{1}{A}\int_A \left(\frac{v}{V}\right)^2 dA \quad (34)$$

It will be seen that Eq. 33 differs considerably from Eq. 8, for the several reasons already discussed.

In an essentially parallel manner, the general energy equation may be derived by equating to the rate of change of energy the rate at which work is done upon the elementary free body of Fig. 4 by the same external forces just considered. It is now necessary, however, to write the component of these forces in the direction of displacement s. Furthermore, it must be noted that the energy of the particle will change as the result of both acceleration and dissipation; the former, of course, is directly expressible in terms of the increase in kinetic energy, but a simple expression for the latter can be written only in terms of the decrease in the total head $H = \frac{v^2}{2\,g} + \frac{p}{\gamma} + Z$ of the element. Thus,

$$\Sigma(dF)_s\, v = \rho\, ds\, dA\, \frac{d\left(\frac{v^2}{2}\right)}{dt} - \gamma\, ds\, dA\, \frac{dH}{dt} \quad (35a)$$

which is the differential equation of energy suggested to replace Eq. 3. Rearranging the terms on the right side,

$$\Sigma(dF)_s\, v = \rho\, ds\, dA\, \frac{\partial\left(\frac{v^2}{2}\right)}{\partial s}\frac{ds}{dt} - \gamma\, ds\, dA\, \frac{\partial H}{\partial s}\frac{ds}{dt}$$

$$= \rho\, \frac{\partial\left(\frac{v^2}{2}\right)}{\partial s}\, ds\, dQ - \gamma\, \frac{\partial H}{\partial s}\, ds\, dQ \quad (35b)$$

Since dQ, unlike dA, is a constant along the stream tube, Eq. 35b may be integrated at once with respect to s:

$$\int_{s1}^{s2} \Sigma(dF)_s\, v = \frac{\rho}{2}\left[(v_2)^2 - (v_1)^2\right] dQ - \gamma\, (H_2 - H_1)\, dQ \quad (35c)$$

Were it practicable to express each individual member of the summation at the left, division by γ and substitution of $v\,dA$ for dQ would yield the correct form of Eq. 14. As in the case of the momentum derivation, however, it should be noted that internal pressures along the stream tube have equal and opposite reactions at points of the same velocity, and hence the net work done by pressures upon the fluid filament will consist only of that at the two end sections. The work done by shear along the filament cannot be evaluated easily, of course; but, in the process of summation over the entire volume, the net internal work done by both pressure and shear is evidently equal to zero. Moreover, since the boundary velocity is zero at all points, the work done by boundary shear must also be zero, leaving at the left of the final equation only the terms for pressure and elevation at the two end sections. The general volume integral, written in terms of the several heads, thus takes the rather obvious form,

$$\int_{A1}\left(\frac{p_1}{\gamma}+Z_1\right)v_1\,dA-\int_{A2}\left(\frac{p_2}{\gamma}+Z_2\right)v_2\,dA$$

$$=\int_{A2}\left[\frac{(v_2)^2}{2\,g}-H_2\right]v_2\,dA-\int_{A1}\left[\frac{(v_1)^2}{2\,g}-H_1\right]v_1\,dA\ldots\ldots\ldots(36)$$

Like the general momentum equation, the foregoing general energy equation may be applied to a given state of flow only if the distribution of velocity and pressure (and hence of total head) is known at both end sections. Unlike the momentum equation, on the other hand, the energy equation involves only the magnitudes of the velocities; however, the velocity distribution here affects both sides of the equation, with the result that the energy principle may usually be applied only if both sections under consideration are located in essentially uniform zones; the pressure distribution is then known (that is, hydrostatic), and the energy equation reduces to the more familiar form of the Bernoulli theorem,

$$C_{e1}\frac{(V_1)^2}{2\,g}+\frac{p_1}{\gamma}+Z_1=C_{e2}\frac{(V_2)^2}{2\,g}+\frac{p_2}{\gamma}+Z_2+H_l\ldots\ldots\ldots(37)$$

in which

$$C_e=\frac{1}{A}\int_A\left(\frac{v}{V}\right)^3dA\ldots\ldots\ldots\ldots\ldots\ldots\ldots(38)$$

and

$$H_l=\frac{1}{A_1}\int_{A1}H_1\frac{v_1}{V_1}dA-\frac{1}{A_2}\int_{A2}H_2\frac{v_2}{V_2}dA\ldots\ldots\ldots\ldots(39)$$

Since Eq. 37 lacks certain unnecessary factors of Eq. 16, yet further clarifies the term for energy dissipation, it is considered preferable by the writers.

From a comparison of the general momentum and energy relationships it will be evident that, although both may describe correctly the same state of flow, the mathematical terminology of the description must be quite different in the two cases. In other words, the inherent distinction between energy as a

scalar and momentum as a vector quantity, as well as the presence of a term for lost head in the one equation and terms for boundary pressure and shear in the other, makes it completely impossible to interchange the two equations by any conceivable transformation process. Therefore, despite the author's statements to the contrary, the Bernoulli theorem can in no way be regarded as a form of the momentum equation. As a matter of fact, it is this very difference between these two integral forms of the basic equation of motion which makes them so effective when used in combination. Hence, it is scarcely logical to attempt to show their similarity, but rather to emphasize the distinctions which make them such powerful complementary tools.

In the basic application of these equations to rapidly accelerated flow, pressure changes are generally so great that boundary-shear, lost-head, and velocity-distribution terms may be neglected without appreciable error; the equations of momentum, energy, and continuity then permit direct solution for both the pressure change and the resultant boundary force accompanying any rate of flow. They have also proved to be a useful combination in another extreme type of motion—the gradual establishment of uniform flow beyond the entrance of a pipe or conduit, in which acceleration and boundary pressure become the terms which are ignored. If these various effects are of comparable magnitude, of course, then none can be neglected safely. Typical of such conditions is the problem of rapidly decelerated flow, which still awaits a satisfactory solution by any but empirical methods. In a word, it is still impossible to obtain a quantitative analysis of the general problem of fluid motion without recourse to measurement, because other means are not yet at hand for obtaining the distribution curves for velocity and pressure which are required in the general momentum and energy equations,

Although the type of motion considered by the author is far simpler than the general case, in that acceleration is negligible and the pressure distribution is therefore hydrostatic, the distribution of velocity still remains unknown. Without actual field measurement, in other words, the coefficients C_m and C_e can be estimated only roughly, for scarcely enough is known about boundary resistance and energy dissipation in nonuniform flow to permit even an approximate evaluation by analytical means. To be sure, such estimates will yield average values which are perhaps closer to the truth than the magnitude of unity customarily assumed, but this still provides no trustworthy clue as to the change in either coefficient from section to section. Fortunately, however, the error involved in assuming these coefficients to equal unity is probably no greater than that arising from the present lack of knowledge regarding boundary shear and energy dissipation under nonuniform conditions. Indeed, the common failure to distinguish between the evaluation of boundary shear and energy dissipation in gradually varied flow is actually tantamount to assuming that $C_m = 1 = C_e$, for under such circumstances (but only then) the energy and momentum equations become identical.

Much is made by the author of the asserted fact that the familiar resistance equations stem from momentum (boundary-shear) rather than energy (dissipation) considerations. Actually, the equations in question have been derived solely for the case of uniform flow; under these conditions, of course, boundary

shear and energy dissipation are directly proportional, which robs the author's point of all significance. The customary loss coefficients of nonuniform flow, on the other hand, are simply means of evaluating the quantity H_l of Eqs. 37 and 39 in terms of the boundary geometry and the Reynolds number, and hence have no source whatever in the momentum principle. Which type of coefficient to apply in the analysis of gradually varied flow must depend upon whether the momentum or the energy principle is utilized when the true functional relationships for the coefficients are eventually determined. Either principle, of course, would be satisfactory for this purpose; although it is to be noted that the momentum principle is not necessarily the simpler of the two because of the difficulty in evaluating the longitudinal component of the boundary pressure.

From the foregoing discussion it would appear that such further refinements in applying the momentum and energy equations to even the relatively simple case of gradually varied flow must await considerable progress in analyzing the phenomena of velocity distribution and either boundary shear or energy dissipation. Fortunately, the solution of any one of these three phases of the resistance problem will almost automatically clarify the other two, for all three are closely interrelated—not only through the mechanics of fluid turbulence, but through the correct integral forms of the momentum and energy relationships.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ANALYSIS OF RIGID FRAMES BY SUPERPOSITION

Discussion

BY ODD ALBERT, AND W. C. SPIKER

ODD ALBERT,[11] ASSOC. M. AM. SOC. C. E.[11a]—Structural analysis by superposition, as proposed by Professor Wilson, is an excellent method. An improvement in procedure, however, is suggested by Example 2, an unsymmetrical three-column bent subjected to an off-center vertical load. The author solves this frame by determining the moment caused directly by the vertical load and then adjusts for the effect of sidesway in a separate operation. Final moments are obtained by adding the two sets of moments. A suggestion that results in more simple mathematics and one that is easier to visualize is presented herein, adapting the principle of slope deflection, as applied to a cantilever beam. For this purpose the deflections of the free end of a cantilever beam under various loadings are given in Fig. 25.

The sign conventions preferred by the writer are illustrated in Fig. 26—that is, when the end moment is negative, the arrow is drawn clockwise.

Vertical Load on Span BC.—If span BC, Fig. 27, is loaded as shown with the frame fixed to prevent sidesway, the moment transmitted to joint B travels to point A, and the moment transmitted to joint C separates and travels through point E to point F.

Referring to column EF, Fig. 27, the angle θ_E causes a deflection of $\theta_E L_{EF}$ at F_1, and the horizontal force H_F causes a deflection in the opposite direction (see Fig. 25):

$$\theta_E L_{EF} - \frac{H_F L^2_{EF}}{3E} = 0 \quad \text{(51)}$$

Since $H_F = \dfrac{M_E}{L_{EF}}$,

$$\theta_E = \frac{M_{EF}}{3E} = -\frac{M_{EC}}{3E} \quad \text{(52)}$$

NOTE.—This paper by David M. Wilson was published in February, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1944, by F. S. Merritt, Ralph W. Stewart, and John E. Goldberg; and June, 1944, by A. W. Fischer, Leon Blog, Arthur B. McGee, and Jaroslav Polivka.

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[11a] Received by the Secretary April 27, 1944.

Referring next to beam CE, Fig. 27, if end C were free to deflect:

$$\theta_E L_{CE} + \frac{V_C L^2_{CE}}{3\,E \times 2} + \frac{M_{CE} L_{CE}}{2\,E \times 2} = 0 \quad \ldots\ldots\ldots\ldots (53)$$

Substituting $V_C = -\frac{M_{CE}}{L_{CE}} - \frac{M_{EC}}{L_{CE}}$,

$$M_{EC} = \frac{M_{CE}}{6} \quad \ldots\ldots\ldots\ldots (54)$$

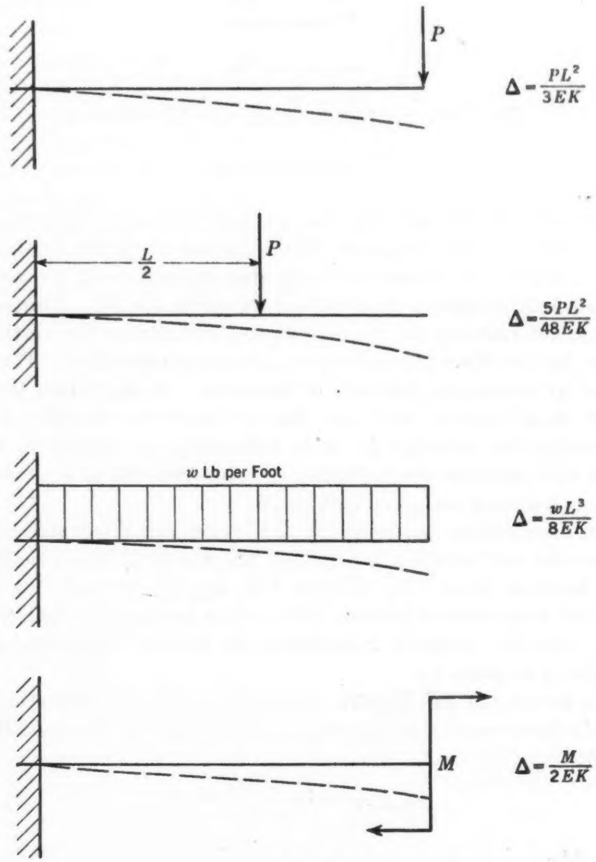

Fig. 25

In the same beam, if end E is free to deflect:

$$\theta_C L_{CE} + \frac{V_E L^2_{CE}}{3\,E \times 2} + \frac{M_{EC} L_{CE}}{2\,E \times 2} = 0 \quad \ldots\ldots\ldots\ldots (55)$$

In this case,

$$V_E = -\frac{M_{CE}}{L_{CE}} - \frac{M_{EC}}{L_{CE}} = -\frac{7\,M_{CE}}{6\,L_{CE}} \quad \ldots\ldots\ldots\ldots (56)$$

Substituting Eq. 56 in Eq. 55 and solving for θ_C:

$$\theta_C = \frac{11\,M_{CE}}{72\,E} \qquad (57)$$

Column DC is treated similarly. Assuming point C free to deflect:

$$\frac{H_C\,L^2_{CD}}{3\,E} + \frac{M_{ED}\,L_{CD}}{2\,E} = 0 \qquad (58)$$

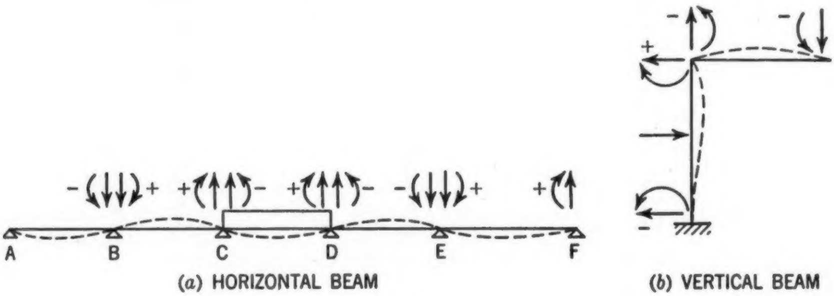

FIG. 26.—SIGN CONVENTIONS

Substituting $H_C = -\frac{M_{CD}}{L_{CD}} - \frac{M_{DC}}{L_{CD}}$,

$$M_{DC} = \frac{M_{CD}}{2} \qquad (59)$$

Assuming point D free to deflect, similarly:

$$\theta_C\,L_{CD} + \frac{H_D\,L^2_{CD}}{3\,E} + \frac{M_{DC}\,L_{CD}}{2\,E} = 0 \qquad (60)$$

With $H_D = -\frac{M_{CD}}{L_{CD}} - \frac{M_{DC}}{L_{CD}}$:

$$\theta_C = \frac{M_{CD}}{4\,E} \qquad (61)$$

Since each member meeting at joint C rotates through an angle θ_C, Eqs. 57 and 61 must be equal; that is, $\frac{11\,M_{CE}}{72\,E} = \frac{M_{CD}}{4\,E}$ and

$$M_{CD} = \frac{11\,M_{CE}}{18} \qquad (62)$$

Distribute the moment M_{CB} to beam CE and column CD as M_{CE} and M_{CD}, respectively. In other words, since $M_{CB} = M_{CE} - M_{CD} = -M_{CE} - \frac{11\,M_{CE}}{18} = -\frac{29}{18}\,M_{CE}$,

$$M_{CE} = -\frac{18}{29} M_{CB} \qquad (63a)$$

and

$$M_{CD} = -\frac{11\,M_{CE}}{18} = -\frac{11}{18} \times \frac{18\,M_{CB}}{29} = -\frac{11\,M_{CB}}{29} \qquad (63b)$$

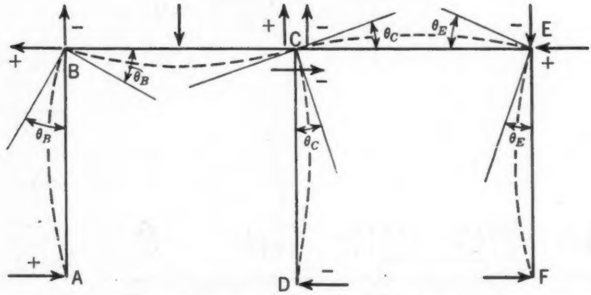

FIG. 27.—VERTICAL LOAD ON A FRAME FIXED AGAINST SIDESWAY

Finally (see Eqs. 57 and 61),

$$\theta_C = -\frac{11\,M_{CB}}{116\,E} = \frac{M_{CD}}{4\,E} = \frac{11\,M_{CE}}{72\,E} \qquad (64)$$

Because column AB is identical to column CD, $M_{AB} = \frac{M_{BA}}{2}$, and:

$$\theta_B = \frac{M_{AB}}{4\,E} = -\frac{M_{BC}}{4\,E} \qquad (65)$$

If beam BC is free to deflect at end C (compare Eq. 53):

$$\theta_B\,L_{BC} + \frac{5\,P\,L^2_{BC}}{48\,E \times 2} + \frac{M_{CB}\,L_{BC}}{2\,E \times 2} - \frac{V_C\,L^2_{BC}}{3\,E \times 2} = 0 \qquad (66)$$

Substituting $V_C = \frac{P}{2} + \frac{M_{BC}}{L_{BC}} + \frac{M_{CB}}{L_{BC}}$:

$$-5\,M_{BC} + M_{CB} = \frac{3\,P\,L_{BC}}{8} = 7{,}500 \qquad (67)$$

If beam BC is free to deflect at end B, an equation similar to Eq. 66 $\left(\text{substituting } V_B = \frac{P}{2} - \frac{M_{BC}}{L_{BC}} - \frac{M_{CB}}{L_{BC}}\right)$ yields:

$$-91\,M_{CB} + 29\,M_{BC} = -\frac{3\,P\,L_{BC}}{8} \times 29 = -7{,}500 \times 29 \qquad (68)$$

Solving Eqs. 67 and 68 simultaneously: $M_{BC} = -\frac{77{,}500}{71}$; $M_{CB} = +\frac{145{,}000}{71}$; $M_{BA} = +\frac{77{,}500}{71}$; $M_{AB} = \frac{M_{BA}}{2} = \frac{1}{2} \times \frac{77{,}500}{71} = +\frac{38{,}750}{71}$; M_{CD} (Eq. 63*b*) $= -\frac{55{,}000}{71}$; M_{DC} (Eq. 59) $= -\frac{27{,}500}{71}$; M_{CE} (Eq. 63*a*) $= -\frac{90{,}000}{71}$; M_{EC} (Eq. 54) $= -\frac{15{,}000}{71}$; and $M_{EF} = +\frac{15{,}000}{71}$.

The horizontal force H_B is equal to $\frac{38{,}750 + 77{,}500}{71 \times 15} = \frac{7{,}750}{71}$. Similarly, $H_C = -\frac{5{,}500}{71}$ and $H_E = +\frac{1{,}000}{71}$. The horizontal force at the top required to prevent horizontal movement, then, is the sum of the three, or $\frac{3{,}250}{71}$.

Sidesway.—When columns are of equal height, the sidesway, or lateral deflection of the horizontal members, is equivalent to the corresponding lateral displacement of the column bases, assuming the girders to be fixed. The problem is approached by analyzing the effect of moving each column base separately.

For example (see Fig. 28), if support A is moved to the left a distance Δ, the moment generated in joint B will be distributed along the two paths

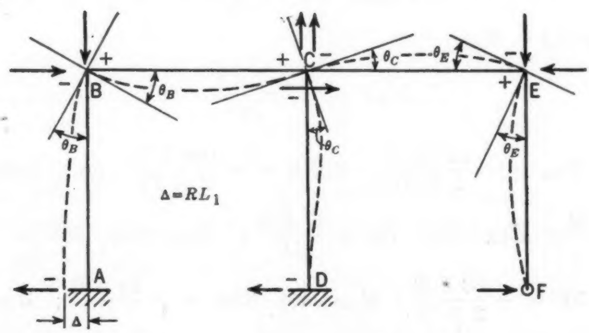

FIG. 28.—SUPPORT A MOVED TO THE LEFT A DISTANCE Δ

C-E-F and C-D. By Eq. 64, $\theta_C = -\frac{11\ M_{CB}}{116\ E}$ in terms of beam BC. If this beam is cut at end B,

$$\theta_C\, L_{BC} - \frac{V_B\, L^2_{BC}}{3\,E \times 2} + \frac{M_{BC}\, L_{BC}}{2\,E \times 2} = 0 \quad \text{(69)}$$

As before, substituting $V_B = \frac{M_{BC}}{L_{BC}} + \frac{M_{CB}}{L_{BC}}$:

$$M_{CB} = \frac{29\ M_{BC}}{91} \quad \text{(70)}$$

Conversely, if joint B rotates with end C cut:

$$\theta_B L_{BC} - \frac{V_C L^2_{BC}}{3 E \times 2} + \frac{M_{CB} L_{BC}}{2 E \times 2} = 0 \qquad (71)$$

Substituting $V_C = V_B$, Eq. 71 becomes:

$$\theta_B = \frac{51 M_{BC}}{4.91 E} = - \frac{51 M_{BA}}{4.91 E} \qquad (72)$$

Isolate column AB, cut end B, and rotate support A, then:

$$\frac{H_B L^2_{AB}}{3 E} + \frac{M_{BA} L_{AB}}{2 E} = R L_{AB} \qquad (73)$$

Substituting $H_B = - \frac{M_{AB}}{L_{AB}} - \frac{M_{BA}}{L_{AB}}$:

$$M_{AB} = \frac{M_{BA}}{2} - 3 E R \qquad (74)$$

Conversely, with end A cut:

$$\theta_B L_{AB} + \frac{H_A L^2_{AB}}{3 E} + \frac{M_{AB} L_{AB}}{2 E} = R L_{AB} \qquad (75)$$

Substituting $H_A = H_B$:

$$M_{BA} = - \frac{273 E R}{71} \qquad (76)$$

Therefore: $M_{AB} = - \frac{273 E R}{2 \times 71} - 3 E R = - \frac{699 E R}{2 \times 71}$; and, similarly: $M_{BC} = + \frac{273 E R}{71}$; M_{CB} (Eq. 70) $= \frac{87 E R}{71}$; M_{CD} (Eq. 63*b*) $= - \frac{33 E R}{71}$; M_{DC} (Eq. 59) $= - \frac{33 E R}{2 \times 71}$; M_{CE} (Eq. 63*a*) $= - \frac{54 E R}{71}$; M_{EC} (Eq. 54) $= - \frac{9 E R}{71}$; and $M_{EF} = + \frac{9 E R}{71}$.

Deflecting support D to the left a distance Δ yields, by the same general procedure: $M_{CD} = - \frac{327 E R}{71}$; $M_{DC} = - \frac{327 E R}{2 \times 71} - 3 E R = - \frac{753 E R}{2 \times 71}$; $M_{CB} = + \frac{55 \times 327 E R}{109 \times 71} = + \frac{165 E R}{71}$; $M_{BC} = \frac{1 \times 165 E R}{5 \times 71} = + \frac{33 E R}{71}$; $M_{BA} = - \frac{33 E R}{71}$; $M_{AB} = - \frac{33 E R}{2 \times 71}$; $M_{CE} = \frac{54}{109} \times \frac{327 E R}{71} = \frac{162 E R}{71}$; M_{EC} (Eq. 54) $= \frac{1}{6} \times \frac{162 E R}{71} = \frac{27 E R}{71}$; and $M_{EF} = - \frac{27 E R}{71}$.

Finally, deflecting support F a distance Δ to the left, and solving as before: $M_{EF} = -\frac{150\,E\,R}{71}$; $M_{EC} = +\frac{150\,E\,R}{71}$; $M_{CE} = \frac{8}{25} \times \frac{150\,E\,R}{71} = \frac{48\,E\,R}{71}$; $M_{CD} = -\frac{3}{8} \times \frac{48\,E\,R}{71} = -\frac{18\,E\,R}{71}$; M_{DC} (Eq. 59) $= -\frac{1}{2} \times \frac{18\,E\,R}{71} = -\frac{9\,E\,R}{71}$; $M_{CB} = -\frac{5}{8} \times \frac{48\,E\,R}{71} = -\frac{30\,E\,R}{71}$; $M_{BC} = -\frac{1}{5} \times \frac{30\,E\,R}{71} = -\frac{6\,E\,R}{71}$; $M_{BA} = +\frac{6\,E\,R}{71}$; and $M_{AB} = \frac{1}{2} \times \frac{6\,E\,R}{71} = +\frac{3\,E\,R}{71}$.

The moments caused by a sidesway of Δ will then be equal to the sum of the moments generated by the separate movements of the supports; that is,

$$\left.\begin{aligned} M_{AB} = -\frac{363\,E\,R}{71}\,;\; M_{BA} = -\frac{300\,E\,R}{71}\,;\; M_{DC} = -\frac{402\,E\,R}{71}\,; \\ M_{CD} = -\frac{378\,E\,R}{71}\,;\text{ and } M_{EF} = -\frac{168\,E\,R}{71} \end{aligned}\right\} \quad \ldots (77)$$

By adding up the end column moments $\left(\frac{1{,}611\,E\,R}{71}\right)$, the horizontal force at the top of the frame required to move it sidewise a distance Δ will be $\frac{1{,}611\,E\,R}{71 \times 15}$ lb; but this force has already been found equal to $\frac{3{,}250}{71}$ lb (see text following Eq. 68). Equating the two quantities: $E\,R = \frac{16{,}250}{537}$ lb, a value that can be substituted in Eqs. 77.

The final moments are obtained by adding the moments caused by the vertical load (following Eq. 68) and those caused by sidesway; thus: $M_{AB} = +\frac{38{,}750}{71} - \frac{363 \times 16{,}250}{71 \times 537} = \frac{2{,}500 \times 28}{179}$. Similarly, $M_{BA} = +\frac{2{,}500 \times 69}{179}$; $M_{DC} = -\frac{2{,}500 \times 40}{179}$; $M_{CD} = -\frac{2{,}500 \times 67}{179}$; and $M_{EF} = +\frac{2{,}500 \times 10}{179}$.

Summary.—The writer can offer no criticism of the basic thesis presented by Professor Wilson. The foregoing comments are offered merely in the hope of suggesting an improvement in the complete procedure.

W. C. Spiker,[12] M. Am. Soc. C. E.[12a]—As the author states (see heading, "Synopsis"), in textbooks on structural analysis the application of the principle of superposition is not emphasized as much as its importance warrants. The writer has studied only one of the examples in the paper—Example 1. To that demonstration he wishes to suggest the following simplification: Omit

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[12a] Received by the Secretary June 21, 1944.

Eqs. 5*a* and 5*b* of the simultaneous solution and proceed thus (omitting E for the sake of brevity):

At joint B—

$$4\,\theta_B - 6\,R + 20\,\theta_B + 10\,\theta_C = 24\,\theta_B - 6\,R + 10\,\theta_C = 0 \quad (78)$$

Therefore,

$$\theta_B = 0.25\,R - 0.417\,\theta_C \quad (79)$$

At joint C

$$20\,\theta_C + 10\,\theta_B + 8\,\theta_C - 8\,R = 0 \quad (80)$$

In Eq. 81, substituting the value of θ_B from Eq. 80, and simplifying,

$$23.83\,\theta_C - 5.5\,R = 0 \quad (81)$$

From the foregoing it appears to the writer that when practicable it is much simpler to substitute the value of θ from each preceding joint in the next succeeding equation than to carry "unknowns" through the fundamental equations and then write new equations and solve for the "unknowns."

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

PRINCIPLES OF DEPRECIATION

REPORT OF THE SPECIAL COMMITTEE AUTHORIZED BY THE BOARD OF DIRECTION TO ANALYZE AND DISCUSS THE 1943 REPORT OF THE NATIONAL ASSOCIATION OF RAILROAD AND UTILITIES COMMISSIONERS' COMMITTEE ON DEPRECIATION

Discussion

By C. Beverley Benson, E. M. T. Ryder, Wallace B. Carr, J. L. Campbell, Paul L. Holland, J. Kappeyne, C. Terry duRell, Anson Marston, Terrell Bartlett, E. E. Hart, and Luther R. Nash

C. Beverley Benson,[1] M. Am. Soc. C. E.[1a]—In view of the increasing emphasis on depreciation in modern accounting and by taxing and regulatory bodies, the Society undoubtedly would be well advised to have the report of its 1915 Committee on Depreciation[2] revised and amplified. It seems to the writer, however, that the decision of the Board of Direction and its Special Committee to take a partisan position on the Report of the Committee on Depreciation of the NARUC is particularly ill advised. The term "partisan position" is used by the writer deliberately after careful study of the report of the Society's Special Committee. The penultimate paragraph of this report ends with a sentence which sums up a hostile attitude toward the work of the NARUC committee with these words: "It would be fine if a crystal ball could be used, and the future accurately determined, but this has not been possible to date and, as far as can be foreseen, never will be." This sentence and many that go before it do not represent the dispassionate summing up of objective considerations that should be given to an engineering or scientific problem by

Note.—This report was published in June, 1944, *Proceedings*.

[1] Cons. Engr., New York Public Service Comm., New York, N. Y. (Guest Member, NARUC Committee on Depreciation).

[1a] Received by the Secretary May 15, 1944.

[2] *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 1311.

disinterested investigators. It seems to the writer to represent an intolerant attitude toward the work of a group of conscientious men who worked for nearly four years upon a difficult problem involving important relationships between the public and corporations which operate for the most part under legal monopolies, and also involving highly specialized aspects of depreciation theory and practice.

Nearly all of the criticisms the Special Committee of the Society levels against the NARUC report have been made by utility counsel time and again before regulatory bodies in the past fifteen years and have usually been rejected by the commissions. Furthermore, these criticisms echo those which have been presented repeatedly to the NARUC Committee on Depreciation during its deliberations by the American Gas Association, The Edison Electric Institute, The Institute of Water Supply Utilities, and the American Water Works Association. There is not a thought in the Special Depreciation Committee's report which has not been carefully considered already by the NARUC committee.

The individual members of this Special Depreciation Committee are certainly within their rights in standing up for their own convictions and for the positions of the clients whom they may represent. However, a great divergence of opinion among the membership was reflected in "Fundamental Aspects of the Depreciation Problem: A Symposium,"[3] and it should be pointed out that this Special Committee does not include a single member of the committee which arranged the symposium. It must be kept in mind that a large number of Society members represent regulatory bodies, and many other members have the public rather than the utility point of view.

In the following comments the term "committee" will be used to refer to the Special Depreciation Committee of the Society appointed in February, 1944; and the term "report" will be used to refer to the 1943 Report of the Committee on Depreciation of the NARUC.

In general, the committee appears to regard a conclusion based upon engineering judgment as a "fact" (see heading, "The Concept of Depreciation") and to regard a conclusion based upon mathematics as "theoretical" or arbitrary. It states further that "* * * it is absolutely impossible to arrive at any reasonable answer by any other process * * *" (that is, any other method than engineering determination). This seems to belittle unnecessarily the functions of management and accounting.

The first matter in the report to which the committee "takes exception" concerns the definitions of depreciation. The committee appears to have centered its attention on paragraph 2, page xiv of the report, where some of the factors which appear under the heading, "Current Concepts of Depreciation," in Chapter II are summarized (see conclusion 2, Appendix). The report does not attempt to redefine depreciation. In its eighteen-page discussion of the matter the report starts (page 24) with the very quotations which the committee accepts under the heading "Definition of Depreciation." No attempt whatever is made in the report to change the definition which appears in most "uniform systems of accounts"; rather an attempt has been made to

[3] *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1235.

clarify this field and to discuss some of the many factors which make the arbitrary application of any definition of doubtful value. The committee appears to have devoted all its attention to the summary and hardly to have read the main text.

The committee goes on to state, in connection with its remarks on definitions, "The Society's committee cannot subscribe to the implied opinion of the NARUC committee that remaining service life and remaining service value are synonymous, although there is usually a relationship (not direct, however) between the two." This appears to be more an inference of the committee than an implication of the report. Although certain statistical methods described in the report use estimated remaining service life as a basis for measuring depreciation, nowhere in the report is there a recommendation that a statistical estimate of depreciation based upon any mathematical formula be used by itself as a measure of depreciation (see report, pages 95–96).

Under the heading, "The Concepts of Depreciation," the committee states that the report "* * * fails to present adequately * * * that the extent of that loss of value at any date should be the basis of all accounting and financial policies with reference to depreciation." It is difficult to understand what part of the report is being criticized, since paragraph 12 (page xv of the report) clearly states, "If the depreciation reserve has been properly determined, it measures the accrued depreciation" and paragraph 40 (page XIX of the report) states, "In fixing public utility rates adequate depreciation expense should be allowed according to the service life basis and, in principle, the reserve requirement determined on the same basis should be deducted in determining the rate base."

Under the same heading, the committee goes on to state, "In the last analysis the estimate of depreciation for any period must rest on human judgment." Again the committee appears to have neglected the main text of the report, which, on page 95, uses almost the same words:

> "The estimation of service life of property which has not yet been retired lies, in the last analysis, in the judgment of the estimator. In the exercise of his judgment he should be guided by such factors as his experience, his knowledge of the art and of the particular plant, his knowledge of the plans of the management and of public authority, and, probably most important if available, statistical studies of past retirement experience (including retirements from all causes) derived from the subject property or from similar properties where conditions are reasonably comparable. The weight to be given to each of these factors will depend upon their relative importance in the particular case."

The committee continues (see heading, "The Concept of Depreciation"),

> "Moreover, the property should be re-examined * * * in order to take into account any changes in the probable future usefulness of any of the units. * * * but it is subject to determination * * * by men experienced in the design, construction, and operation of utility properties. It is basically an engineering determination * * *."

Compare these comments with those on page 96 of the report:

> "The weight to be given to the results of statistical studies varies with the completeness and consistency of the data available. Even where these

data are meager they are usually of considerable value. The weight to be given past experience depends also upon the extent to which conditions affecting service life in the future are expected to be similar to or different from those in the past. It is at this point that knowledge of past and present technical [and] economic factors is important. Where changing conditions are evident, it is better in most cases to start with the results derived from past experience and to modify them in order to give reasonable effect to the known or expected developments, than it is to make forecasts without consideration of the past.

"Judgment enters into the statistical estimation of service life, not only in the allowance made for expected future conditions, but also in the statistical analysis of past experience. However, the judgment involved in allowances for future conditions should be based largely upon technical knowledge of the industry, * * *."

It is difficult to see how the report could be in closer agreement with the committee without using identical words.

The committee comments under the heading, "Straight-Line Depreciation": "Also, the data that are available, generally, cover items that are different and usually inferior to those presently existing in the usual utility property." Must one assume that progress has stopped and that items presently in use will not be "different and usually inferior" to those in the future? Must one also ignore the fact that the remaining service lives of items presently in use will be governed on the one hand by their own physical characteristics and the inertia of their own mass which will resist displacement while they still have economic value, and on the other hand by the pressure resulting from changes in the arts, requirements of public authority, etc.? Where does the report state that appraisers are not to consider that "* * * a large percentage of retirements are made for functional reasons * * *" or that they should not consider "the character of the community served, its rate of growth, and the cost of fuel * * *"? Where does the report state that the results found for one locality are to be applied arbitrarily to some other locality? The quotations already made from pages 95–96 of the report indicate that the NARUC committee had these factors in mind and provided for their consideration.

Under the heading, "Straight-Line Depreciation," the committee states, "Straight-line depreciation does not conform to the facts for long life property whose accrued depreciation experience more nearly parallels the accumulation of a sinking fund." But does it? Consider a pipe line delivering a million gallons a day and having an estimated life of 200 years. In one sense, if its capacity remains undiminished, all its depreciation may be said to take place on the day it is taken out of service. In another sense it depreciates 1/200 for each year it is in service. Surely it can never again perform years of service which are past; but a 4% sinking fund would accrue 32.5% of the cost in the last 10 years of the life. In other words, one would have to speculate on interest rates and economic conditions nearly two centuries in the future for almost one third of the investment. Common sense and equity between present and future customers would seem to require that the cost be spread uniformly over the service rendered. The committee appears to overlook the fact that the report is dealing with a special problem in depreciation which has to do with a legal monopoly. The regulatory bodies have the duty to ensure that each

group of consumers pays its fair share of the cost and, having done so, that it shall not be called upon to do so again either in whole or in part, and that it shall not pay a return to the investors on amounts which the customers have contributed toward depreciation.

The committee's comments under the heading, "Straight-Line Depreciation Not Cheapest for Customers," seem naive. There is not the slightest question but that the " '* * * total dollar charges to customers in the long run * * * will be greater under the sinking-fund or compound-interest methods than under the straight-line method' * * *" as shown in the table on page 83 of the report; but, if present value considerations indicate that all methods of depreciation accounting will have identical results, then either all methods are equally acceptable and the committee should have no criticism of the report, or the present value criterion is not a proper one to use in evaluating depreciation methods. Of course, the latter conclusion is inescapable. He who invests his money in a scheme to provide utility service does so with the knowledge that he must pay now for facilities to render future service and to produce future income; but the consumer pays for this service as he obtains it. When the consumer pays for a kilowatt-hour used this month, he is not concerned with the one he paid for last month or the one he will pay for next month. He has no more interest in the present value of future payments for kilowatt-hours than the housewife who buys a pound of spinach today has in the present value of the spinach she will buy next year.

It would seem that the report and the committee agree on the depreciation reserve. The committee states (see heading, "Depreciation Reserve") that

> "* * * the balance in it [the reserve] at any one time should at least equal the amount of accrued depreciation existing at the same time. * * *
>
> "It is submitted that the actual depreciation should control the size of the reserve, rather than that the reserve should determine the actual accrued depreciation."

These statements are to be compared with paragraph 12, page xv of the report: "The depreciation reserve measures that part of the cost of plant still in service which has been written off, usually as an operating expense. If the depreciation reserve has been properly determined, it measures the accrued depreciation." Thus, the report states that the reserve measures the accrued depreciation "if the reserve has been properly determined."

Under the heading, "Retroactive Adjustment of the Depreciation Reserve," the committee appears to depart from principles it has previously stated. In the first place it has stated that (see heading, "The Concept of Depreciation") "Accrued depreciation * * * is a matter of fact and not of theory"; that (see heading, "Depreciation Reserve") "the balance in it [the reserve] at any one time should at least equal the amount of accrued depreciation existing at the same time"; and that (see heading, "Depreciation Reserve") "It is submitted that the actual depreciation should control the size of the reserve * * *." However, under the heading, "Retroactive Adjustment of the Depreciation Reserve," the committee states that "The reserve requirements should be measured by the actual needs of the utility * * *."

It would be difficult to obtain a better estimate of accrued depreciation than that obtained by applying the principles stated on pages 95 and 96 of the report. However, the reserves of many utilities are lamentably low by any standard. This condition has resulted from poor judgment, misfortune, requirements of regulatory bodies, the payment of inexcusably high dividends (particularly to affiliated companies), and in many cases from just plain negligence. The committee appears to think that no matter how the deficiency arose, it should always be made good by the customers and that it is ethically unjust even to consider that the stockholders should pay for the negligence of their management.

Entirely aside from the question of the legality of charging future customers for the services received by past and often different customers, there is the practical question of feasibility. In a recent case before the New York Commission the estimated deficiency was so great that the annual depreciation charge would have to be increased by 50% for the next 12 or 15 years in order to make it up. Such a large future charge seems almost impossible to justify, yet unless something is done the company's books will not represent the facts for many years; and in the meantime the law requires that the rates be fixed in the light of the actual depreciation.

The report recognizes the complexity of the problem and it is difficult to state it more emphatically than in paragraphs 30 and 31 on pages XVII and XVIII

> "30. The depreciative reserve should not be readjusted gradually through modification of the annual depreciation rates when the difference between the book reserve and the proper reserve is substantial. When the difference is not substantial it is satisfactory to spread the remaining net cost of the properties over their remaining lives.
>
> "31. In principle any necessary correction of depreciation reserves should be made through surplus or a special section of the income account. However, the adjustment of inadequate depreciation reserves, while sound in theory, presents many practical difficulties. Where such deficiencies are serious, it is desirable to make every effort to adjust them, although it is recognized that the application of a uniform rule without regard to what is equitable and feasible under the circumstances of individual cases might cause injury to security holders out of proportion to the long-range benefits. Therefore, it is concluded that the objective of correcting inadequate reserves should be approached with appropriate consideration of the practical effects of alternative courses of action."

The adjustment of an inadequate depreciation reserve is a real and, in many cases, a serious problem. Whether one elects to define depreciation as "loss in service value" or "loss in service life" is relatively immaterial. All authorities agree that plant (and therefore the investment in it) becomes impaired or depreciates whether the loss is measured by a straight line or some other method, and all agree that an adequate reserve is desirable. No single method of adjusting inadequate reserves has been suggested by any thoughtful person. Solutions of the problem should be approached in the light of, and be based upon, the conditions surrounding each individual case. The report states this principle in unequivocal language.

E. M. T. RYDER,[4] M. AM. SOC. C. E.[4a]—Several ideas in the NARUC report are criticized by the Society's committee, including "the straight-line method" and the NARUC definition of depreciation as "consumption of service life" instead of "loss in service value."

Life Versus Value.—The Society's committee emphasizes "the engineering method of estimating actual depreciation." However, in its detailed analysis, the Society's committee includes evidence of "elements that would affect probable future usefulness." To discount these elements, thus making them elements of "actual" depreciation is obviously an attempt to determine remaining life—precisely the fundamental objective of the NARUC committee.

Granting that life is actually determined by both methods, the only virtue in determining the condition at any intermediate date is in case it is desired to accrue a depreciation reserve on that basis. Obviously, the depreciation curve will vary greatly with circumstances. For a piece of electrical equipment it may be a straight line of high efficiency abruptly cut off by obsolescence. For steam railroad rails the curve may drop uniformly with the wear of the rails. For a utility with many property items there is little reason to be concerned about the shape of the curve. What is of interest is to have a depreciation reserve large enough to take care of replacements.

For a concern that has been in operation for many years an important factor (probably the major one) is experience. There is no justification for demanding a depreciation reserve equal to the accrued depreciation, which depreciation tends in time to reach a 50% condition for a utility with many depreciable items.

Valuation for Rate Making Purposes.—In the opinion of the writer the U. S. Supreme Court is slowly—very slowly—"turning toward the light." For utilities such as a transit company a return should be allowed on the cost of all property used, irrespective of how much it is depreciated, provided the property as a whole is kept in proper operating condition. This idea is dramatized in the following hypothetical memorandum:

The One Bus Corporation Versus the Supreme Court.—

Enter the Bus Corporation.—Assume that the One Bus Corporation purchased, for $10,000, a bus with a life expectancy of 10 years and went into the transportation business with a rate of fare that provided—

(*a*) Operating and maintenance costs;
(*b*) A depreciation reserve of $1,000 per yr; and
(*c*) A fair return on the original investment.

At the end of 10 years the bus was worn out, the corporation had a reserve of $10,000, and had received an annual fair return on its original investment. It was now in a position to—

(*d*) Go out of business with its capital unimpaired, having earned a fair return on it; or

(*e*) Continue in business under original conditions.

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[4] Way Engr., Third Ave. Transit Corp., New York, N. Y.

[4a] Received by the Secretary July 5, 1944.

Enter the Supreme Court.—The dictum of the Supreme Court is that the foregoing business operation was improper because the corporation should have received a return only on—

(*f*) The original value of its investment less accumulated depreciation on its property; or
(*g*) Historical value less depreciation; or
(*h*) Fair value less depreciation; or
(*i*) Whatever value means; but anyway less depreciation.

If depreciation is assumed to be zero at the beginning, uniform through life, and 100% at the end, the Supreme Court dictum would be fair at the beginning, half fair at the middle, and no fair at the end, and would average a fair return on only half of the capital honestly and necessarily invested in the business.

To meet the argument that no return should be allowed on capital originally invested but returned through the depreciation reserve, it should be noted that realism requires the observation that money in the bank today does not draw interest, and that an investment must be salable at par to buy the replacement bus when needed. Short term loans to municipalities bring only a fraction of 1% interest. If preferred, on the other hand, the depreciation reserve may be accumulated on an actuarial basis so that including interest it reaches a total of 100% just at the minute the bus wears out.

Now consider a more complex property: A 200-mile street railway with rails having a useful life of 20 years. The tracks are in such condition that 10 miles require rebuilding each year, at a cost of $40,000 per mile. Assume an original investment of $8,000,000 with an annual charge to the depreciation reserve (5% of $8,000,000) of $400,000.

How about the argument that interest on the depreciation reserve offsets the loss of return on original investment due to deducting depreciation at 50%? Is it claimed that a depreciation reserve of $4,000,000 should be built up? The money originally put into the property has not been taken out. It is still there. There is no question of "water" or prior profits or lax management or failure to maintain. There is no way of returning the money to the investors, whose only hope is to maintain a going concern that will pay them a fair return on their entire investment, not on half of it.

A depreciation reserve should be merely a protective account which could just as well be omitted entirely by direct charges to maintenance accounts except for the fact that in a property large or small physical items do not wear out uniformly, receipts do not come in uniformly year after year, and unforeseen events do happen. The proper size of a depreciation reserve is a matter of good judgment (and good luck). Rates should be set to encourage making it large enough. Income from it should be added to receipts from the business in the determination of proper rates. Nothing should be allowed to obscure the fact that a fair return should be permitted on the entire amount of money honestly and necessarily invested in the property under reasonable business judgment. A property under public regulation should be safeguarded as well as supervised by such regulation.

A. B. C. Transit Company.—An hypothetical 2,000-bus, or 2,000-car, transportation company has a property that tends to get into a 50% depreciated condition if efficiently managed. Some property items are new, some are worn out, and some are in between. The property as a whole is only above a 50% condition when it is relatively new, or is constantly expanding, or is extravagantly maintained. Depreciation should figure in a rate case only when it has not been possible to keep the property up to a 50% condition, and then the receipts should be increased to remedy this condition.

Now consider the comparison of a fair return on the investment, with a return on present value, reproduced new, less depreciation:

(*j*) With constantly rising prices (such as did generally prevail for many years past) the difference may not be great. Over a period of 20 years, assume a 60% increase in cost and a depreciation of 30%. Then the original cost being 100, the present value produced now is $(100 + 60) - (160 \times 30\%) = 112$. The investor has gained by the theory.

Assuming a cost increase of 50% and a depreciation of 50%, the corresponding present value is $(100 + 50) - (150 \times 50\%) = 75$. In such a case the investor has been cheated only 25%. With a decrease in cost of 20%, with 50% depreciation, however, the comparable present value is $(100 - 20) - (80 \times 50\%) = 40$. In the latter case the investor has "lost more than half his shirt."

Obsolescence, Inadequacy, Municipalities, and Murder.—Assume a street railway track built in a street under normal conditions, with a life expectancy of 20 years. Five years after its completion the city authorities notify the corporation that the track must be removed at the end of the next 5 years.

(*k*) For this case the Supreme Court says that depreciation must be computed on a straight-line basis, which means that the value on which a return may be allowed is reduced immediately from 75% to 50%, and 5 years later (at the end of 10 years) to zero. The normal 20-yr life depreciated condition at the end of 10 years would be 50%, and at that time the track would be just as useful in the service of the public as it was when new.

Why should not the Supreme Court go further and declare that the computations for the previous 5 years should be recomputed on a 10-yr basis also because it turned out that way and one must be logical? Suppose, at the end of the second 5-yr period the city officials changed their minds and permitted the track to remain for its full life of 20 years as planned. Can any one imagine the corporation receiving a rebate?

WALLACE B. CARR,[5] M. AM. SOC. C. E.[5a]—A concise exposition of the errors in several fundamental conceptions of the NARUC report is presented in the report of the Society's committee. The writer will amplify and support somewhat the use of an interest factor in calculations of depreciation and also will present an argument against deduction of the depreciation reserve, a principle

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[5a] Received by the Secretary July 14, 1944.

advocated in the NARUC report but which did not seem to be adequately discussed in the Society committee's report.

In reference to an interest factor in depreciation calculations, the objective of any investment in a capitalistic economy is twofold—namely, the preservation of the integrity of the investment and the earning of a fair rate of return on the investment, while the property representing it is usefully employed. The progressive decrease of the expectancy of the continued earnings must therefore reflect the "present worth" of such expectancy. When the rate of interest employed in the "present worth" calculations is the same as the objective or fair rate, the progressively decreasing present worths will represent the true transition, in a capitalistic economy, of an asset from its initial worth or value to a state of complete worthlessness. When the time element, within reasonable knowledge, is applicable to the problem and an appreciable period is involved, the interest factor is of considerable importance. The usual wide disparity between informed judgment and straight-line calculations will be materially narrowed.

The NARUC report recommends that depreciation should be based upon cost, which, according to most current systems of accounts for utilities, means original cost, and also that the "reserve requirement" should be deducted in determining the rate base. Because it seems probable that, to a large extent, the deduction principle springs from a long train of decisions of the United States Supreme Court—all more or less founded on the oft-quoted cases of Smyth versus Ames (U. S. 169) and Knoxville versus Knoxville Water Company (U. S. 212)—it is pertinent to recall the specific attention given to depreciation in those two decisions. The former simply enumerated, as elements of fair value of property used for the convenience of the public,

> "* * * the original cost of construction, the amount expended in permanent improvements, the amount and market value of its bonds and stock, the present as compared with the original cost of construction, the probable earning capacity of the property under particular rates prescribed by statute and the sum required to meet operating expenses."

The decision indicated other matters that might be given consideration and, finally,

> "What the company is entitled to ask is a fair return upon the value of that which it employs for the public convenience."

No mention of depreciation was made.

The latter decision stated

> "The cost of reproduction * * * diminished by the depreciation which has come from age and use * * * is one way of ascertaining the present value."

and further

> "It [the company] is entitled to see that from earnings the value of the property invested is kept unimpaired, so that, at the end of any given term of years, the original investment remains as it was at the beginning."

It is significant that the NARUC rule of deduction substitutes "original cost" for "value" in the Smyth versus Ames decision, and for "reproduction cost" in the Knoxville decision, and also that this rule becomes the only basis of determining a rate base whereas it was originally expounded as only one element of the process of finding present value.

Assuming for the moment that the calculated "reserve requirement" is equal to the depreciation reserve and that the entire reserve has been reinvested in the utility to provide part or all of the required growth, the deduction principle of course limits the earnings to an amount represented by the allowable rate of return on the original and subsequently invested capital—that is, essentially bonds, stock, and surplus. This does not measure the entire property actually used and useful in the public service. There would be no compensation for the larger risk assumed in the management and operation of the larger property created by the combined capital and reserve. The reserve can be used only temporarily in this way because it must be made available to renew the property upon its retirement and to continue the required service. Since the accruals to the reserve are only sufficient for this renewal or replacement obligation, they cannot cover the cumulative growth and their use for this purpose must be temporary. Additional capital financing must inevitably be required so that the temporary use of the reserve becomes merely a borrowing operation substantially equivalent to any other similar financing transaction. Since the reserve is dedicated to the continuation of the same or equivalent service, this service must have a continuous value as of its date of origin until, by retirement of outmoded property, equivalent or better service can be furnished at equal or less cost. The mere physical deterioration of property, or its gradual approach to a state of inadequacy for the service required, does not in any way affect the current value of the service, provided such deterioration or inadequacy is not permitted to accrue to a degree actually affecting the quality of the service.

Attempts to justify the deduction principle have sometimes been made on the basis that the result represents the net investment of the security owners. This inescapably subtracts from the property as a whole that part of the total represented by the reserve. The investment thus deducted is denied earning power. It is difficult to understand what difference, so far as over-all earning power is concerned, there can be between any items of property in a single enterprise created by any particular investment or source of capital. In further support of this principle, if it is suggested that the earning power of the reserve is reflected in reduced rates, then a non-risk investment is credited with the composite rate allowed to risk and non-risk capital.

The practical result of deduction of the "reserve requirement" based upon straight-line depreciation is easily evaluated in a normal electric utility. The "reserve requirement" will be from 30% to 35% of the fixed capital (plant and property) so that earnings at an allowable rate of 6% on net fixed capital after deduction would mean an actual rate of return on the property of only 3.9% to 4.2%. There are, of course, many other factors affecting the flow of capital in investment channels, but it is questionable whether the utility industry can

successfully compete in the capital market for its required funds on such a limited return basis.

Since the use of the reserve by the utility for purposes other than property renewals is merely a borrowing operation, it remains to consider the essential character of that transaction and to evaluate properly the equities involved. Certainly, as owners of the enterprise, the security holders assume the entire risk in the successful investment of the reserve funds; the consumers assume no risk at all, particularly when protected by regulatory commissions. Thus, the terms of the borrowing should reflect its non-risk character. The equity of the consumers in the reserve or in its normal earnings should not exceed a non-risk rate of interest and the remainder of these earnings should be applied to the benefit of those who have assumed the risk. This principle is well stated in the Report to the Board of Public Utility Commissioners of New Jersey on the Rate Adjustment Plan for the New Jersey Power and Light Company by J. Rhodes Foster, consultant, and H. J. Flagg, M. Am. C. E., chief engineer of the Commission, as follows:

> "The Utility is entitled to compensation for management of the investment and for performance of the risk-taking function. Unless the enterprise is reasonably compensated, management might be expected, in the absence of regulatory restraint, to reduce the risk and responsibility by investment in government bonds or other relatively risk-free securities. Deduction of the balance in the depreciation reserve is, in effect, the same as charging the income of the Utility with interest at the overall rate of return."

J. L. Campbell,[6] M. Am. Soc. C. E.[6a]—Differences between methods of finding depreciation are not as important as the use made of the findings. The sponsors of the railway valuation act believed the railways were greatly overcapitalized and that a fair return upon the cost of reproduction new would call for large reduction in railway rates. Great was their astonishment when federal valuation showed that such reproduction would be greater than existing capitalization.

Ignoring the sound dictum that depreciation is the "loss of service value not restored by current maintenance," cost of reproduction new less depreciation was set up as the maximum upon which a fair return should be allowed.

On the day a railway plant is new, complete, undepreciated, and ready for service, it has service capacity and value which increase year by year during a development period of solidification, consolidation, and adaptation of plant and personnel to the service rendered until maximum service capacity and value are attained and maintained thereafter by adequate current upkeep. Were railway plant and personnel brand new every morning, it would never—and never could—attain that maximum.

Now mark a conclusive fact. During the foregoing development, the undepreciated plant becomes a depreciated one while its service capacity and value increase. This depreciation, however, is applicable only to the individual units of the plant and is not applicable to the plant as a going, serving concern.

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[6] Chf. Engr., Northwestern Pac. R. R. (Retired), Oakland, Calif.

[6a] Received by the Secretary June 28, 1944.

It averages a relatively small percentage and continues thereafter on a substantially level plane in all adequately maintained railroads—a common degree of maintenance prevalent throughout the United States. So maintained, the railway plant is immortal, and its service capacity and value remain undiminished by the depreciation in its parts.

It follows that deduction of such depreciation as a basis for rate making is wrong.

Paul L. Holland,[7] M. Am. Soc. C. E.[7a]—It is unfortunate that this committee, appointed to express, in a formal way, the opinion of the Society on a subject of such importance, did not include representation from the large number of members who are daily engaged in some phase of this vexatious problem of utility depreciation. Perhaps the report might have been of more value in finding a solution to this problem if the point of view of this segment of the Society had been presented.

In the "Foreword" to the report appears the expression, "the contrary views of civil engineers." It is pertinent to note at this point that three of the seven engineers on the NARUC Committee on Depreciation are members of the Society, as are the chief engineers of the regulatory commissions in Massachusetts, Connecticut, New York, New Jersey, Pennsylvania, Maryland, and West Virginia, not to mention the chief engineer and many members of the staff of the Federal Power Commission, a number of commissioners, and others. Probably not one of these engineers will agree with the report of the Special Committee. Perhaps the label, "the views of civil engineers," is a misnomer.

Despite the wide latitude granted to administrative and semijudicial bodies by the recent decisions of the Supreme Court (namely, in the Hope Natural Gas Case, the Natural Gas Pipe Line Case, and less directly, but just as effectively, in the cases of Burford vs. Sun Oil, National and Columbia Broadcasting Companies, F.S.A. vs. Quaker Oats Company, and others), utility regulation is still a most complicated process. Methods followed until quite recently have been ineffective in that final decisions in some cases have been delayed ten to fifteen years (see Lone Star Gas Case, the McCardle Case, the Illinois Bell, and the Oklahoma Packing Company). Under such circumstances, the changes in economic conditions, the turnover in customers, and the lack of confidence engendered by such delays militate against equitable treatment of consumers and investors. When the costs of such rate cases are permitted as operating expenses, the principal beneficiaries in the proceedings are legal and engineering consultants. Those who hope to see a continuation of privately owned and operated utility industry will do well to recognize certain aspects of the problem and be realistic in the approach to a solution. They can ill afford to permit personal or private bias to warp professional theory. Prompt, effective, and equitable regulation, as envisaged by the Supreme Court in its recent decisions, calls for a change in methods. The action of the NARUC Committee on Depreciation in amplifying the definition of deprecia-

[7] Chf. Engr., Public Service Comm. of Maryland, Baltimore, Md.

[7a] Received by the Secretary July 14, 1944.

tion in accordance with the present concept of regulatory bodies and in framing a practical and equitable method of applying the theory represents a long step forward in the realistic approach. Likewise, the suggested method of dealing with the depreciation reserve, as regards the determination of a rate base, is equitable to all. A careful study of the report will reveal this intent of equitable dealing, as evidenced by frequent explanatory notes, calling attention to the difficulty of rigidly applying a sound theory to practical problems, and by provision for the exercise of commission judgment in those cases where inequity might result from strict adherence to the recommendations. All of these explanations and comments, found in the text of the NARUC report, which indicate the thorough manner in which the work was done and the earnest attempt to deal fairly with a problem forced upon regulatory bodies by past abuses, have been ignored by the Special Committee, at least so far as its report indicates.

In the specific critical comments by the committee, there are four principal points of divergence, as follows:

1. Fundamental Concept of Depreciation.—It is perhaps fortunate that this committee is not charged with the formulation of such a concept. That is a function of the regulatory bodies. The committee quotes with full approval the definition of depreciation given by Mr. Chief Justice Hughes in the Lindheimer case (292 U.S. 167). Additional light would have been thrown on the entire subject if the committee had quoted also pages 168 and 169 of this decision. To indicate the pertinence, see the following:

> "If the predictions of service life were entirely accurate and retirements were made when and as these predictions were precisely fulfilled, the depreciation reserve would represent the consumption of capital, on a cost basis, according to the method which spreads that loss over the respective service periods. But if the amounts charged to operating expenses and credited to the account for depreciation reserve are excessive, to that extent subscribers for the telephone service are required to provide, in effect, capital contributions, not to make good losses incurred by the utility in the service rendered and thus to keep its investment unimpaired, but to secure additional plant and equipment upon which the utility expects a return."

Under the present accounting method the accuracy of depreciation charges is entirely up to the utilities. Instruction 10—General, of the Uniform System of Accounts prescribed for electric utilities, provides that the utility shall record as at the end of each month the estimated amount of depreciation accrued during that month. Mr. Chief Justice Hughes, in the case just referred to, enumerated certain elements to be considered in determining the currently accruing depreciation. There is no prescribed limit to the elements which may be considered, nor to the frequency of consideration.

The hair-line distinction between "consumption of service life, capacity, or utility" and "loss in service value not restored by current maintenance" is not of importance when compared to the committee's partisan condemnation of the report as a whole. As to the "implied opinion" that service life and service value are synonymous, the NARUC committee apparently does not

subscribe thereto to any greater extent than the Special Committee. In fact, one of the principal differences between the views of the two groups as to the concept of depreciation lies in the measure or extent of the annual loss and the accumulated or accrued loss at any specific time. This can be determined each year or each month by any method the utility may deem proper, subject only to such reasonable limit as may be fixed to prevent extravagance or fraud.

2. Straight-Line Depreciation.—The comment as to preference "ostensibly on the grounds of simplicity" is incomplete, as will be apparent from a careful reading of Chapter V of the NARUC report. Pages 86 to 91 refute in full the criticisms as to inaccuracies or inequities.

3. Straight-Line Depreciation Not Cheapest for Customer.—This comment is wholly unnecessary, since the report sets out the entire story, whereas the excerpt quoted by the committee only tells a part. Furthermore, the inference drawn from a single sentence is distorted. The mathematical calculations in the NARUC report are correct, and the statement is neither incorrect nor misleading; nor would it be if founded upon the assumption that rate payers are less provident in regard to the payment for utility service than the utilities themselves. If the customers' ledger of a utility company remained unchanged over the service lives of the property consumed, it might be feasible for customers to purchase annuities with which monthly bills could be paid. Such a scheme must be what the committee had in mind. However, this just is not done, even in the best regulated communities—a fact well known to the members of the Special Committee.

4. The Depreciation Reserve.—Regardless of the opinion of the committee as to what the reserve for depreciation should include, the rules legally promulgated in connection with the Uniform System of Accounts provide that no sums other than the estimated amounts of currently accruing depreciation shall be charged to operating expenses and credited to this reserve. The accumulation in this reserve of excessive amounts, thus representing accretions to capital, was condemned by Mr. Chief Justice Hughes in the Lindheimer case. The committee's comments are not consistent with those of the Chief Justice, whose definition of depreciation is approved so thoroughly. As to deduction of the reserve, properly computed, the committee is referred to a more recent case, decided on January 6, 1944. This opinion, in the Hope case, is and will be considered by regulatory commissions as of more validity than that of the Special Committee. Such treatment of a reserve accumulated from charges to customers, which include reasonable sums for current consumption of the property in use, is not a new or novel idea, nor did it originate with the NARUC committee. In 1912, the late J. E. Allison, M. Am. Soc. C. E., chief engineer and member of the Public Service Commission of St. Louis, Mo., in a report to that body, had this to say:

> "But after depreciation charges have been established by legal regulation and are paid by the consumer, then for the future any accumulation which is or should be in the depreciation fund as the result of these charges is a repayment of investment by the consumer and should be deducted from Gross Value to determine the Just Amount upon which to base reasonable returns."

Testimony before public service commissions in this state and elsewhere during quite recent years leads to the conclusion that, as to accuracy and realism, the methods suggested by the Special Committee are sometimes quite inferior to the mathematical calculations involving statistical data. Some of this testimony relates to fantasies rather than to facts.

Aside from its purely technical aspects, the report is disappointing in that the committee failed to make a practical or constructive contribution toward the solution of a problem of national interest and importance. Utility regulatory commissions are among those administrative agencies upon which Judge John J. Parker, U. S. Circuit Court of Appeals, 4th Circuit, in an address before the Maryland State Bar Association on July 1, 1944, commented, stating

> "We might as well face that situation realistically. These agencies are here to stay. They have been developed as the result of the need for some manner of control over the economic life of the nation. The people realize that, without the exercise of some such power by the Government, they are helpless in the hands of those who would otherwise direct the course of economic forces. * * * The choice is not between laissez-faire and Government regulation but between Government regulation and some form of collectivism. If we expect to preserve our system of free enterprise, we must make our system of administrative agencies work. And the question to which the thinking lawyer will address himself is not whether the administrative agencies should be abolished, not how they may be hamstrung and crippled, nor even how they may be made like courts, but how there may be developed for them an efficient procedure that will preserve the spirit of fair play and equal justice which is the glory of our democratic institutions."

The Society has repeatedly proffered its services to the government in connection with matters of professional interest. The Special Committee had a wonderful opportunity to suggest such modifications in current procedures and techniques of these regulatory bodies as would lighten or expedite their tasks. In this aspect, it is of interest to examine the committee's report.

The total book cost of utility property for which depreciation accounting is now prescribed is approximately 50 billion dollars. To determine accrued depreciation, and monthly accruals to the depreciation reserve, the committee suggests engineering studies at intervals of five to ten years, such studies to be made in accordance with the plan outlined under the heading, "The Concept of Depreciation." This plan, as set out in the report, is as follows:

> "The modern methods of determining depreciation include some observations of the property, but they include much more. This modern method might properly be called the engineering method of estimating actual depreciation.
>
> "The engineering determination of accrued depreciation involves many operations and analyses in addition to what has been referred to as 'observing' it. Operating records are studied, including past and probable future growth of the system, the types of equipment, their functional features such as adequacy and obsoleteness, and many other elements that would affect probable future usefulness. Poles are inspected and butts measured to determine their actual condition; maintenance records are examined to determine the care that has been given to the upkeep of the property; deferred maintenance, if any, is ascertained; records of past

retirements are analyzed, and in fact all data having a bearing on the future usefulness of the units involved in the particular property in question are studied.

"When the depreciation of a property has been determined in the aforedescribed manner, the actual depreciation thus established is factual and affords a sound basis for appropriate financial and accounting procedures. The Society's committee contends that accrued depreciation can only be ascertained accurately by this method."

There is no doubt that determination as made in accordance with this thorough plan will be of value. In fact, the suggested procedure includes not only what the Chief Justice proposed in the Lindheimer case, but also what the NARUC committee suggests as the basis for determining these charges. The plan is all inclusive.

Assuming such studies to be made at the mean of the periods suggested—namely, at intervals of 7½ years—involves the detailed study, inspection, and report on approximately 7 billion dollars worth of property each year, a sum almost exactly equal to the total capital expeditures of the entire electric light and power industry from 1929 to 1944, inclusive. The committee comments upon the "unrealistic" results of the recommendations contained in the NARUC report!

Such statements as those submitted by the Special Committee, and the concomitant implications, if unchallenged, may bring upon the Society such condemnation as that recently laid upon another learned society—that its activities partake of the nature of a trade association rather than of a professional society.

J. Kappeyne,[8] M. Am. Soc. C. E.[8a]—Accounting is the art of recording costs. Proper accounting requires the recording of all costs incurred. As capital assets eventually must be written off, it is proper that the accounting therefor be recorded during the period in which such assets are useful in the business. In many instances the straight-line method has certain advantages over others for recording these costs.

Depreciation is a loss in value. Existing depreciation is a fact, the determination of which is an engineering function rather than an accounting problem. Costs do not depreciate.

Hence, the writer cannot subscribe to the statement of the Society's committee that (see heading, "The Concept of Depreciation") "* * * loss of value * * * should be the basis of all accounting * * * with reference to depreciation," because so-called depreciation accounting is a process of allocation and not of valuation.

It is unfortunate that such terms as "depreciation" and "value" have been used in prescribing systems of accounts. Instead of "depreciation reserve" the term "amortization reserve" appears to be less open to misunderstanding. For the term "service value," "net cost to be amortized" or another suitable term might well be substituted.

[8] Cons. Engr., Brooklyn, N. Y.

[8a] Received by the Secretary June 22, 1944.

The question of "value" and hence of "depreciation" in public utility regulation arises when the utility claims that its property has been taken without just compensation, whether in a compulsory sale or in a rate adjustment case.

The determination of "costs of assets" and related "reserves" is required in reorganization and security issue cases and of course in the establishment of proper accounting methods.

These are two separate concepts, requiring different applications of related but not identical dollar amounts. The failure to distinguish properly between these two concepts of "costs" and "value" has resulted in the fundamental objection to the NARUC report. It is suggested that the Society's committee should stress these points more fully in any subsequent report.

The statement (under the heading, "Depreciation Reserve") that "* * * the reserve * * * is in a sense a trust fund * * *" is poorly worded. A fund is an asset, a reserve is a liability. A reserve is not a fund. The assets offsetting the reserve might be considered in the nature of a trust fund. The writer agrees that such assets "* * * should not be considered a return of capital * * *."

Finally, the writer wishes to compliment the Society's committee for its expression (see heading, "Retroactive Adjustment of the Depreciation Reserve") that "* * * the adjustment of the current reserve upward so as to equal those computed on the straight-line basis by withdrawal from surplus or otherwise is not only generally financially impracticable but is also ethically unjust."

C. Terry duRell,[9] M. Am. Soc. C. E.[9a]—The report of the Special Committee of the Society is highly commendable for its accuracy although it might have contained proof of some of the statements that may appear self-evident to many. For instance, consider the reference to the NARUC statement under Conclusion 24 concerning the straight-line method. The committee states that it "is incorrect and misleading," which is perfectly true, but people, not so familiar with depreciation, might wonder which to accept as correct. Had the committee inserted a hypothetical table, its statement would have been proved at a glance. Table 1 illustrates the point. A graph showing both straight-line depreciation and actual depreciation would have given eye proof of the statement, "Straight line depreciation does not conform to the facts * * *."

Definition of Depreciation.—The committee approaches the question of definition, selects correct ones but omits the important financial aspect of "Why is depreciation." Several factors in the definition indicate great hazard. Therefore in its application to accounting, some factor of safety should be applied. A very fundamental of business is that not only must there be compensation for the use of capital and the risk of losing it but that it must be returned intact. It is assumed that real estate is not a wasting asset except for contained mineral, the capital invested in which is returned through

[9] Constr. Engr., Engineered Constr., Inc., Redondo Beach, Calif.

[9a] Received by the Secretary July 15, 1944.

depletion. Provision for the return of capital in intangible assets is through amortization. Current assets are continually being liquidated so there remains that portion of capital represented by assets being lost in the five or seven ways enumerated in the definitions and which cannot "be seen with the naked

TABLE 1.—Comparison of Annual Income Taxes, in Dollars, as Affected by Methods of Computing Depreciation (Cost of Property $1,000; Return Fixed at 8%, 25-Yr Useful Life)

Year	Annual Depreciation, 6% Sinking Fund			Straight line	Net earnings	Taxable Income		Annual Taxes—12%	
	Annuity	Interest	Total			Sinking fund	Straight line	Sinking fund	Straight line
1	18.23	....	18.23	40.00	98.23	80.00	58.23	9.60	6.9876
2	18.23	1.09	19.32	40.00	98.23	78.91	58.23	9.47	6.9876
3	18.23	2.25	20.48	40.00	98.23	77.75	58.23	9.33	6.9876
4	18.23	3.48	21.71	40.00	98.23	76.52	58.23	9.18	6.9876
5	18.23	4.78	23.01	40.00	98.23	75.22	58.23	9.03	6.9876
6	18.23	6.17	24.40	40.00	98.23	73.83	58.23	8.86	6.9876
7	18.23	7.63	25.86	40.00	98.23	72.37	58.23	8.68	6.9876
8	18.23	9.18	27.41	40.00	98.23	70.82	58.23	8.50	6.9876
9	18.23	10.83	29.06	40.00	98.23	69.17	58.23	8.30	6.9876
10	18.23	12.57	30.80	40.00	98.23	67.43	58.23	8.09	6.9876
11	18.23	14.42	32.65	40.00	98.23	65.58	58.23	7.87	6.9876
12	18.23	16.38	34.61	40.00	98.23	63.62	58.23	7.63	6.9876
13	18.23	18.45	36.68	40.00	98.23	61.55	58.23	7.39	6.9876
14	18.23	20.65	38.88	40.00	98.23	59.35	58.23	7.12	6.9876
15	18.23	22.93	41.16	40.00	98.23	57.07	58.23	6.85	6.9876
16	18.23	25.51	43.74	40.00	98.23	54.49	58.23	6.54	6.9876
17	18.23	28.14	46.37	40.00	98.23	51.86	58.23	6.22	6.9876
18	18.23	30.82	49.05	40.00	98.23	49.18	58.23	5.90	6.9876
19	18.23	33.82	52.05	40.00	98.23	46.18	58.23	5.54	6.9876
20	18.23	36.83	55.06	40.00	98.23	43.17	58.23	5.18	6.9876
21	18.23	40.24	58.47	40.00	98.23	39.76	58.23	4.77	6.9876
22	18.23	43.71	61.94	40.00	98.23	36.29	58.23	4.35	6.9876
23	18.23	47.39	65.62	40.00	98.23	32.61	58.23	3.91	6.9876
24	18.23	51.41	69.64	40.00	98.23	28.59	58.23	3.43	6.9876
25	18.23	55.57	73.80	40.00	98.23	24.43	58.23	2.93	6.9876
..	455.75	544.25	1,000,000	1,000,000	2,455.75	1,455.75	1,455.75	174.69	174.69

eye" for which some sure and definite return must be made. Repairs and maintenance, increasing in cost with age, can never completely restore this plant asset to its original condition. Possibly it may have a final salvage or scrap value which, in a small measure, may restore part of this capital. Some items of this asset will have to be replaced long before the complete return of this capital. The time for such replacement has been expressed carefully in a formula by P. B. Bucky.[10] While there is agreement that some method should be devised for the return of this capital represented by plant (buildings, structures, machinery, etc.), there is much disagreement as to the method, although all advocates of any method term it "depreciation" for lack of a specific name for each method. The source of this capital, be it individuals, stockholders, or trustees, is not concerned as much with the method of its return as with the sureness.

Methods of Depreciation.—The following list of depreciation methods has been grouped and tabulated from those described by Earl A. Saliers:[11]

[10] *Mining and Metallurgy*, February, 1930, p. 99.

[11] "Depreciation," by Earl A. Saliers, 3d Ed., Ronald Press, New York, N.Y., 1939.

A. Formula methods
 1. Straight line
 2. Reducing balance
B. Formula methods requiring interest calculations
 1. Sinking fund (compound interest method)
 2. Annuity (sinking fund actually created)
 3. Equal annual payment (sinking fund not created)
 4. Unit cost (another form of sinking fund formula)
C. Methods requiring no mathematical formula
 1. Based on return of invested capital
  a. Amortization (cost spread over a definite period)
  b. Production (cost spread over units to be produced)
  c. Revenue (by utilities for inadequacy, supercession and obsolescence)
  d. Appraisal (at beginning and end of a period)
 2. Based on replacements only
  a. Replacement cost (for financing replacement units)
  b. Renewal expense (renewals all charged to expense)
  c. Retirement expense (as replaced, original cost charged to expense)
  d. Retirement reserve (no depreciation if plant efficiency is cared for)

It is a misnomer to call any of the foregoing methods of depreciation in the light of the definition for depreciation as given because the curve of none of them corresponds to a true depreciation curve or to a mortality curve. The report states that "* * * depreciation experience more nearly parallels the accumulation of a sinking fund" and favors the use of the sinking fund or the annuity methods. A method that fails to recognize depreciation as defined, and which fails to provide for the return of this loss to an investor by sound business and accounting methods, is unworthy of consideration. This eliminates methods 1c, 2a, 2b, 2c, and 2d in class C. Amortization 1a, class C, is ideal for fixed life assets of comparatively short lives, such as patents, leases, etc. Production method 1b, class C, fails to recognize depreciation during periods of little or no production. It is applicable where future units can be determined fairly accurately, as in mining operations, but not with such indeterminate units as miles, kilowatt hours, cubic feet, etc. Its actual application is not as simple as first-sight appearance indicates. Appraisal 1d, class C, at the frequent accounting intervals necessary is too apt to be left to someone incapable of such appraisals and will cause inconsistent fluctuations in manufacturing or production costs. The reducing or declining balance method 2, class A, has the good point of returning capital rapidly during the early stages of operation when repair and maintenance charges are low to give a more even operating cost throughout the entire period, but it is the extreme opposite of interest or sinking-fund methods.

The picture is not so good for short-lived items, as is shown by an example. Using this method for a $5,000 asset and a scrap value of $100 at the end of a useful life of 5 years, the formula gives an initial charge or deduction of 54.27% or nearly triple the amount determined by the straight-line formula. This is

hardly sound business and the method is rarely used. There then remains for consideration the four methods using interest calculations and the straight line. The committee states that "it is manifestly sound business to provide for the obligation maturing at a remote time by an annuity * * *." Theoretically this procedure may be correct but it is followed so seldom, using an actual sinking fund, as to be almost negligible. Business men give various reasons for not using compound interest or annuities, such as, "We can't be bothered with that," "Too much bookkeeping," etc., but fail to give valid reasons. However, there are good reasons favoring the straight-line method besides simplicity.

Operating Cost.—The nature of its definition shows depreciation to be a true operating cost which should appear with other manufacturing or production costs, such as labor, light, heat, power, fuel, repairs, etc. Any of the foregoing methods not eliminated furnishes a means for the determination of an amount to be thus debited. With these methods, the original bookkeeping entry for doing this may be: "Depreciation (Dr.) to Depreciation Reserve (Cr.)." This puts it into the unit (monthly) period cost sheet but shows that no money was actually paid out of the business for that period for this item; so there is no voucher for it as there are for other cost items. From Table 1 it is seen that the annuity is the same yearly as is the case with the straight-line method, although the amounts differ. In either of these methods (1 in class A or 2 in class B), this amount can be obtained by taking a percentage of the asset cost, the percentages differing. An additional entry must be made to take the annuity out of the business and also with reference to the interest to be reported as income. With the other interest methods (1, 3, and 4, class B), the formula must be applied to each depreciable asset for each (yearly) unit period of time. Also, more time is required for the calculation—more and larger numbers (logarithms) must be used on the calculating machine.

Straight-Line Method.—It was shown that five of the outlined methods should be eliminated, that one had little merit for cost accounting, that two were good in certain instances, that one wrote off the assets too fast, and that a choice was left between the four interest formula methods and the straight line. It is claimed that, because of the interest factor, depreciation curves from the use of these four methods more nearly resemble the true depreciation curve of which the straight line has nothing but the starting and end points in common. Why then is the straight-line method better for the purpose of determination of the annual or unit period charge, called "depreciation" for want of a specific name, in the total cost or production or service? Some of the most important reasons are: (1) Cost accounting, (2) depreciation reserve, and (3) sinking-fund interest.

1. Cost Accounting Reason.—Certainly it is poor business for a firm or corporation to delude itself or stockholders with the belief that withdrawals or dividends represent interest on capital invested if, in reality, part represents return of capital itself. Look to the details to see how easily this may happen. In manufacturing or production costs an amount must be included, termed "depreciation," which, according to definition, represents five to seven variable

items that are likely never to be the same for two consecutive unit periods of time. Accounting should care for this hazard, somewhat as follows:

Step	Accounting Procedure	Resulting information
1	....	Income from sales or service
2	Deduct the total cost including depreciation	Gross profit
3	Deduct selling expense	Selling profit
4	Deduct total administrative expense	Operating profit for the period
5	Add any outside income	Total income
6	Deduct local taxes, insurance, bond interest, note interest, etc.	Net profit for the period
7	Deduct income taxes, donations, etc.	Profit and loss statement
8	Close out profit and loss account to surplus account	....

Withdrawals are made, or dividends paid, from Item 8, the surplus account. In the event that the item known as "depreciation" was too small, withdrawals or dividends were too great. In other words, part of the depreciation was return of capital and the receipients were deluded. Quite true, engineers insist on working things out in accordance with facts but after using all the facts in a formula to determine the ultimate load, a large factor of safety is applied to determine a safe working load. "Straight-line depreciation does not conform to the facts," but it does give a good safety factor for these seven immeasurable variables. It would seem better business to follow along this straight line aimed at the same proposed end point than to follow along on a beautiful curve slightly below the rupture limit. The cost item deducted as depreciation includes an amount, in addition to actual depreciation, in the same way that the term interest is made up of two items: (*a*) Rent or the return for the use of money, and (*b*) the risk of losing the money. The actual depreciation is not the only factor to be considered in making this cost deduction. The straight-line method takes care of these other conditions better than any other known method. A similar analogy may be used with reference to the fixation of a unit cost or selling price for manufactured goods or service.

2. Depreciation Reserve Reason.—Certainly it is poor business to borrow money on inflated assets. It is the same as buying stocks on too small a "margin." The crash may come at any time. Consider the details to observe how this happens. As was stated previously, a depreciation reserve account is merely an "earmarked" part of surplus account out of which no dividends or withdrawals may be paid. If created by the straight-line method, no money represented by it was paid out, although it was included with manufacturing or production costs for which money went out of the business. With some of the interest formula methods, money is paid out into a sinking fund. An overstatement or an understatement of depreciation reserve account either increases or decreases surplus account by that amount. Although possible.

it is unlikely that the amount shown by the depreciation reserve account ever represents the actual accrued depreciation to that day no matter what method was used in its determination. What difference does it make whether it does or not and why all this perturbation concerning it? The depreciation reserve is absolutely necessary for accounting purposes. Although it may appear as only a single item or account on financial statements or in the general ledger, there should be a plant ledger, possibly in several volumes, showing every depreciable item. For accounting purposes, this depreciation reserve shows the progress being made toward the return of the capital cost of the various items and prevents over-return. By this means, overstatement of depreciable assets and misrepresentation to the firm or to the stockholders may be prevented. If the total in this account be low compared to the total cost of offsetting assets, it may be possible to borrow more money through notes or bonds, or it may be possible to secure a higher rate for service because of claimed necessity for interest rate on greater capital. For this reason, many utility companies have been willing to pay higher income taxes than they should in order to leave accounts undisturbed which might be the cause for lowering their rate and giving them a smaller income. The straight-line method is here seen to build up a greater reserve during the early life, to present a more conservative basis for loans, and to more truly represent a balance between it and cost for which the property might be sold in case of liquidation. There are comparatively few old firms or corporations in the United States. It is the exception when one exists beyond a period of twenty-five years without at least being refinanced. Thus depreciable assets should be returned during the earlier years. This money then remains on the debit side to offset credits in the way of stock and bonds. Obsolescence wrecks many an engineer's long-life estimate. F. W. Altstaetter,[12] M. Am. Soc. C. E., has well illustrated this fact in describing how double-track bridges of the Mexican Central Railroad became obsolete before time to use them. Charles Schwab, in speaking of a steel mill he built, stated that Andrew Carnegie, accompanying him through it shortly after its completion, remarked, "Charlie, you do not seem very enthusiastic about it." He told Mr. Carnegie he had made a mistake and that the production costs were too high. Mr. Schwab was told to tear it down and rebuild it, which he did. Witness over the United States "ghost plants"—enduring concrete buildings with nothing but echoes of the once beautiful electric generating machinery that became obsolete at the close of World War I when production costs had to be lowered by a lesser fuel consumption and whose machinery went into furnaces as scrap during the past few years. Also witness the abandonment and scrapping of both steam and electric railways, main line and interurban, since the 1920's because of obsolescence and the disrespect of many men connected therewith who were previously respected citizens of those communities—all this because of unsound finance methods. Therefore, the straight-line method seems more applicable for long-lived items in order to offset early obsolescence.

The difference between accrued true depreciation and cost is no criterion of present worth of a plant that can only be shown correctly by its earning

[12] *Civil Engineering*, June, 1944, p. 264.

power. In other words, an evaluation of any merit must be made by the "present value method" which discounts future expectancies over a reasonable period based on past experience. For a long period, astronomical figures of no value for natural resources are obtained. Present worth curves of 4% through to 20% flatten preceptibly after 30 or 40 years, so that an arbitrary life of no more than 40 years should be the limit. There are so many cases of profits too low even to return the cost of plant and equipment without consideration of a fair interest rate on capital invested. In one particular industry of more than 5,000 operations by individuals, firms, and corporations, fewer than 150 operations yielded sufficient earnings to return the plant and equipment, which meant that the other enterprises had no value. However, this is no reason for not determining what is termed "depreciation" and "depreciation reserve" for which there is no short cut over the method outlined by the committee, not once and for all but at intervals, preferably of much less than the 10-yr limit mentioned. Many depreciable assets are being discarded continually without proper book adjustment. Also, over-depreciation is apt to occur and improper adjustments for replacements are apt to be made.

Sinking-Fund Interest Reason.—Recommendation of a sinking-fund method is prima-facie evidence of lack of faith in the management or lack of faith in the ability of the business to pay an interest rate commensurate with the risk. Liquid assets, consisting of cash and securities readily converted into cash, are necessary for the everyday transactions of the business and, in part, may have been derived from loans listed among the liabilities as notes and bonds. Depreciation (accounting sense) represents a transition from the fixed assets through operating cost to liquid assets. It is quite easy for a poor or dishonest management to dissipate at least a part of the liquid assets. Because of risk, the business must pay interest at a rate far above a safe rate on the capital invested which is represented by the sum of the fixed and liquid assets on the one hand and by obligations to investors on the other. Taking money out of liquid assets to put in a fund at a low rate of interest is antagonistic to the very fundamentals of that business, especially if part of the liquid assets are offset by loans secured by means of mortgages on the fixed assets. If an engineer or any one fears dissipation of liquid assets or an interest rate to investors at or below a safe rate and recommends such a procedure, he had better be consistent and go a step further by recommending the liquidation of the business and the placing of all moneys from the proceeds in funds at a safe rate of interest so as to surely return all possible to the investors at some distant future date. This fact alone is sufficient justification for the elimination of any of the four interest formula depreciation methods involving such procedure. This applies especially to method 2, class B, annuity method, and to methods 3 and 4, class B, in case, for these two, a sinking fund is actually created outside the business.

With formula 3, class B (equal annual payment), the sinking-fund method is used to get a deduction each year which is also deducted from the remaining balance on which the interest is computed. Then the depreciation deduction from income is the sinking fund (assumed annuity) plus interest. There would have to be a number of extra columns in the plant ledger so as to list

remaining balance, annuity, interest, and total in addition to the ordinary columns in connection with every depreciable asset item. An assumption of an interest rate must be made and amounts computed as illustrated in Table 1. This interest rate means but little as it will be at variance with the interest rate earned by the business except as a mere coincidence. If it be intended as representing the interest rate of the business, it is like assuming an answer to get the correct answer. There seems to be no fact to justify the elimination of this method, the curve of which more nearly approaches the straight line. It is merely a question as to whether or not the results obtained justify the extra expense incurred over expenses incurred by use of the straight line.

With reference to method 3, class B (sinking fund) and method 4, class B (unit cost methods), where amounts deducted through depreciation are left in and not put outside the business in a sinking fund, the case is far different. Omit the question as to whether this item of liquid assets draws simple or compound interest and also the question of the necessity for immediate investment of the annuity as well as its interest. There remains the fact that only the annuity increments are deducted as depreciation. Unlike the annuity in method 2, class B (annuity method) which remains constant throughout the period, these gradually increase from year to year. In the case of method 1, class B (sinking fund), the initial annuity is more than doubled during a 25-yr period at 6%, and is almost one third of the total accrual. In such a case, nearly two thirds is represented by the assumed interest accruals. This highly problematical factor is an added risk to the seven variables of the definition. Thus gradual increasing accrual occurs during the period of gradual increase in repairs and maintenance to create unsafety in the business. The low safety factor condemns these two methods even on the assumption that the annuity is left in the business.

For true accounting purposes and to show the facts clearly, the interest from any of these interest formula methods must be reported as income to the business. Such an entry shows it as a debit which should be posted to a separate account from cost of property. Otherwise the total cost of property appearing on the books is not the true cost and is misleading, as is sometimes the case when the sinking-fund method is used.

Method of Determination.—A mere sketchy outline of things to be done in the determination of depreciation is given by the committee. Certain definite things must be done and failure to realize what they are results in many engineers making a mess of the whole thing. It may be assumed of course that engineering ethics will prevent engineers from attempting to establish an unreasonably high plant value for purposes of sale, loan, rate determination, tax evasion, etc. What then are the steps necessary for the basis of engineering judgment leading to an unbiased recommendation? As stated by the Society's committee "it cannot be determined with mathematical accuracy by formula" and "it is impossible to determine depreciation, either annual or accrued, by means of theoretical actuarial calculations." Here are some definite things that must be done. First, the engineer must study the plant ledger carefully to familiarize himself with each and every depreciable item. If none exists he must first go to the field for an inventory with which to make

one. He is then ready to start work. He will have found many items needing special investigation from the original vouchers and the original invoices which may or may not be in the same file. While this is going on, there should be typed (on a wide-carriage machine, so as to give plenty of space for extra columns) a list of all depreciable items with columns showing date of purchase, past life in years, cost, depreciation reserve for each period of the same rate used, the total for all periods, salvage value (if used), column for adjustment of depreciation taken, column for total adjusted depreciation, reserve and remaining cost, in addition to a column for adjusted differences at the time of replacement. The adding-machine totals of this tabulation must coincide with plant and general ledgers. The voucher examination may be cause for adjustments to be made and shown in the column provided on these sheets. An analysis must be made of profit and loss account to be sure that no depreciable items have been charged out direct, that adjustments on retirements are handled correctly, and that there have come into this account correct amounts representing depreciation charged through operating costs. An analysis of repairs and maintenance accounts may show items that should have gone to plant ledger and an analysis of additions and betterments accounts may show items that should have gone to expense in the year contracted. With this and any other office study deemed necessary, the engineer is now ready to go to the field, probably with a better knowledge of the depreciable plant than any single man connected with it. Taking with him the plant ledger and being accompanied by an accountant familiar with it and the foremen or superintendent of the particular department being investigated, he identifies each depreciable item, investigates its present condition, and sets down, in a separate column opposite, a number representing the years before the item will be removed from service. This expectancy depends entirely upon his judgment based on all obtainable information. Now his work as an engineer is practically finished except for the dictation of a prelude to his report and its recommendation. After showing a comptometer operator how to determine a few amounts of depreciation for the unit period and the depreciation reserve, these sheets can be retyped with these new data, checked by a calculating machine, and become the body of his report from which his recommendations are made. This is not any swivel chair theory but one that is based on actual practice.

It is neither sound business nor sound practice to correct past errors by the use of future errors which is the case if correction is attempted through the depreciation rate. The rate of depreciation for future years is determined by two things only—the remaining cost and the remaining years—and should not include any amount representing any past error correction. An error is an error and should be recognized as such and not covered up by any false accounting. It should be corrected by an entry showing the true facts at the date of its discovery. If too little depreciation has been deducted during past years, the amount will be shown in the adjusted depreciation column as will also any amount of excess depreciation of any item in past years. The net difference shown by the adjusted depreciation column should be debited or credited, as the case may be, to the profit and loss account for that year. Of course this

will decrease or increase the surplus account by that same amount, having nothing whatever to do with the depreciation reserve account. In the event that this amount is a debit to surplus to the effect of decreasing dividends too much in one year, it can be charged to an account termed "adjustment to depreciation" or the like and be amortized over a number of years through profit and loss. With reference to this procedure, the Society's committee states that it "is not only generally financially impracticable but is also ethically unjust." Its practicability has just been shown. Why isn't it just? Dividends are paid from surplus. Therefore, if the depreciation reserve needs to be increased, surplus was formerly too large and dividends too great in past years by this difference. In other words, through error, dividends were paid in part out of capital because part of the asset disappeared through actual depreciation without being accounted for.

The means of determining the depreciation reserve, with reference to all facts found by the engineer, depends upon the depreciation method that he is using. His estimate or expectancy of the number of years an item will remain in use has been fixed by his judgment. The past life in years is known. Therefore, one method is by means of mortality curves.[13] If it is for income tax purposes, the yearly rate of depreciation is found by dividing the cost by the sum of years of past and expected life and multiplying by the number of years of past life (straight line). If this product is greater than the book depreciation, it is used; otherwise the book reserve is used. Of course this is arbitrarily unjust, being merely one of the countless inequities in connection with income tax laws.

ANSON MARSTON,[14] PAST-PRESIDENT AND HON. M. AM. SOC. C. E.[14a]—Much of interest is presented in the NARUC report. It contains many encouraging proofs of progress since 1909, when the writer began his own engineering valuation practice, and since 1917 when the Society's Valuation Committee said in its report[15] that "* * * the art of valuation is still in formative condition, * * *."

The NARUC report discards "retirement reserve" and a number of other outmoded depreciation practices, and states correctly that only "straight-line," "sinking-fund," and "present-worth" methods of computing utility depreciation "are generally recognized or accepted today." (In its 1917 report the Society's Valuation Committee used the misnomer "compound-interest method" for "present-worth method." The evident reason was that then it was not yet discovered that the true basis of the method is that depreciated values at different dates are equal to the present worths of the probable future operation returns.) Furthermore, the report (on page 33, NARUC report) recognizes that "depreciation is a certainty—a current fact; * * *." The report discards the worn-out practice of claiming that depreciation is one thing for determining the annual depreciation expense and something entirely different for deciding the size of the depreciation reserve. Also the report

[13] *Bulletin No. 103*, Iowa State College, Ames, Iowa.
[14] Dean Emeritus of Eng., Iowa State College, Ames, Iowa.
[14a] Received by the Secretary July 20, 1944.
[15] *Transactions*, Am. Soc. C E., Vol. LXXXI (1917), p. 1312.

contains valuable historical data on the development of recognition of depreciation, including rulings of the U. S. Supreme Court at different dates, and brings to the front the remarkable researches of the past quarter century into the use of mortality data of industrial property items in forecasting service lives and computing depreciation.

In spite of the good features of the NARUC report the writer finds himself in agreement with most, although not all, of the criticisms in the report of the Special Committee of the Society.

Definition and Concept of Depreciation.—The writer is in entire agreement with the Society's Special Committee in the choice of definitions of depreciation. That by Chief Justice Hughes in the Lindheimer case (292 U.S. 151) is authoritative as well as good.

In general, depreciation is the "loss in service value" due to decrease with age in service in the present worth of the probable future operation returns. As age increases, the period remaining until the probable future retirement date decreases.

In effect, the NARUC report assumes throughout that the actual service lives of physical property items can be forecasted correctly at the dates of their installations. This is not true. The most certain thing about any such forecast is that it will not turn out to be true—even engineers are not divinely inspired prophets. The writer is in complete agreement with the statement of the Society's committee (see heading, "The Concept of Depreciation") about the necessity for repeated re-examinations of the property items by qualified engineers, who make such revisions of forecasted expectancy as are indicated from time to time by the facts of the actual service lives as they unroll with time. Thus, the correctness of three essentials is assured:

1. The final estimated service life of each property item becomes its actual service life;

2. The final total depreciation expense allowed always equals the value new of the item, less its actual net salvage value; and

3. The total annual depreciation expense allowances for the item will have been distributed between the different years of its actual service life in strict accord with the actual facts of its service life history.

Although the foregoing description applies especially to individual unit depreciation accountancy, entirely similar re-examinations and revisions of forecasts of expectancies and average lives should and can be made readily in group depreciation accountancy procedures.

Straight-Line Depreciation Versus True Actual Depreciation.—The writer agrees with the Society's committee in its criticisms of the NARUC report's advocacy of straight-line depreciation. The straight-line depreciation assumption is incorrect, for it assumes that the present worth of future returns is equal to their total amounts. In 1943[16] the writer stated the fact

> "* * * that in every one of the many billions of purchases and sales, the world over, each year, the exchange value is determined by the present worth of the forecasted (not the recorded), probable (not the actual),

[16] *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1265.

future (not the past or the present) annual operation return values of the services yet to be rendered by the item purchased."

This statement applies equally to seller and to purchaser, for they are equal parties who must reach agreement in all determinations of exchange value. This is true whether the nature of the anticipated services to the seller is the same or not as those to the purchaser. The present worth of probable future returns is the fundamental basis of all exchange values; at each service age of a physical property item the depreciated value is the then present worth of the then probable future operation return values of the then probable future services by the item.

The fact that the revised forecasts of the expectancies of the property item at different dates may not agree with former forecasts does not at all invalidate the former estimates of depreciation at former dates. At the former dates, the present worths of the then probable, then future, operation returns correctly fixed the then depreciated values. The U. S. Supreme Court ruled in 1929 that:

> "But the value of property at a given time depends upon the relative intensity of the social desire for it at that time, expressed in the money that it would bring in the market (see International Harvester Co. vs.Kentucky, 234 U. S. 216, 222, 34 S. Ct., 853, 58 L. Ed. 1284). Like all values, as the word is used in law, it depends largely on more or less certain prophecies of the future, and the value is no less real at that time if later the prophecy turns out to be false than when it comes out true." (Ithaca Trust Co. vs. United States, 279 U. S. 151, 49 S. Ct. 291, 292.)

This opinion was written by Mr. Justice Holmes. At the end of the foregoing quotation, he further cites Lewellyn vs. Electric Reduction Co., 275 U. S. 243, 247, 48 S. Ct. 63, 72 L. Ed., 262; City of New York vs. Sage, 239 U. S. 57, 61, 36 S. Ct. 25, 60 L. Ed., 143.

The Fallacy of the Claim of "Simplicity" for the Straight-line Depreciation Method.—The condition percentages of physical property units have been computed and published for all three of the depreciation computation methods still not outmoded—straight line, sinking fund, and present worth—for a wide range of service ages and forecasted service lives, stated in years (up to 100 years).[17,18]

For each of the aforementioned three depreciation computation methods, the computation process need consist merely in looking up the condition percentage in the tables (some interpolation is required for group depreciation accountancy) and then multiplying it by the value new less salvage value of the item (the surviving items in group depreciation accountancy). This process is "simple" for each of the three methods.

For all three methods, repeated re-examinations and corresponding revisions of forecasted expectancies are equally necessary; without such re-examinations and revisions the computed depreciations are practically certain to be incorrect.

The Cost of Straight-Line Depreciation to Customers.—In the 1917 report[19] of the Society's Valuation Committee it is demonstrated that the costs to

[17] *Bulletin No. 155*, by Robley Winfrey, Iowa Eng. Experiment Station, Ames, 1942.

[18] *Bulletin No. 156*, by Robley Winfrey, Iowa Eng. Experiment Station, Ames, 1942.

[19] *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), pp. 1477–1480.

customers are exactly the same by the straight-line depreciation method as by the sinking-fund method and by the present-worth method, when (as they should be) the dates of the payments by customers for net returns plus depreciation expense are correctly taken into account, with compound interest allowed (as it should be) at the fair rate of net return on the depreciated values. Both the present worths at the dates of installation and the accumulations at the dates of retirement are the same for all three methods, and even for retirement accountancy in which no depreciation is charged until retirement.

Robley Winfrey, M. Am. Soc. C. E., has verified this procedure for these and other methods.[17,18]

The Best Current Practice in Determining the Annual Depreciation Expense and the Total Accrued Depreciations of Physical Property Units.—Almost universally at the present time the "depreciation bases" for computing depreciation expense are the "original costs new" of the existing physical property units less their net salvage values. A recent rather extensive personal interview study by Charles V. Armstrong,[20] made under the general direction of the writer, of the physical property accounting practices of a number of large utility corporations demonstrated that most of those studied were engaged in the difficult and expensive process of breaking down their plant accounts (amounting to from $31,000,000 to $1,249,000,000 in original costs for different utilities studied) to show the original costs of their separate individual physical property units. The trend toward making and keeping constantly up to date complete and continuous physical property inventories seems to be becoming more and more recognized as inevitable.

In the best depreciation expense practice, all the physical property units and depreciation groups of units are re-examined regularly by qualified employees, with engineering (and preferably also accounting) experience, who make such revisions of former forecasts of expectancies and efficiencies as are indicated in each case by:

(1) The then physical condition—excellent, good, average, poor, or bad (E, G, A, P, B)—for its age;

(2) and (3) The past and probable future service conditions—mild, favorable, average, unfavorable, severe (M, F, A, U, S); and

(4) The probable future operation return ratio (a numerical estimate) compared with average for the unit (or group) throughout service life.

The revised forecasts of expectancy and (for depreciation groups) average life can best be made with the aid of mortality survivor type-curves, such as those made available by Lieutenant-Colonel Winfrey in 1935.[21] Mortality data of past experience with the property of the particular enterprise should be used in the first selections of the type-curves to be tried; the re-examinations supply data to select better types if they are later found to be needed. For each depreciation group, its actual survivor curve is plotted year by year from its actual retirements.

[20] "Industrial Physical-Property Valuation Records," by Charles V. Armstrong, thesis presented to Iowa State College at Ames in 1941 in partial fulfilment of the requirements for the degree of Master of Science.

[21] *Bulletin No. 125*, by Robley Winfrey, Iowa Eng. Experiment Station, Ames, 1935.

Changes at the dates of the re-examinations in previously forecasted expectancies do not invalidate or change the depreciated values estimated at former dates. The earlier determined depreciated values at the dates of changes in forecasts, plus the costs of any current partial replacements (or other betterments), are simply spread over the remaining service lives.

The actual service life histories of such individual physical property units as buildings, bridges, roads, and many other structural and equipment items show that repeatedly these are partly reconstructed or otherwise bettered. There are many residences more than 100 years old still in good condition.

In thinking of the depreciations of industrial properties, care should be taken not to confuse the general nature of the depreciation of a single property unit (and of a single group of like units) with the general nature of the total depreciation of the entire property. A large industrial property contains thousands of individual items whose service lives and dates of installation, retirement, and replacement vary so greatly that a continual exchange of old items for new (often better) goes on from year to year. In addition, many new items usually are being added from time to time.

If, by the process of re-examination and by the revisions previously described, the total depreciation charged off for each unit to the date of its retirement is made just equal to its depreciable value new, its average annual depreciation is the same by either the straight-line, the sinking-fund, or the present-worth depreciation method.

Original-Cost Value, Fair-Cost Value, and Reproduction-Cost Value Depreciations.—The condition percentages of physical property units and depreciation groups of units are the same whether the "depreciation bases" for computing depreciations are:

1. Original-cost value new less net salvage, which may have to be used for depreciation expense accountancy;
2. Fair-cost value new less net salvage, which may have to be used in estimating depreciated fair-cost values of physical property units; or
3. Current reproduction-cost value new less net salvage, needed for insurance adjustments.

In case 2 the values new should be determined item by item, repricing only those occasional items so old that it has become certain that real and probably permanent changes in price levels justify some conservative repricing.

For case 3 the reproduction-cost prices should be those most certain to prevail during the immediate future—3 to 5 years.

For all items, repriced or not, only the actual overhead cost percentages actually paid when they were installed should be added to the direct costs of such items.

In the view of the writer, the imaginary, impossible process of estimating reproduction-cost values heretofore so much used should be abandoned. It sometimes has been so abused as to justify the appellations "speculative" and "fanciful."

The Size of the Depreciation Reserve.—At all times the depreciation reserve balance should just equal the accrued depreciation on the existing physical

property units. The writer cannot agree with the expressed view of the Society's committee that (see heading, "The Depreciation Reserve") "The reserve should normally exceed the accrued depreciation by a reasonable margin, * * * to absorb casualties (not covered by insurance) which are properly not included in the determination of accrued depreciation." The Society's committee admits that: "The depreciation reserve is created by credits from annual charges against operations for depreciation." How, then, can the reserve be greater than the accrued depreciation?

If a reserve is needed to provide for casualties "which are properly not included in the determination of accrued depreciation," such casualty reserve should be entirely separate from the depreciation reserve.

Depreciation and the Rate Base.—There can be no doubt at the present time that the total accrued actual depreciation on existing physical property items must be deducted in determining utility rate bases. It now seems a far cry from the time, as late as about 1928, when numerous valuators still argued that no depreciation at all should be deducted in determining rate-base, original-cost values.[22]

It follows that the depreciation reserve should always be deducted if it is just equal (as it always should be) to the accrued actual depreciation, but, as the Society's committee states (see heading, "Depreciation Reserve"):

> "It is submitted that the actual depreciation should control the size of the reserve, rather than that the reserve should determine the actual accrued depreciation."

The NARUC report supports the requirement that utilities use the straight-line depreciation method in their depreciation account; this, as the NARUC committee states, will require depreciation reserves greater than those called for by the present-worth method. It is the view of the writer that the present-worth method gives the true actual depreciations, and hence that depreciation reserves equal to straight-line accrued depreciations are greater than the true accrued actual depreciations.

Nevertheless, when the straight-line method is used, the entire corresponding depreciation reserve should be deducted to obtain the rate base. In the typical case, the annual depreciation appropriations from income have all been invested in replacements or additions to the utility property, and are earning the same rate of net return as the remainder of the property. (Exceptions to this disposition of the depreciation expense appropriations are minor, and of such character as to justify the assumption that the sums involved earn, or at least are capable of earning, at the fair rate of net return.) Since the utility's customers have had to pay excess depreciation charges to provide the excess depreciation reserve, they will have been treated unjustly unless the rate base is decreased by the same amount the depreciation reserve is inflated.

In the Lindheimer case (292 U. S. 151), the Illinois Bell Telephone Company had built up a depreciation reserve of approximately twice the accrued depreciation determined by the Illinois Commerce Commission. This was done by

[22] "Valuation of Public Service Corporations," by Robert H. Whitten and Delos F. Wilcox, 2d Ed., 1928 Section 825, Vol. II, pp. 1709–1714.

collecting straight-line annual depreciation expense charges to customers in addition to net returns on a rate base for which the depreciation reserve had not been deducted. The U. S. Supreme Court ruled the reserve excessive, and upheld the reduction of rates ordered by the commission.

The rate-base question most debated currently is whether the rate base should include the depreciated "fair-cost value" of the physical property, as heretofore so often ruled by the U. S. Supreme Court, or its depreciated "original-cost value," which is the as yet unrepaid investment. Although the Court has refused to overrule rate orders based on original-cost rate bases in at least two instances (Los Angeles Gas and Electric Co. vs. United States, 289 U. S. 287, 1933; and Federal Power Commission vs. Hope Natural Gas Co., U. S. Supreme Court, January 3, 1944), it has not as yet clearly reversed its long line of decisions that the rate base should be present fair value.

In the Los Angeles gas case, the Court gave dominant weight to "historical cost" (original cost less depreciation) because "* * * the outstanding fact is that the development of the property had for the most part taken place in a recent period * * *." Previously in the decision the Court had reaffirmed the doctrine that, in general: "* * * the cost of reproducing the property is a relevant fact which should have appropriate consideration."

In the case of the Federal Power Commission versus Hope Natural Gas Company, the Court refused to overrule a rate order made upon a rate base which the commission found "* * * represented the 'legitimate cost' of the company's interstate property less depletion and depreciation and plus unoperated acreage, working capital and future net capital additions." Consideration of the company's estimated reproduction cost had been ruled out as "not predicated upon facts," and as "too conjectural and illusive to be given any weight in these procedures."

In both of the foregoing cases, the Court emphasized that it is not the function of courts to make valuations.

Adjustments of Depreciation Rates and Reserves.—There are two possible cases in which depreciation rates and reserves may require adjustment: (1) When the existing reserve is too large; and (2) when the existing reserve is too small.

In the New Jersey telephone case (271 U. S. 23), the existing reserve was too large, and the U. S. Supreme Court ruled in 1926 that over-collections of depreciation expense charges in the past do not justify less than full fair net return charges to future customers. "It is a poor rule that will not work both ways," so it is logical to say that, for the second condition, under-collections of depreciation expense charges in the past do not justify more than fair net return charges to future customers. It would seem that in both cases the equitable way to make "adjustments" of depreciation rates and reserves is by transfers between surplus and the reserve—to surplus when the reserve is to be decreased and from surplus when the reserve is to be increased. In the latter case, the transfers can be spread over a reasonable term of years if the utility finances require.

Utility Property Depreciation Compared With Depreciation on Other Physical Property.—The NARUC report, and by far the greater part of the published

discussions of depreciation, have been devoted wholly or mainly to depreciations of utility physical property. It should be recognized, however, that the depreciation of any physical property unit is just a fact to be ascertained, and is no different for utility than for other physical property.

The total of the depreciations of utility property undoubtedly is much less than that of other property in the United States; also it is of less importance.

The success and security of all industrial and other business enterprises require sound depreciation accountancy practices.

Income tax and excess profit laws in the United States require annual certified detailed depreciation reports from many million citizens, in addition to corporations. Business men, and even farmers, must be depreciation experts in some degree, or hire the services of persons who claim to be experts.

Furthermore, all taxes on real estate property are supposedly based on present depreciated values. State boards of assessment and review are endeavoring to train tax assessors in the technique of correct assessment procedures. Municipalities find it necessary at times to hire high-grade valuation experts to review their assessed valuations, property by property.

The field of engineering valuation has expanded and changed greatly since the Society's Valuation Committee made its classic report in 1917.

TERRELL BARTLETT,[23] M. AM. SOC. C. E.[23a]—In all the hundreds of thousands of words written on the subject of depreciation, embodied in court and commission opinions and in discussions by representatives of the owners of properties and by technical societies, there is one vital aspect of the subject which, if it has been stated clearly, has escaped the attention of the writer.

In any utility property, units that have not been replaced or retired to purely stand-by service—regardless of how near they may approach the end of their useful lives, and regardless of the accrued depreciation that actually exists—nearly always continue to serve and earn at 100% capacity right up to the end of their lives. Indeed, in most cases such units render greater service in their later years than when originally installed. In some cases, as they approach the end of their lives they may require additional maintenance, but that is an operating cost independent of, and outside of, the cost of aging commonly included within the definition of pure depreciation.

If a unit or component that is still serving and earning on a 100% basis, just as when new, is depreciated on a straight-line basis, either for rate purposes or for prospective sale valuation, then the owner of that property is deprived of part of the earnings, or capitalized earnings, which the unit is producing. For instance, consider a motor-driven water-works pump with a life of 20 years, half of which has elapsed. The pump may well be delivering its full rated discharge on the same electric consumption as when new. If it be depreciated by the straight-line method at 50% for rate purposes, then the public gets 100% service for 50% value, not 50% service for 50% value. If the 50% straight-line depreciation be applied for fixing a sale value, the seller obtains less than a fair price because the purchaser continues to get service and earnings for the ensuing 10 years on the excess depreciation so deducted from the sale price.

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[23a] Received by the Secretary July 25, 1944.

Consider a structure costing $100,000 built by Owner A on a 50-yr leasehold for a tenant who leases the property for assumption of taxes, maintenance, and ground rent, for amortization of cost in the 50 years, and for $5,000 per year being a 5% return on the $100,000 investment in the building. To amortize the cost on a net 5% return as contemplated would make the annual payment for that purpose included in the rent contract $477.67 and the uniform annual building rent $5,477.67.

At the end of 25 years Owner B buys the property and leases from Owner A. At that time Owner A has received twenty-five amortization payments. The rental being fixed at 5% on the original $100,000, he gets 5% compound return on the amortization payments which then amount to $22,798, leaving his investment in the building at $77,202. If Owner B pays $77,202 for the remaining twenty-five annual payments of $5,477.67, he will receive 5% on his purchase price and will also have his investment in the building returned to him intact by the end of the remaining 25 years. If instead, the property is conveyed on the basis of a straight-line depreciation, the seller would lose $27,202, or more than 27% of his investment, and the buyer would gain the same amount.

In the foregoing illustration it will be argued by advocates of straight-line depreciation that the annual charge for depreciation should be one fiftieth each year or $2,000 instead of $477.67. In the first place, the ability of the owner to make his lease acceptable to any good business man on that basis is challenged, but, if the lease is so made, what will happen? Owner A will receive $7,000 a year and, at the end of 25 years, a $50,000 purchase price. The accumulation of $2,000 at 5% for 25 years is $95,454, so Owner A has his 5% return plus a profit of $45,454 above his original investment. Owner B buys the $7,000 annual payment on the building for $50,000 and starts with $4,500 a year over and above a 5% return on his purchase price. The accumulation of $4,500 at 5% for 25 years is $214,772, thus returning to Owner B his $50,000 with interest plus $164,472. The two owners A and B have received an aggregate of $210,226 above the return of their original investment with 5% interest, and Owner A has, in addition, the accumulation of 5% on $45,454 for the last 25 years, amounting to $108,470—or a total above the return of investment and 5% thereon of $318,696. This is the accumulation of $2,000 a year at 5% less the original $100,000 amortized. Both owners have fared quite well; but how about the lessee, who in this illustration is in a position corresponding to that of a user of a public utility service?

A leading court decision on depreciation indicates a preference for a depreciation fixed by the judgment of an experienced and qualified engineer who examines the property with care, rather than for a depreciation arrived at by theoretical life tables and formulas. This decision is entirely correct in one aspect and entirely erroneous in another. It is correct in that the actual service condition of each unit of property and its remaining adequate serviceable life can be fixed better by qualified examination than by a mere knowledge of age against some theoretical and possibly inapplicable life table. When the present age is known and the remaining years of serviceable life are competently fixed, as contemplated by the decision, the determination of the

accrued depreciation becomes a matter for calculation solely by mathematical formula, taking account of time and interest, and the decision is wrong in rejecting such necessary precise means of giving final effect to the desired experienced judgment.

When the present age of a depreciable piece of property is known and competent judgment has determined the probable remaining years of useful service —their sum being the effective life of the unit—then the accrued depreciation is the present value, at the going rate of interest, of the future obligation to replace the unit when necessary. The determination of the present value of the deferred expenditure for replacement is a mathematical problem in compound-interest discount which can be computed only by formula.

There are situations in which judgment does not enter the problem of determining the remaining life of a property, when contractual conditions limit that life; for example, a building constructed on a leasehold of fixed tenure, as in the preceding illustration, or where the life of a utility property is limited by the expiration of a franchise in states which do not recognize any continuing rights to operate such property beyond a fixed franchise period.

Considered as the present value of a deferred obligation to buy a new unit or component, accrued depreciation assumes its true commercial value. The straight-line method of depreciation is commercially sound only when the earning or interest value of money is zero. This condition prevails for the owner of capital assets in a commune. That an organization of responsible governmental regulatory bodies should advocate the determination of accrued depreciation on the straight-line basis—that is, without taking into account the ordinary commercial discount applicable when an obligation is deferred to a future date (generally many years in the future)—is evidence of the extent to which the viewpoint of governmental agencies has become socialized or communized.

E. E. Hart,[24] Esq.[24a]—The Special Committee of the Society has possibly overlooked the statement in the NARUC report (Conclusion 17, Appendix) which recommends that the basis for depreciation allowances shall always be the cost of the assets being depreciated. Presumably the intention is that cost shall be the basis regardless of when the asset being depreciated was acquired.

This requirement may definitely be a hardship to investors in the case of assets acquired prior to World War I at costs substantially below their present replacement value. It must be remembered that, although, as a rule, cost is the safest long-term determinant of value, after a period of great inflation such as that of World War I, it may cease to be such a determinant, and values should be restated and depreciated in terms of new value. It must be remembered that the value of a physical property is being depreciated and not a list of figures in a book of accounts; the owner is entitled to the value of the asset being depreciated.

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[24] Supervisor, Material Standards, John Inglis Co., Ltd., Ordnance Div., Toronto, Ont., Canada.
[24a] Received by the Secretary August 10, 1944.

LUTHER R. NASH,[25] ESQ.[25a]—The viewpoints of the Society's committee and of the NARUC committee on this much discussed subject are naturally different. The former visualizes the problem as one of engineering and economics, the latter approaches it from the accounting and regulatory angle. The differences are not irreconcilable, but the Society's committee rightly concludes that "administrative convenience" should not be a controlling factor where it is in conflict with engineering principles.

The NARUC committee had the problem of clarifying and elaborating the depreciation provisions of the accounting systems adopted in 1937 which undertook to bring about greater uniformity than was included in the retirement provisions of the earlier accounting systems adopted in 1922. It is generally agreed that depreciation does not accrue uniformly and that there may be abrupt and radical changes in the extent of existing depreciation in an active property. There is also general agreement that accounting for depreciation should not be subject to wide fluctuations but rather should seek to smooth variations out in an approach to uniformity. Apparently the Society's committee is not opposed to such procedure although it rejects the NARUC method of accomplishing it.

The useful-life method of determining annual depreciation and creating reserves, advocated by the NARUC committee, is vulnerable for more reasons than those listed in the Society committee's report. Probable lives of units to be retired in the future will be different from those of the past not only because of improved types, efficiency, preservatives, and repairability but also because of fundamental economic changes, some of them with divergent effects but all generally tending to prolong life. The Society committee's report states (see heading, "Straight-Line Depreciation") that a "large percentage" of past retirements has been due to nonphysical causes. Recent surveys made by the electric power and manufactured gas industries showed that over an extended period of years more than 80% of all retirements, measured in dollars, was due to such causes. Less than 20% was due to wear, decay, rust, and erosion—causes that operate with some approach to uniformity.

Most prominent among the nonphysical causes in the past has been obsolescence, due primarily to radical improvements in production efficiency. Further improvements of a similar order cannot be expected in the future as the margin between attained and theoretical efficiency has become narrow. There will be less "firing" of present units for such a reason, with longer service life as a result.

Another prominent cause of past retirements was inadequacy, lack of capacity to keep up with growing demands for service. This too may be subject to substantial curtailment. In the electric power industry, with which the writer is most familiar, the curtailment of growth within the past 30 years (roughly a life cycle of its property) has been most striking, and probably has not been paralleled in other industries, but companies in this industry make up the largest group to which the NARUC program applies. Assuming the growth in property investment in the entire 30 years as 100, that in the more recent 20

[25] Ridgefield, Conn.

[25a] Received by the Secretary August 14, 1944.

years was 71 and that in the final 10 years dropped to 30. These figures are from a large, widely distributed group of electric power properties growing through self-expansion rather than consolidations and are believed to be typical. It is true that the past ten years included a most severe depression, war priorities, and regulations which restricted growth, but there were also unusual war expansions and the industry will enter the postwar period with a 25% margin of capacity over its peak war requirements. It appears probable, therefore, that growth in the coming years will not return to the rate of the earlier years. As growth has declined, so also have retirements, indicating an extension of service life. The growth in 1943 was the lowest in many years, as was also the percentage of retirements.

Aside from these changing conditions which invalidate the application of past service lives to the future, there are other infirmities in past records which render them misleading. Such records apply largely to the very many small units included in the typical utility property. In fact, it has been estimated that useful life records that show even apparent consistency and reliability cover not more than 20% of the investment of a typical electric power property. Because of the multiplicity of these small units and lack of individual life history, it has been customary to determine, as a substitute, the life of the dollars in groups of like units. Useful life of the units could be derived from such values in the absence of price changes, but the many, sometimes violent, changes in practically all utility prices during the life of surviving units have caused a wide diversity between dollar ages and unit ages.

In addition to such illustrations of the limitations of the useful-life program, it is possible to give an over-all indication of its unreliability. Applying the conventional estimates of life as now visualized by the proponents of the NARUC program to a typical growing utility property, the so-called accrued depreciation is found to be about 30%. This is obviously far in excess of the depreciation that would be found by engineers who followed the method outlined in the Society committee's report. Incidentally, because a unit is old and relatively inefficient it does not follow that its usefulness and value are gone. Such units are quite as useful for emergency and peak loads, where their efficiency is immaterial, as are new units. It may also be observed that under the NARUC program, with rate base reduced to the extent of the computed depreciation reserve, it would be possible to increase the income authorized under prevailing regulatory procedure by replacing an old, but still satisfactorily useful old, unit by an equivalent new one.

The Society committee's condemnation of retroactively computed reserves has been actively supported by utility and accounting authorities. There are reasons beyond those listed in the report for these views. Regulatory authorities have been demanding consistency in utility accounting, particularly that relating to depreciation. Retroactively computed reserves are in striking violation of such consistency. As the Society committee's report states, all past retirement accounting, largely under commission supervision, is "thrown out of the window," and present estimates of useful life are substituted for those believed from time to time over the years to be adequate. However, such estimates are related, not to investment recomputed in accordance with present

accounting methods, but rather to that kept meticulously in conformity with the records and methods in effect when the property was originally acquired, without amplification for obvious inadequacies but with exceptions resulting in the exclusion of many questioned items. As an example of such excluded items, if a company should employ a designing and construction organization that is defined as an affiliate, only the bare cost of services to the affiliate is allowed, but, if an independent agency with equal skill and service fees were employed, full fees including profits are not questioned. If fixed capital were recomputed strictly in accordance with present standardized methods, consistent with the proposed depreciation accounting, the over-all effect would be far different from that now contemplated by some of the regulatory authorities. One finds nothing in the present accounting systems that provides for amortization of investment such as, in effect, is the purpose of the proposed NARUC program which implies that contributions by customers for depreciation constitute a return of capital instead of advance provision for continued maintenance of service.

The assertion by the Society's committee (see heading, "Depreciation Reserve") that the reserve should be "at least" equal to accrued depreciation in order that it may take care of contingent retirements, for which no other specific accounting provision has been made, clearly indicates that such a reserve should not be used as a measure of existing depreciation. Retirements that occur as the result of storms, floods, careless or drunken driving, fires, and government requirements amount in many cases to a material part of all retirements.

The contention of the Society's committee that the proper procedure in the depreciation program is first to fix the size of the reserve and then to determine the credits necessary to maintain it will have general approval in utility circles as being in accordance with long-standing and tested practice. Definite records to support this procedure are available covering a long period of years. It is a logical reversal of the NARUC plan of starting with estimated credits without limitations on reserve size.

The determination of the credits necessary to maintain a defined reserve is a simple matter. Elsewhere,[26] the writer has expressed such credits in a formula in which the only necessary factors are reserve size, retirement charges, and property growth—all expressed as percentages of investment averaged over a period of years sufficient to represent normal operations and trends. Adjustments arising from the moving average of years, together with a further adjusting or accelerating factor where needed to provide for abrupt change in property characteristics, serve automatically to maintain the reserve within appropriate limits. The reserve itself may be similarly determined and expressed with certain constants, determined by engineering methods such as are outlined in the Society committee's report, and adjusted to fit the character of particular properties.

Only one minor feature of the Society committee's report remains for comment in this discussion. Two definitions of depreciation are listed and quoted in full as "best" (see heading, "Definition of Depreciation"). The one taken

[26] *Public Utilities Fortnightly*, August 3, 1944, p. 146.

from the NARUC system of accounts excludes property protected by insurance. Such exclusion is inconsistent with common accounting methods under which final debit or credit balances after adjustment of insured losses are handled through the depreciation reserve. Under the NARUC plan, a company that insured all its depreciable property against retirement from any cause would need no reserve, and its rate base would not be subject to any depreciation deductions. It may be noted that a company has been organized to write such unusually comprehensive insurance.

The Society committee's report, being limited in its scope to that of the NARUC report, is necessarily brief and omits many important phases of an increasingly controversial subject, including the effects of recent Supreme Court decisions on this and other segments of regulatory practice. It is to be hoped that a more comprehensive study of depreciation will be made by the Society in due time, supplementing and bringing up to date the exhaustive studies made many years ago.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ERECTION OF HYDRAULIC TURBINES AND GENERATORS

Discussion

By J. F. Roberts, and D. E. Brainard

J. F. Roberts,[7] M. Am. Soc. C. E.[7a]—This description of the field erection practice of the TVA on large hydraulic turbines of both the Francis and Kaplan types is interesting and will be of great assistance to engineers with similar problems. During wartime when the privately owned power companies are not able to do any appreciable amount of extension work or to develop new power facilities, it is gratifying that government owned corporations and commissions are allowed to publish data on their projects. This opinion has been expressed by several engineers connected with private power companies.

Mr. Komora's brief comments regarding the grouting of embedded parts could have been amplified considerably. The writer was intimately connected with the experiments conducted at Wilson Dam by the TVA in an effort to study the results of various grouting procedures. Some special compounds were tested, especially those which tend to cause expansion on setting. Although in some cases these gave excellent results, it has been found difficult in practice to obtain uniformly good results because construction forces object to the careful measurements and accurate proportioning of ingredients that seem necessary. In certain cases a slight excess of water entirely ruined the effects of the agents which were supposed to cause expansion on setting. In the final analysis it was found that more uniform results could be obtained with a straight cement mixture, provided that such a mixture was rodded for a sufficient length of time to work all the free water out of the mixture.

Mr. Komora mentions that mechanical vibrators should not be placed closer than 10 ft to the unit to avoid possibility of distortion of the parts of the machinery. Originally, the writer was in accord with this opinion, but further experience has indicated the advisability and extreme usefulness of the small sized vibrators especially in grouting work where rodding and tamping are

Note.—This paper by Andrew M. Komora, M. Am. Soc. C. E., was published in March, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: June, 1944, by Paul E. Gisiger, and Frank I. Morgan.

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[7a] Received by the Secretary May 22, 1944.

difficult. These small vibrators, which will pass through a $2\frac{1}{4}$-in. hole, have been of material assistance in compacting the grout, and, if the embedded parts are properly anchored down and the height of the pour or lift of concrete is kept to reasonable limits, as outlined by Mr. Komora, no difficulties should be encountered in moving, shifting, or distorting the embedded castings.

Closer cooperation between the designers of the machinery itself and the designers of the concrete structures would be helpful. Wherever possible the undersurface of castings that are to be grouted in should be constructed with a slope such as to avoid dead pockets, as it is very difficult to produce a satisfactory grouting job under a large horizontal surface.

Mr. Komora mentions the use of temporary coupling bolts for lifting the turbine runner and shaft into position. Although this idea works satisfactorily, using a quicker and easier method the turbine shaft and runner are brought up to the generator coupling by means of four or more hydraulic jacks equally spaced around the coupling. These can be supported on the head cover or in some cases satisfactory results have been obtained by mounting the jacks on horizontal plates suspended with two long bolts through the generator coupling bolt holes. Turbine runners and shafts weighing as much as 200 tons have been lifted into final position in less than one hour with this arrangement. By checking the gap between the couplings at frequent intervals, or placing dial indicators to observe the movement, the couplings can be brought up absolutely parallel with no danger of binding.

Many civil engineers fail to appreciate the extreme flexibility of large castings when subjected to hydrostatic pressures such as those which may be exerted by large masses of liquid concrete. Turbines with runner openings from 20 to 24 ft in diameter, having speed rings with over-all dimensions of from 30 to 37 ft in diameter, are easily distorted and have to be securely held in order to maintain a true circle. Runner clearances on large Kaplan and propeller type units vary from 0.120 in. to 0.180 in. on a radius. Few large installations can be maintained truly round, the average discrepancy being 0.020 in. to 0.030 in. on the radius. Where an insufficient number of radial struts are used to hold these castings round, flat spots may be found between the braces. It is advisable to take twenty to thirty equally spaced readings around the circumference to secure an accurate picture of the condition of such rings. Temperature also is an important factor. Daily readings on embedded parts show a gradual distortion as the maximum temperature in the setting concrete is obtained and then a gradual return to the original conditions as this heat is dissipated. Continuous sprays of cooling water on embedded metal parts help to reduce this distortion.

This paper and similar ones materially assist manufacturers in giving field construction engineers a better understanding of the problems to be encountered and of the accuracy of the desired results.

D. E. Brainard,[8] Esq.[8a]—Proper installation is an essential prerequisite for successful operation of hydroelectric machinery. Designers may profit

[8] Motor and Generator Eng. Div., General Electric Co., Schenectady, N. Y.

[8a] Received by the Secretary July 26, 1944.

greatly from the experience of those who erect their equipment. Therefore, the author's orderly recording of salient points is welcome. The following supplementary comments are offered from the point of view of the generator manufacturer.

The author has outlined a procedure for minimizing the tendency of the side arms of a girder-type thrust bearing bracket to pinch the guide bearing. In a number of cases, the side arms are attached to the main beams with a hinged joint so that the function of transverse stiffening may be performed without any twisting of the main beams.

Since 1933, at the insistence of certain operating groups, the use of a jacking fit on coupling bolts has flourished. Most manufacturers saw no benefit from the tight fit and the resulting tendency to score the bolts, and preferred to make the bolts and the holes "line and line" so that the bolts could be driven in by a rawhide mallet. With few exceptions, customers now accept the "light driving fit."

Coupling bolts should be stretched in excess of the load imposed on the bolts by the weight of the water-wheel parts, the hydraulic thrust, and the cylinder oil pressure, if any. For Francis-type wheels, however, this stretch may be as small as 0.002 in. The bolts should not be stretched more than necessary in a given case, as excessive tightening may lead to galling the threads.

The author questions the value of factory fitting of coupling bolts and alinement of water-wheel and generator shafts. Certainly, there is a size of unit below which the additional expense of this operation is not justifiable. For large units there appears to be an advantage in employing factory facilities to handle the heavy and sometimes cumbersome parts involved in fitting the bolts. The assurance that the parts will run true when reassembled appears to be a good investment.

The intricate procedure outlined for obtaining proper distribution of load among the shoes of a thrust bearing is required by only one type of bearing. One of the generator manufacturers originated and produces a type of thrust bearing that provides inherent distribution of the load over the bearing surfaces and also automatically compensates for changes in the line-up of the unit which would disturb the loading of a rigidly supported bearing.

The rotation check described in the paper as "customary" has been required by only one customer. It is possible to determine whether a shaft is straight and plumb by checking from tight wires without the laborious process of turning the shaft and rotor. The almost universal practice of making a factory check with the thrust collar on the shaft has been found in practice to give adequate protection against inaccuracies.

A thrust collar forged integral with the shaft does have certain definite advantages, but this feature can be used only when the thrust bearing is below the rotor. This "overhung" design with no upper guide bearing and often no exciter was developed on moderate-sized, low-speed units where the stator and rotor could be made accessible for inspection or repair by disassembling a minimum of related parts without disturbance to the shaft alinement and bearing assembly. This size of unit allowed space for reasonably

convenient disassembly of the thrust bearing. As the size and resulting importance of units increased and as the runaway speeds were raised by water-wheel developments, upper guide bearings were added, largely eliminating the element of simplicity. When the output and speed of a generator become such as to require a relatively small diameter, space does not permit reasonably convenient disassembly of the thrust bearing if it is located below the rotor. So, except for a very limited range of size and speed, it is desirable to use the "conventional" mechanical arrangement with the thrust bearing above the rotor. Top location of the thrust bearing reduces its size and permits handling it with the crane.

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DISCUSSIONS

FLOOD FORMULAS BASED ON DRAINAGE BASIN CHARACTERISTICS

Discussion

BY L. R. BEARD, AND M. J. ORD

L. R. BEARD,[34] JUN. AM. SOC. C. E.[34a]—A phase of hydrology that deserves close attention from those interested in hydrologic design is treated in a practical manner in this paper. The Los Angeles (Calif.) office of the U. S. Engineer Department adapts the principles of the summation hydrograph (S-graph) as presented by Mr. Langbein[11] and Franklin F. Snyder,[35] Assoc. M. Am. Soc. C. E., for relating flood characteristics to drainage area characteristics. This relation is accomplished through the medium of "lag," which is a function similar to "time of concentration." Lag is related to the length and slope of the longest stream and the length of the stream up to the center of the drainage area. All other variations are considered negligible within a given runoff region.

The function of length used by the authors appears to be a much better measure of lag or other time functions than the length of the longest stream, which often is not a representative measure for the entire drainage system. It is interesting to note that the peak discharge of Eqs. 6*a*, 6*c*, and 6*d* is inversely proportional to the length raised to the 0.7 power. This result is in substantial agreement with Mr. Snyder's equation which relates the discharge inversely to the length raised to the 0.6 power, and is in slightly better agreement with studies made in the Los Angeles Engineer Office which relate the discharge to the length raised to the 0.76 power.

The function of slope used by the authors also appears well chosen and should prove to represent the slope factor better than more simply computed functions. Although the slope factor is omitted in the ordinary application of the summation hydrograph, it is almost mandatory that a slope function be

NOTE.—This paper by H. B. Kinnison, M. Am. Soc. C. E., and B. R. Colby, Esq., was published in March, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1944, by Clarence S. Jarvis.

[34] Associate Engr., U. S. Engrs., War Dept., Los Angeles, Calif.

[34a] Received by the Secretary June 2, 1944.

[11] "Channel-Storage and Unit-Hydrograph Studies," by W. B. Langbein, *Transactions*, Am. Geophysical Union, Pt. II, 1940, pp. 620–627.

[35] "Synthetic Unit-Graphs," by Franklin F. Snyder, *ibid.*, Pt. I, 1938, pp. 447–454.

introduced into formulas that relate runoff and drainage area characteristics if such formulas are to have wide application. Heretofore, application of the S-graph has been restricted within relatively small hydrologic regions, within each of which the slope function probably does not vary sufficiently to affect results and has therefore been omitted. The lag equation derived by Mr. Snyder for Appalachian areas shows the lag to be several times as long as lags developed for Pacific slope areas where stream slopes are much greater. The slope factor should account in a large part for these discrepancies.

Variation of the shape of the unit hydrograph with flood magnitude is usually overlooked or avoided although its existence is reasonable to assume. The curves of Fig. 3 showing the relation between the peak of the unit hydrograph and peak runoff are in substantial agreement with the findings of R. L. Gregory and C. E. Arnold,[36] Assoc. Members, Am. Soc. C. E., who expressed the time of concentration as inversely proportional to the discharge raised to the one-fourth power. As stated by the authors in substantiation of the data shown in Fig. 5(*a*), the peak of the unit hydrograph is inversely proportional to the time of concentration or to lag. (E. P. Bennett, who has reviewed this discussion, has stated that this relation does not hold strictly unless the unit-rain interval is proportional to the lag. However, this relation is approximate for cases in which the unit-rain interval is short compared to the lag.) Thus, the peak of the unit hydrograph, in accordance with the equations of Messrs. Gregory and Arnold, should be directly proportional to the discharge raised to the one-fourth power, as approximated by the curves in Figs. 3(*a*), 3(*b*), 3(*c*), and 4. (Fig. 5(*b*) does not show this relation because factors other than flood discharge have an effect.) The incorporation of a function of discharge in lag equations may improve the relationships considerably, but, before application of the results shown in Figs. 3, 4, and 5, it should be ascertained whether subtraction of subsurface flows from observed hydrographs has involved sufficient error to affect the peaks of the unit graphs.

Those who work with rainfall and runoff frequency determinations for the Pacific slopes area are aware of the great variations of over-all runoff coefficients that might prevail. The curves of Figs. 1 and 2 are presented for illustration of the great variation of runoff magnitude compared to the variation of rainfall magnitude in some regions (which indicates that accuracy in the determination of rainfall magnitude does not necessarily result in accuracy in the derivation of runoff magnitude). Fig. 15 shows a duration curve of peak runoff for Big Bear Lake Dam, and Fig. 16 shows a duration curve of daily rainfall for the Santa Ana River near Mentone, Calif., which is in the tributary area above Mentone. Inasmuch as these curves are representative of hundreds of rainfall and runoff curves for the region, they serve to illustrate that the largest of 50 annual maximum occurrences of rainfall for any duration is usually about 110 times the smallest, whereas the largest of 50 annual maximum runoff occurrences is usually about 1,000 times the smallest. It is questionable whether such curves could be correlated by the analysis presented in this paper with as much accuracy as the accuracy of the duration curve of runoff.

[36] "Run-Off—Rational Run-Off Formulas," by R. L. Gregory and C. E. Arnold, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1058.

Regarding the great influence of rainfall-runoff relations and their erratic nature, an example will be cited. The most disastrous flood that ever struck Southern California was that of March, 1938. During that flood, hundreds of rainfall and runoff records were broken. In January, 1943, a storm occurred

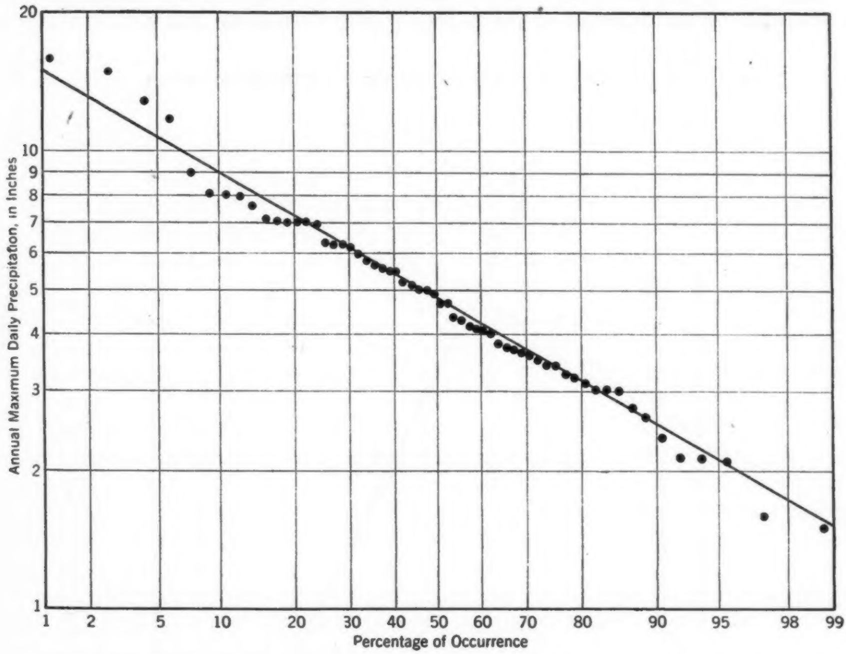

FIG. 15.—DURATION CURVE OF DAILY PRECIPITATION, BIG BEAR LAKE DAM; 1883–1944 (60 YEARS)

whose 24-hr rainfall set a record for the entire nation and which was from 30% to 50% greater than the 24-hr rainfall of the 1938 storm. However, mainly because of the difference in ground conditions, the greater storm of 1943 resulted in a comparatively small flood, which is not generally classed as a major flood.

Of course, the over-all runoff coefficients in the Massachusetts area do not vary as much as those in Southern California, but it is possible that they do vary more than has become evident in the analysis and that curves C and D, Fig. 7, actually should agree more closely with curve A. It is not evident from the presentation just how reliable the precipitation curve of Fig. 6 may be, but it should be expected that errors of that curve and errors in rainfall-runoff relationships combine to produce the errors in curve D, Fig. 7. Curve C or D is not obviously more accurate than curve A. In fact, the occurrence, within a short period, of a flood in excess of the 1,000-yr value is so highly improbable as to cast serious doubt on the accuracy of curves C and D. The inaccuracy of curve B on this basis cannot be reasonably doubted. It would be helpful if the authors would include with their closing discussion the data

that show the probability of a given depth of runoff being generated by a given depth of precipitation. Also, the criteria used in selecting 6-hr and 2-hr runoff amounts from the 72-hr volume have not been indicated.

The writer's only criticism of the analysis and results would be that the scope of the analysis might be too limited even for application within Massachusetts. Some manner of combining runoff records for the entire Atlantic

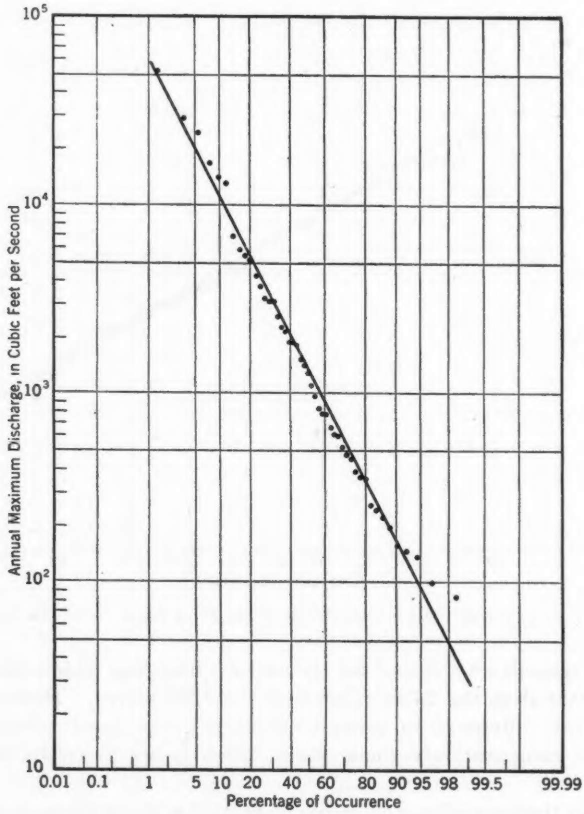

FIG. 16.—DURATION CURVE OF PEAK DISCHARGE, SANTA ANA RIVER NEAR MENTONE 1896–1943 (47 YEARS); 189 SQ MILES

coast of the United States and perhaps for some distance inland should yield results that not only would be more generally applicable but also would be more accurate for each small region as a result of the combination of many independent records. In the opinion of the writer, the advantage of resorting to rainfall-runoff analysis lies not in the superiority of rainfall data to runoff data but in that the results of such an analysis permit the combination and correlation of runoff records. Extrapolation of individual runoff frequency curves by a study of rainfall is generally unsound because of the inadequate

knowledge of the frequency variation of runoff factors and the effect of such variation on the runoff-frequency curve.

The analysis presented is straightforward and simple. It includes wisely chosen general expressions such as average elevation of an area (which includes the factors of soils, vegetal cover, temperature, and rainfall distribution) and size of area (which includes short-time variations in precipitation and diminution of average rainfall with size of area). The degree of "breakdown" into the various component factors can be too high, and often much simplified functions such as those used in this paper are adequate for application within a limited region.

M. J. ORD,[37] JUN. AM. SOC. C. E.[37a]—Flood characteristics of Massachusetts streams have been analyzed thoroughly in this paper. However, the writer questions the wisdom of presenting flood formulas based on what the authors admit to be inadequate basic data. Although limitations have been given to their formulas, once presented, such formulas are frequently used without considering their limitations. It might have been preferable to present the relationships in the form of curves together with plotted values to define the curves.

The minor, major, rare, and maximum floods given in Table 1 can be expressed as percentages of the major flood (see Table 3). In view of the fact that reasonably consistent and uniform rainfall and runoff factors were used in the development of the floods, the percentages for each magnitude flood developed by unit hydrographs should be expected to be reasonably uniform. The percentages for the minor and rare floods do show only a small variation—for the most part within 10% of the average percentage. The percentages for the maximum flood have a greater variation. The floods represented by the extreme variations should be questioned.

The average percentages for the minor, major, and rare floods (based on unit hydrographs) were plotted versus their probable recurrence intervals of 15, 100, and 1,000 years. The resulting curve is shown in Fig. 17. It is of interest to note that the percentage for the maximum flood has a recurrence interval of 1,000,000 years determined by a conservative extrapolation of the curve.

In Table 1 there are fourteen stations for which the maximum recorded flood exceeds the computed 100-yr flood. Such a condition is not reasonable considering the short period of record, without further evidence. More weight should have been given the 1936 and 1938 floods in determining the frequencies. Thus, the slope of the frequency curve should be increased. However, accepting the authors' probabilities of occurrences, Fig. 17 will give results as acceptable as those determined by the formulas presented by the authors, and it has the advantage of permitting the determination of a flood of any frequency once the 100-yr flood has been determined. The 100-yr flood can be determined by the authors' formula or, in the writer's opinion, preferably by unit

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[37a] Received by the Secretary June 2, 1944.

hydrograph, assuming that the distribution of unit runoff amounts has been correctly defined.

For areas having unknown rainfall-runoff relationships, synthetic unit hydrographs can be determined by any one of several methods. The writer prefers a method, developed by the Los Angeles Office of the U. S. Engineer Department, which is similar to that proposed by Mr. Snyder[35] and modified by Mr. Langbein.[11]

In this method "lag" is plotted against a factor $\frac{L \times L_{ca}}{S^{0.5}}$, in which L = length of the longest watercourse, in miles; L_{ca} = length along a watercourse, measured upstream, to a point opposite the center of area, in miles; and S = overall slope of drainage area, in feet per mile. Lag is defined as the elapsed time (in hours) from the beginning of unit rainfall to the instant that the corresponding summation hydrograph reaches 50% of ultimate discharge. The summation hydrograph for a given stream at a given point is the hydrograph that would result from the application of the unit hydrograph for that location to a continuous uniform supply of effective rainfall over the tributary area. The **S**-graph, first introduced by Mr. Langbein,[11] is the summation hydrograph with the coordinates expressed in dimensionless form, with the discharge scale as a percentage of ultimate discharge, and with the time scale as a percentage of lag. With the lag curve and **S**-graph determined for a region, synthetic unit hydrographs can be de-

TABLE 3.—FLOOD PEAKS FROM TABLE 1 EXPRESSED IN PERCENTAGES OF FLOOD PEAKS FOR MAJOR FLOODS

No.[a]	FROM FLOOD FORMULAS				FROM UNIT HYDROGRAPHS			
	Minor (9)	Major (10)	Rare (11)	Maximum (12)	Minor (13)	Major (14)	Rare (15)	Maximum (16)
1	39	100	169	593	47	100	170	602
2	40	100	158	548	47	100	172	632
3	52	100	164	645	50	100	164	670
4	54	100	178	750	....	....	....	....
5	46	100	158	600	48	100	165	643
6	48	100	174	701	....	....	....	....
7	53	100	172	664	48	100	169	556
8	48	100	166	676	46	100	167	686
9	41	100	175	681	42	100	174	1,012
10	37	100	169	590	....	....	....	....
11	38	100	175	585	....	....	....	....
12	38	100	166	608	....	....	....	....
13	47	100	166	562	....	....	....	....
14	63	100	165	558	....	....	....	....
15	41	100	165	592	....	....	....	....
16	50	100	162	532	....	....	....	....
17	43	100	172	628	....	....	....	....
18	43	100	174	654	....	....	....	....
19	41	100	177	650	....	....	....	....
20	41	100	177	655	49	100	159	596
21	41	100	177	620	....	....	....	....
22	38	100	168	557	49	100	156	490
23	46	100	175	733	48	100	169	860
24	43	100	161	650	49	100	153	652
25	40	100	173	635	48	100	164	618
26	39	100	169	671	53	100	162	778
27	66	100	161	542	....	....	....	....
28	66	100	172	594	....	....	....	....
29	56	100	173	612	....	....	....	....
30	48	100	163	576	....	....	....	....
31	45	100	168	606	....	....	....	....
32	40	100	175	633	48	100	159	571
33	36	100	160	538	48	100	195	697
34	41	100	166	531	40	100	181	668
35	38	100	166	572	44	100	167	566
36	36	100	164	567	47	100	195	912
37	54	100	167	572	53	100	152	407
38	55	100	168	548	82	100	159	418
39	57	100	162	584	53	100	174	680
40	53	100	168	601	53	100	157	528
41	41	100	165	592	....	....	....	....
42	46	100	177	628	52	100	173	645
43	54	100	176	781	....	....	....	....
44	38	100	166	606	....	....	....	....
45	43	100	175	626	54	100	154	498
46	40	100	160	527	54	100	160	444
47	58	100	181	673	56	100	177	934
48	58	100	179	658	52	100	164	767
Average	46	100	169	615	50	100	167	649

[a] See Fig. 10.

veloped easily for areas of unknown runoff characteristics from the factors L, L_{ca}, and S.

In the development of the lag relationship for mountain streams in Southern California, numerous physical characteristics were investigated, including values corresponding to M and L in the paper. The factor $\frac{L \times L_{ca}}{S^{0.5}}$ gave as satisfactory a correlation with lag as any combination of drainage basin

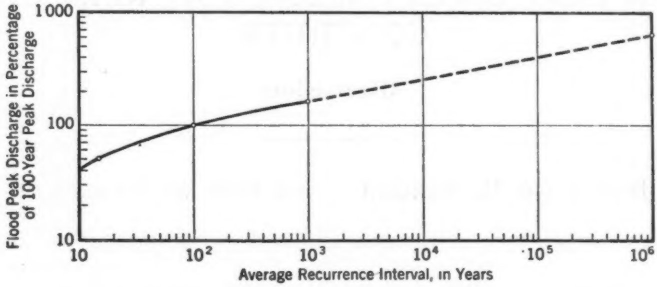

FIG. 17.—FREQUENCY CURVE OF FLOOD PEAKS, STREAMS IN MASSACHUSETTS

characteristics and, in addition, had the advantage of simplicity of determination. For the mountain region of Southern California the improvement in the relationship resulting from introducing the slope factor S is small. The necessity for introducing the slope in the lag relationship is much more apparent in correlating the lag for areas of steep slope with those of gentle slope. A lag relationship applicable to all regions would probably be very complex. The factor of roughness n may be an important item, especially where there is a large variation in the hydraulic efficiency of the channels of different basins.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

SHEAR AND BOND STRESSES IN REINFORCED CONCRETE

Discussion

BY VICTOR R. BERGMAN, AND PHIL M. FERGUSON

VICTOR R. BERGMAN,[20] ASSOC. M. AM. SOC. C. E.[20a]—In calling attention to the errors that can result from a careless application of the commonly used formulas for shear and bond stresses to beams subjected to thrust as well as bending, the authors have performed a useful service. However, their paper contains a few questionable points which the writer wishes to discuss. For example, $j\,d$ is not in general exactly equal to $\left(1 - \frac{k}{3}\right) d$, even for beams without thrust.[20b] The relationship does not hold quite true for T-beams or even for prismatic beams with compressive steel. For the case of beams carrying thrust, the expressed relationship is very definitely in error. The writer would prefer to substitute for the authors' definition of $j\,d$ that given in the Joint Committee Code[21]—"$j\,d$ = the moment arm of the internal force couple."

Fig. 2 is drawn incorrectly, because the compressive stress triangles are depicted as if the $k\,d$-distances were equal for the two sections. Such is not the case, because, as the authors state under the heading, "Member with Thrust," "Any change in the value of the thrust or moment changes the location of the neutral axis." This fact is illustrated in Fig. 10, which shows a definite shift in the position of the neutral axis in the distance dx.

NOTE.—This paper by Stanley U. Benscoter and Samuel T. Logan was published in March, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1944, by F. R. Shanley, B. J. Aleck, Dean Peabody, Jr., and William E. Wilbur; and June, 1944, by L. E. Grinter, and Anders Bull.

[20] Structural Designer, The Kellex Corp., New York, N. Y.

[20a] Received by the Secretary May 10, 1944.

[20b] Correction for *Transactions:* In March, 1944, *Proceedings*, page 296, line 4, correct the definition of "j" as follows: "$j\,d$ = internal moment arm = $\left(1 - \frac{k}{3}\right) d$."

[21] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., Pt. 2, June, 1940, p. 53.

From Fig. 10, using the notation shown, and taking moments about any point at the level of the force $C + \Delta C$, it is possible to write (approximately):

$$\Delta T \left(d - \frac{k\,d}{3} + \frac{h}{3} \right) = V\,dx - C\,\frac{h}{3} \quad \text{.............. (42)}$$

and, since $v\,b\,dx = \Delta T$:

$$v = \frac{V\,dx - C\,(h/3)}{\left(d - \frac{k\,d}{3} + \frac{h}{3} \right) (b)\,(dx)} \quad \text{................. (43)}$$

Eq. 43 is slightly approximate, because it assumes the location of each resultant compressive force at one third of its respective $k\,d$-distance from the

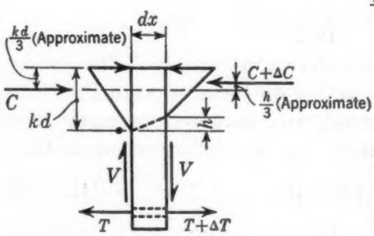

Fig. 10

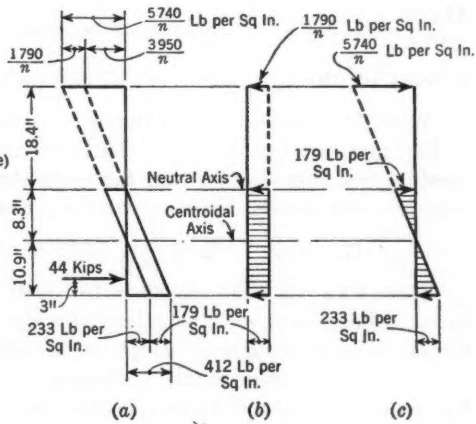

Fig. 11.—Member with Thrust—Internal Stress Distribution (V = 47.5 Kips; and n = 10)

top of the member, thus ignoring the modifying effect of the compressive steel. An accurate determination of the positions of the resultant compressive forces can be made easily by well-known methods, if desired.

The writer has applied Eq. 43 to the case shown in Fig. 3, by first locating the neutral axis of a section parallel to that shown in Fig. 3, but 1 in. closer to the left support (Fig. 4). The neutral axis for this new section is approximately 18.15 in. from the bottom of the beam, thus making $h = 1.05$ in. and $C = 44{,}000 + 1.2 \times 3{,}950 = 48{,}740$ lb. Then, approximately,

$$v = \frac{(47{,}500)\,(1) - (48{,}740)\left(\frac{1.05}{3}\right)}{\left(37.6 - 6.4 + \frac{1.05}{3}\right)(12)\,(1)} = 80 \text{ lb per sq in.,}$$

which agrees fairly well with the authors' determination. An even better agreement could be achieved by: (1) Computing the actual position of the resultant compressive forces; and (2) using a dx-distance smaller than 1 in.

It should be noted carefully that the writer has not used the length $j\,d$ in this determination of v.

Another and probably more instructive and useful approach to this problem is to consider the section (Fig. 3) as resisting: (1) A direct compression of 44,000 lb applied at the center of gravity of the transformed area; and (2) a bending moment whose value is 44,000 (10.9 − 3.0) = 348,000 in-lb.

The stress diagram of Fig. 3 (slightly modified by corrections) is reproduced in Fig. 11(*a*) where the combined stress diagram is resolved into two component parts—that due to direct compression and that due to bending, as shown separately in Fig. 11(*b*) and Fig. 11(*c*), respectively. The direct compressive stress of Fig. 11(*b*) is obtained simply enough by dividing the 44,000-lb thrust by the transformed area A_{TR}. The section shown in Fig. 3 has a transformed area, $A_{TR} = (12)\,(19.2) + (10 - 1)\,(0.44) + (10)\,(1.2) = 230.4 + 4.0 + 12.0 = 246.4$ sq in., and a direct compressive stress (in pounds per square inch) of $\frac{44{,}000}{246.4} = 179\,-$. This stress can be obtained also from Fig. 11(*a*) by a simple proportion; thus (in pounds per square inch): $\frac{8.3}{19.2}\,(412) = 178\,+$.

With this stress value and the relationships shown in Fig. 11(*a*), the stresses of Fig. 11(*c*) are found easily. The stresses acting on the section in Fig. 11(*c*) result solely from flexure and therefore should, and do, produce equal compressive and tensile forces, C and T, which can be computed as follows: $C = \frac{233}{2}\,(12)\,(10.9) + (233)\,\frac{6.5}{10.9}\,(4.0) = 15{,}800$ lb; and $T = \frac{179}{2}\,(12)\,(8.3) + 5{,}740\,(1.2) = 8{,}920 + 6{,}880 = 15{,}800$ lb. These two forces form a resisting couple whose lever arm, calculated by standard procedure, is 22.0 in. This couple, whose numerical value is $15{,}800 \times 22.0 = 348{,}000$ in-lb, exactly balances the bending moment already referred to, caused by the thrust of 44,000 lb acting at a distance of 7.9 in. from the center of gravity of the transformed section.

The authors state that "A dotted line is shown in Fig. 3 to indicate the shear stress that would be obtained from the formula in the 1940 Joint Committee Code[6] (or Eq. 15*a* using $j = \frac{7}{8}$)." Although it is true that the use of $\frac{7}{8}$ for $j\,d$ in Eq. 15*a* will give the erroneous result indicated by the authors, it should be understood clearly that the error is caused by a mistake in the evaluation of $j\,d$ rather than in the formula, which is correct.

The Joint Committee Code defines $j\,d$ as "the moment arm of the internal force couple." Bearing this definition in mind, a glance at Fig. 3 should make apparent the fallacy involved in taking $j\,d$ as $\frac{7}{8}\,d$ (= 32.9 in.), or even as $\left(1 - \frac{k}{3}\right) d$ (= 31.2 in.). The stresses shown as acting on the section do not produce the equal, opposite forces required for a couple. The writer's value of 22.0 in. for the moment arm of the resisting couple (Fig. 11(*c*)), is the proper value of $j\,d$ for use in Eq. 15*a*.

Substituting numerical values, $v_m = \frac{47{,}500}{12 \times 22.0} = 180$ lb per sq in., which checks exactly the authors' finding for v_m.

[6] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., Pt. 2, June, 1940, Eq. 1, p. 52.

To verify the value of v = 78 lb per sq in. given in Fig. 3, it is necessary first to determine what proportion of the total resisting moment is furnished by the tensile steel (see Fig. 11(c)). The position of the resultant compressive force is found readily to lie at a distance of 3.7 in. from the bottom of the beam (10.9/3 = 3.6 in. could be used with but trifling error). The proportion of the total resisting moment, furnished by the tensile steel, is now found to be: $\frac{(1.2)\,(5{,}740)\,(37.6 - 3.7)}{348{,}000} = 0.67$.

If the tensile steel carried all of the tension involved in the resisting couple, the shearing stress, v, would equal: $\frac{47{,}500}{12\,(37.6 - 3.7)} = 117$ lb per sq in. Actually, $v = 0.67 \times 117 = 78$ lb per sq in., which agrees with the authors' value.

The writer has presented two additional methods of calculating shearing stresses in beams subject to thrust. Whether or not the individual designer will consider either method preferable to that furnished by the authors is a matter of personal choice.

PHIL M. FERGUSON,[22] M. AM. SOC. C. E.[22a]—The limitations that attend the proper use of the ordinary formulas for bond and shearing stress in reinforced concrete beams are discussed in this paper. In view of many discussions in recent years relative to the divergence between calculated and actual steel stresses, especially where direct stress is involved, any one would be cautious about insisting on any value of bond stress as a true value. Shrinkage and flow of concrete alter these stresses significantly, even though designers do not usually include their effect in calculations. This is not to disparage the calculations given in this paper and those presented herewith, but rather to emphasize the artificial base upon which both practice and theory now rest. So long as specifications and practice maintain this artificial base, designers, in order to be consistent, must use calculations of this type.

Since bond stress is measured by the change in tensile steel stress, it is entirely logical that direct compressive stress, as it reduces the steel stress, should also reduce the bond stress. However, differences as great as those shown in Fig. 6 are somewhat startling and might seriously modify a design.

The question immediately arises as to whether there are many situations where the ordinary formulas—

$$u = \frac{V}{\Sigma_o \frac{7}{8} d} \qquad (44a)$$

and

$$v = \frac{V}{b \frac{7}{8} d} \qquad (44b)$$

—are inadequate. These formulas are simple and commonly known. They fit beams not subject to direct stress. Does the small direct stress which may be present in many beams in a continuous frame generally limit their usefulness? If not, designers are not apt to adopt the authors' formulas for general use. The proposed formulas are simple enough in appearance, but contain a very

[22] Chairman, Civ. Eng., Univ. of Texas, Austin, Tex.

[22a] Received by the Secretary June 19, 1944.

troublesome term S_n. It will be recalled that S_n is based on a centroid which shifts with the ratio between M and P. Although not a complex calculation, the determination of S_n is lengthy enough to cause one to avoid it wherever possible. No simpler equation seems to be available for those cases where direct stress really modifies bond or shear significantly. Investigation reveals, however, that small axial compressive loads acting on ordinary beams do not modify bond and shear stresses seriously enough to be a matter of concern. This might be more accurately stated by saying that no compressive force will modify shear or bond stresses as much as 10% unless it is large enough, relative to the moment, to produce an e/h-ratio (Fig. 12) of less than unity.

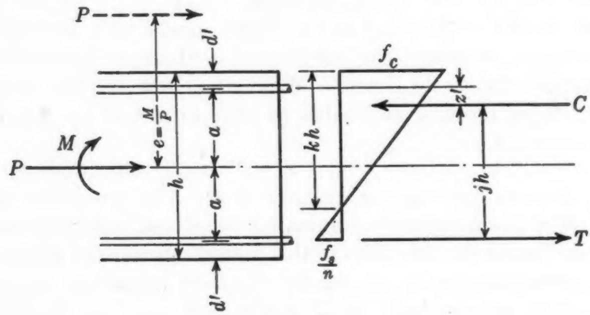

FIG. 12

Although some of this might have been ascertained by applying the authors' method to a few sample cases, it seemed desirable to investigate the theoretical relationships more thoroughly in order to establish the general area of significant difference. The following proofs include the familiar terms k and j as fractional parts of the over-all depth h, the total depth, instead of the effective depth d. This is in accordance with the notation commonly used in combined direct stress and bending.

The applied M and P (at middepth) are held in equilibrium by a resultant compression C and a tension T (Fig. 12). Summation of moments about the tension steel gives $C\,j\,h = M + P\,(0.5\,h - d')$, and

$$C = \frac{M + P\,(0.5\,h - d')}{j\,h} \qquad (45)$$

Since $C = T + P$, $\frac{dC}{dx} = \frac{dT}{dx}$. Also $dT = u\,\Sigma_o\,dx$. Hence, $u\,\Sigma_o = \frac{dT}{dx} = \frac{dC}{dx}$ $= \frac{1}{j\,h}\frac{dM}{dx} - \frac{M + P\,(0.5\,h - d')}{(j\,h)^2}\,h\,\frac{dj}{dx} = \frac{V}{j\,h} - \frac{P\,e + P\,(0.5\,h - d')}{j^2\,h}\frac{dj}{dx}$; and

$$u = \frac{V}{\Sigma_o\,j\,h}\left[1 - \frac{P}{V}\frac{(e + 0.5\,h - d')}{j}\frac{dj}{dx}\right] \qquad (46)$$

Although Eq. 46 might seem to indicate that the correction term in the bracket depends upon V and possibly upon P, it will be shown that a proper evaluation of the term $\frac{dj}{dx}$ results in the elimination of all the load terms from this expres-

sion, with the exception of e, the equivalent eccentricity of the load. An expression for $\frac{dj}{dx}$ will now be found.

The resultant compression C can be located by taking moments of the compressive forces about the center line of the compression steel:

$$\left(\frac{f_c}{2}\,b\,k\,h\right)\left(\frac{k\,h}{3} - d'\right) = C\,z' = \left[\frac{f_c}{2}\,b\,k\,h + p'\,b\,h\,n\,f_c\left(\frac{k\,h - d'}{k\,h}\right)\right]z'$$

$$z' = \frac{k^2\left(\frac{k}{3} - \frac{d'}{h}\right)}{2\,p'\,n\left(k - \frac{d'}{h}\right) + k^2}\,h \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

It will be noted that $p'\,n$ is used instead of the value $p'\,(n - 1)$, sometimes preferred. The value of n is not usually accurate enough to justify any distinction. Since $z' + j\,h$ is the distance between tension and compression steel, which is fixed, $\frac{dz'}{dx} + h\,\frac{dj}{dx} = 0$, and

$$\frac{dj}{dx} = -\frac{1}{h}\,\frac{dz'}{dx} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

Differentiating Eq. 47:

$$\frac{dz'}{dx} = h\left[\frac{k^2 - 2\,\frac{d'}{h}\,k - 2\left(\frac{z'}{h}\right)(p'\,n + k)}{2\,p'\,n\left(k - \frac{d'}{h}\right) + k^2}\right]\frac{dk}{dx} \ldots\ldots\ldots (49)$$

Hence,

$$\frac{dj}{dx} = -\frac{1}{h}\,\frac{dz'}{dx} = -\left[\frac{k^2 - 2\,\frac{d'}{h}\,k - 2\,\frac{z'}{h}\,(p'\,n + k)}{2\,p'\,n\left(k - \frac{d'}{h}\right) + k^2}\right]\frac{dk}{dx} = -R\,\frac{dk}{dx} \ldots (50)$$

In Eq. 50,

$$R = \frac{k^2 - 2\,\frac{d'}{h}\,k - 2\,\frac{z'}{h}\,(p'\,n + k)}{2\,p'\,n\left(k - \frac{d'}{h}\right) + k^2} \ldots\ldots\ldots\ldots\ldots (51)$$

Formulas relating k to $\frac{e}{h}$ are available in several textbooks, but can be derived by writing the equilibrium equation for the moment of T and C about the eccentric position of the load shown in Fig. 12. It is convenient to subdivide C into its components—C_1 on the compressive steel and C_2 on the concrete:

$$\Sigma M_P = C_1\,(e - a) + C_2\left(e - \frac{h}{2} + \frac{k\,h}{3}\right) - T\,(e + a)$$

$$= p'\,b\,h\,n\,f_c\left(\frac{k\,h - d'}{k\,h}\right)(e - a) + \frac{1}{2}\,f_c\,b\,k\,h\left(e - \frac{h}{2} + \frac{k\,h}{3}\right)$$

$$- p\,b\,h\,n\,f_c\left(\frac{h - d' - k\,h}{k\,h}\right)(e + a) = 0 \ldots\ldots\ldots\ldots (52a)$$

or

$$p'\,n\left(k-\frac{d'}{h}\right)(e-a)+\frac{k^2}{2}\left(e-\frac{h}{2}\right)+\frac{k^3\,h}{6}$$
$$-\,p\,n\left(1-\frac{d'}{h}-k\right)(e+a)=0\ldots\ldots\ldots\ldots\ldots(52b)$$

Differentiating Eq. 52*b* with respect to x and collecting terms gives the equation:

$$\frac{dk}{dx}\left[p'\,n\,(e-a)+k\left(e-\frac{h}{2}\right)+\frac{1}{2}k^2\,h+p\,n\,(e+a)\right]$$
$$=-\frac{de}{dx}\left[p'\,n\left(k-\frac{d'}{h}\right)-p\,n\left(1-\frac{d'}{h}-k\right)+\frac{k^2}{2}\right]\ldots\ldots(53a)$$

Since $de=\frac{dM}{P}$, or $\frac{de}{dx}=\frac{dM}{dx}\frac{1}{P}=\frac{V}{P}$,

$$\frac{dk}{dx}=-\frac{V}{P\,h}\left[\frac{p'\,n\left(k-\frac{d'}{h}\right)-p\,n\left(1-\frac{d'}{h}-k\right)+\frac{k^2}{2}}{p'\,n\left(\frac{e}{h}-\frac{a}{h}\right)+k\left(\frac{e}{h}-\frac{1}{2}+\frac{k}{2}\right)+p\,n\left(\frac{e}{h}+\frac{a}{h}\right)}\right]$$
$$=-\frac{V}{P\,h}B\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(53b)$$

In Eq. 53*b*, B is the quantity in the bracket. The value of B simplifies either for symmetrical steel or for no compressive steel. If $p'\,n=p\,n$,

$$B=\frac{2\,p\,n\,k-p\,n+k^2/2}{2\,p\,n\,\frac{e}{h}+k\left(\frac{e}{h}-\frac{1}{2}+\frac{k}{2}\right)}\ldots\ldots\ldots\ldots\ldots(54)$$

The value of $\frac{dk}{dx}$ from Eq. 53*b* may be substituted into Eq. 50:

$$\frac{dj}{dx}=-R\,\frac{dk}{dx}=+\frac{V}{P\,h}B\,R\ldots\ldots\ldots\ldots\ldots\ldots(55)$$

Eq. 46 now becomes:

$$u=\frac{V}{\Sigma_o\,j\,h}\left[1-\frac{P}{V}\,\frac{(e+0.5\,h-d')}{j}\,\frac{V}{P\,h}\,B\,R\right]$$
$$=\frac{V}{\Sigma_o\,j\,h}\left[1-\left(\frac{e}{h}+\frac{1}{2}-\frac{d'}{h}\right)\frac{B\,R}{j}\right]\ldots\ldots\ldots\ldots(56)$$

The bracket might be considered the modification term except that common practice substitutes $\frac{7}{8}\,d$ for $j\,h$ in ordinary beam design and this is not a good substitution here. Including the necessary correction, the equation becomes:

$$u=\frac{V}{\Sigma_o\,\frac{7}{8}\,d}\left\{\frac{0.875}{j}\,\frac{d}{h}\left[1-\left(\frac{e}{h}+\frac{1}{2}-\frac{d'}{h}\right)\frac{B\,R}{j}\right]\right\}=\frac{V}{\Sigma_o\times\frac{7}{8}\,d}$$

times a modification term $\ldots\ldots\ldots\ldots\ldots\ldots(57)$

The modification term is lengthy, but is not otherwise difficult to evaluate. In practice, the authors' equation $u = \frac{V\, A_s\, n}{\Sigma_o\, S_n}$ usually would be preferable if a correction is needed; but it does not indicate what controls the modification and gives no idea of whether this modification is important or otherwise. Eqs. 57, 53*b*, and 50 show that the modification term depends on $p\, n$, $p'\, n$, $\frac{d'}{h}$, and $\frac{e}{h}$; or that, for a given beam, it will vary solely with the $\frac{e}{h}$-ratio, since $\frac{e}{h}$ controls k. Eq. 52*b* can be used to find either $\frac{e}{h}$ or k if the other is known; or almost any textbook contains k-curves for the case of symmetrical reinforcement.

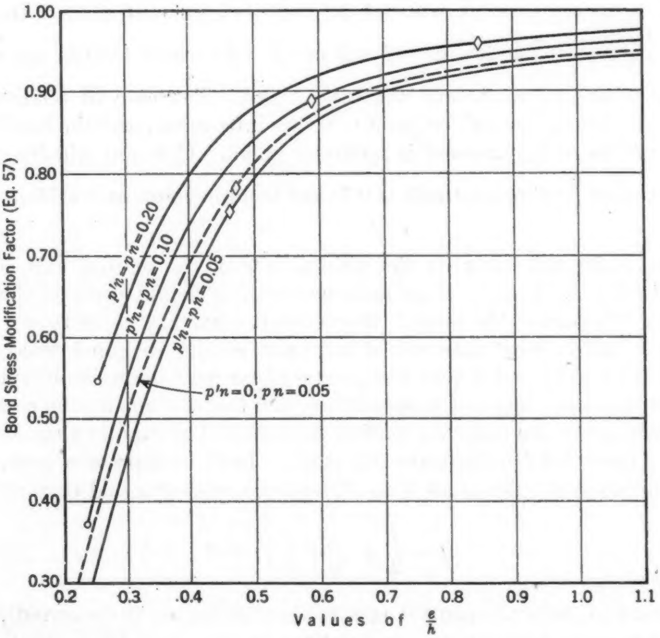

FIG. 13

The modification term is too lengthy to visualize. Fig. 13 shows the modification term $\left(\text{for } \frac{d'}{h} = 0.10\right)$ plotted against $\frac{e}{h}$ for cases of symmetrical reinforcement varying from $p\, n = 0.05$ to 0.20 and for one case of no compressive steel. The value $p = \frac{A_s}{b\, h}$ is based on steel in one face. These curves are terminated on the left at the $\frac{e}{h}$-value which gives zero stress on the tension steel. Any smaller $\frac{e}{h}$-ratio puts this steel in compression. The value of $\frac{e}{h}$ on

each curve that corresponds to a neutral axis at the middepth of the member has been marked by a small diamond.

These curves indicate that for this $\frac{d'}{h}$-ratio, unless $\frac{e}{h}$ is less than 0.7, the modification term will be 0.9 or more, since the steel usually will not be less than that indicated by $p\,n = 0.05$. Few designers will care to make any correction for larger $\frac{e}{h}$-values than this, since it is well known that the usual bond formula is very approximate at best and a 10% change on the safe side is not very important.

If designers retain the use of the usual formula wherever $\frac{e}{h}$-values exceed 0.7, the proposed formula will not be necessary in most cases. In ordinary beams, values of $\frac{e}{h}$ as low as 0.7 will occur only where thrusts are relatively large or where moments are relatively small. The case of relatively large thrusts at sections critical for bond is rather infrequent; and the bond is rarely of interest where the moment is relatively small. Columns will frequently be loaded to give $\frac{e}{h}$-values as small as 0.7; but bond in columns is seldom a serious matter.

Since horizontal shear on the tension side of the neutral axis is directly related to the bond stress, these same conclusions should apply to shear calculations in this part of the beam. However, it seems improper to say that this shear is a satisfactory measure of the most serious diagonal tension in the beam. It must be noted that the greater shear near the centroid will tend to cause an increased diagonal tension there, and that the small axial compression might only cause the diagonal tension to increase less rapidly than v.

Fig. 3 gives data to illustrate this point. In the compressive area, it seems reasonable to apply the usual formula for maximum diagonal tension:

$$t = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + v^2} \quad \text{(58)}$$

At a point 5 in. from the neutral axis (in the direction of the centroid), $f = 108$ lb per sq in., $v = 164$ lb per sq in., and $t = -54 + \sqrt{(54)^2 + (164)^2} = -54 + 173 = 119$ lb per sq in. This is remarkably close to the 120 lb per sq in. given by Eq. 44*b*. This matter needs further study before any modification of present methods is proposed.

Correction for *Transactions:* In March, 1944, *Proceedings*, page 301, in Fig. 5(*c*), extend the arrow denoting "centroidal axis" to the dashed line.

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DISCUSSIONS

PRESSURE ON THE LINING OF CIRCULAR TUNNELS IN PLASTIC SOILS

Discussion

BY M. A. DRUCKER

M. A. DRUCKER,[5] ESQ.[5a]—As a result of his analysis of the horizontal pressure on the Lincoln Tunnel, the author concludes that the Rankine formula should not be abandoned in the design of tunnels. It is not intended to dispute this statement; but the method indicated for deriving the angle of repose, as well as the angle obtained, seems questionable. In obtaining the angle of repose of the silt, the author considers a uniform horizontal water pressure equal to that at the bottom of the river extending downward through the silt and, in addition, pressure due to a mixture of water and solid particles for the depth of the silt—the total pressure at point A (Fig. 9) being equal to 40 lb per sq in. Two objections may be raised to this method. In the first place, the pressure due to the water is assumed to be divided into two parts, and, secondly, the silt is considered to be a fluid mixture having an angle of repose. That it is not a mixture is evident from the depth of clear water above the top of the silt. It is in fact a sediment having its own angle of repose in water.

Generally for structures in soil below water, the horizontal pressures are calculated on the basis of full water pressure plus the effect of the soil, whose angle of repose is either known or assumed and whose weight is considered to be reduced by the buoyant effect of the water. Applying this method to the case under consideration, it may be seen that the unit water pressure at point A would be $\frac{82.5 \times 64}{144}$ or 36.7 lb per sq in. This result leaves 3.3 lb per sq in., from the measured horizontal pressure of 40 lb, for the effective horizontal soil pressure. For the given weight of silt of 104 lb per cu ft, the effective weight of the solid particles, per cubic foot of silt, would be 40 lb. Using the Rankine formula for active earth pressure, the pressure at point A for a height of

NOTE.—This paper by D. P. Krynine, M. Am. Soc. C. E., was published in May, 1944, *Proceedings*.

[5] Structural Civ. Engr.—in Chg., Board of Transportation City of New York, New York, N. Y.

[5a] Received by the Secretary August 9, 1944.

silt of 34.5 ft gave the following relationship: $3.3 = \frac{34.5 \times 40}{144} \times \frac{1 - \sin \phi}{1 + \sin \phi} = 9.6 \times \frac{1 - \sin \phi}{1 + \sin \phi}$. From this it was found that $\phi = 29° 20'$—a rather large angle of repose for the material in question. Pressure readings somewhat smaller than the actual pressures could account for this.

The paper contains information from which the angle of repose may be obtained more directly. As stated by Jacob Feld, M. Am. Soc. C. E., the vertical shear planes, discussed by the author, may be considered as approximately equal to the angle of repose of the material.[6]

Assuming that the shearing surfaces start from the horizontal axis of the tunnel and spread out so that they are 67 ft apart at the bottom of the river, which is 46 ft below water level (as given in the paper), the tangent of the angle they make with the vertical is $\frac{0.5\,(67 - 31)}{82.5 - 46} = \frac{18}{36.5} = 0.49$, giving an angle of 26°. A somewhat smaller angle is obtained by considering the bulge of soil over the tunnel. It is stated that, on July 6, this bulge was 12 ft high and 67 ft wide—apparently an unstable condition—and, on July 8, the bulge was down to 7 ft. Assuming the curve of the bulge to be a parabola, as does the author, its maximum slope would be $\frac{7 \times 2}{33.5} = 0.418$ which corresponds to an angle of 22° 40′. This agrees very closely with the assumed upper limit of 20° used in the design of the Holland tunnels in the same Hudson River silt. For such an angle of repose, the pressure due to the solid particles of the silt would be $9.6 \times \frac{1 - \sin 20°}{1 + \sin 20°} = 4.7$ lb. This pressure, added to the 36.7 lb due to the water pressure, would give a total at point A of 41.4 lb which is only 1.4 lb greater than the measured 40 lb and is within the 2-lb limit of accuracy of the pressure measurements as stated in the paper. Incidentally, in accepting the angle of repose of 9° obtained by the author, one would very likely use the general method for finding the horizontal pressure and would obtain a value of $36.7 + 9.6 \times \frac{1 - \sin 9°}{1 + \sin 9°} = 36.7 + 7.0$ or 43.7 lb per sq in. instead of the 40 lb from which the author obtained the 9° angle of repose.

There is another item in the paper that is of unusual interest. This is the calculated reduction in the measured external pressures due to the internal air pressure which, according to the author, causes the lining to expand. The author finds this correction to be 0.15 lb per sq in. per lb of internal air pressure or 4.2 lb for a maximum air pressure of 28 lb. It is true that after making corrections on this basis the average pressures, p_d and p_u, for the several dates, are brought into close agreement, as shown in Table 1. However, the differences in the measured pressures may be due to entirely different causes or to inaccuracies in the pressure measurements. In support of this view, the writer would like to call attention to the measured horizontal and vertical distortions of the tunnel as given in the paper. It is stated that the increase in diameter in one direction was accompanied by an equal decrease in the

[6] *Proceedings*, Brooklyn Engrs. Club, Vol. 22, No. 2, January, 1924, p. 33.

diameter at right angles to it. Theoretical calculations show that equal increase and decrease in lengths of diameter at right angles to each are due to external loading.

If the internal air pressure induced passive pressures in the surrounding soil, it would be necessary for the tunnel to expand in all directions. This would cause a greater increase in length of diameter that elongated, due to the outer pressures, and a smaller decrease in the diameter that is shortened due to the same causes. For the case of a corrected pressure of 4.2 lb per sq in., the pressure per square foot would be about 600 lb. Although it is not known what the soil constant, in pounds per square foot per inch of soil compression, would be for the silt around the tunnel, an assumed value of 1,000 lb, for the purpose of discussion, would seem reasonable. Thus, the tunnel diameter would have to increase $2 \times \frac{600}{1,000} = 1.2$ in. to develop the passive pressures applied as corrections; also the decrease of the shortened diameter would be reduced by 1.2 in. However, the measurements show that the elongation in one direction was equal to the shortening at right angles and the changes in diameter were about 1.5 in. Had the tunnel stretched in accordance with the assumption of a 1,000-lb soil constant, the total elongation of one diameter would have been 1.5 in. + 1.2 in., or 2.7 in., whereas the net shortening of the diameter at right angles would have been 1.5 in. − 1.2 in., or 0.3 in. If a larger soil constant had been assumed, although the nature of the soil does not seem to warrant it, the necessary differences between the increase and decrease in diameters would have been proportionately less than 2.7 − 0.3, or 2.4 in., but still of a magnitude that would have shown up in the measurements. Even for a soil constant of 5,000 lb, the difference between the elongation of one diameter and the shortening of the one at right angles to it would have been about 0.5 in. Also, the tendency of the tunnel to stretch, caused by the tensile stresses set up by the internal air pressures, is offset by the ring shortening due to the compressive stresses caused by the larger external active pressure. Furthermore, as the internal air pressure did not exceed 28 lb per sq in., which was less than the minimum external water pressure, then, at any point on the tunnel, the active outer pressure was greater than the inner air pressure. It is difficult to understand, therefore, how the internal air pressure could induce a passive pressure in the surrounding soil.

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DISCUSSIONS

CHARACTERISTIC REDUNDANTS USED FOR ANALYZING STATICALLY INDETERMINATE STRUCTURES

Discussion

BY CHARLES A. ELLIS

CHARLES A. ELLIS,[5] M. AM. SOC. C. E.[5a]—The author expresses the hope that the methods presented in his paper will be of interest because they differ so radically from the usual methods. That they differ no one will deny. Whether they will stimulate interest is a matter of individual opinion.

The great amount of labor involved in giving birth to the concept of his paper is a monument to the ingenuity and patience of the author, who, in his "Theory of Statically Indeterminate Structures,"[6] has shown a decided preference for the principles of least work and virtual work which, although sound in theory, are mathematical abstractions. They permit neither physical conception nor space perception which are so helpful in the understanding and application of any principle or method.

Had the author simplified or clarified these two principles he would have done the profession some service; but, on the contrary, his method requires seven pages and thirty-one formulas or equations for its development. Two applications of the method are made to a truss having redundant members. Three pages, seven equations, and two tables are required for the first application in section 12; and three pages, thirteen equations, and one table, for the alternate application in section 17—and all of it is fairly "tough meat."

The writer prefers the principle of deflections in solving all types of statically indeterminate structures. There are at least four methods of application, including area moments, expression of moments in terms of angular rotations (commonly called the slope-deflection method), and moment distribution, all of which are well known to structural engineers. The fourth method—the method of Williot equations—is not so well known. In 1934 he published the

NOTE.—This paper by John B. Wilbur, Assoc. M. Am. Soc. C. E., was published in June, 1944, *Proceedings*.

[5] Prof., Structural Eng., Purdue Univ., West Lafayette, Ind.

[5a] Received by the Secretary July 5, 1944.

[6] "Theory of Statically Indeterminate Structures," by W. M. Fife and J. B. Wilbur, McGraw-Hill Book Co., Inc., New York, N. Y., 1937.

first application of his method to the towers of Golden Gate Bridge;[7] and in the same year he derived the Williot equations for the three most common types of quadrilateral units to be found in parallel and inclined chord trusses and trestle bents.[8] Each of these three units has two diagonal members. The equations are derived from the geometrical properties of a Williot diagram for each unit, and give the relation between the strains of the six members in each unit.

A four-panel parallel chord truss having two diagonals in each panel is solved by the methods of least work, virtual work, and Williot equations, showing a comparison of the labor involved in each solution. This method will be used in solving the stresses in the author's similar redundant structure shown in Fig. 3(*a*), and the following equation, stated in words, is applicable:[9]

> The sum of the strains in the two diagonals multiplied by cosec θ equals the sum of the strains in the two horizontal members multiplied by cot θ, plus the sum of the strains in the two vertical members, where θ is the angle between the diagonal and the horizontal member. . . (73)

Let the quantities, $5\,a$, $5\,b$, and $5\,c$, represent, respectively, the assumed tensile stresses in the three redundant members U_0L_1, U_1L_2, and U_2L_3 caused by the load P at L_1. The computed stresses are as follows:

Member	Stress
L_0L_1	$\frac{1}{2}P - 3\,a$
L_1L_2	$\frac{1}{4}P - 3\,b$
L_2L_3	$-3\,c$
U_0U_1	$-3\,a$
U_1U_2	$-\frac{1}{2}P - 3\,b$
U_2U_3	$-\frac{1}{4}P - 3\,c$
U_0L_0	$-4\,a$
U_1L_1	$\frac{2}{3}P - 4\,a - 4\,b$
U_2L_2	$-\frac{1}{3}P - 4\,b - 4\,c$
U_3L_3	$-\frac{1}{3}P - 4\,c$
U_0L_1	$+5\,a$
L_0U_1	$-\frac{5}{6}P + 5\,a$
U_1L_2	$+5\,b$
L_1U_2	$\frac{5}{12}P + 5\,b$
U_2L_3	$+5\,c$
L_2U_3	$\frac{5}{12}P + 5\,c$

The author assumes a constant value for $\frac{L}{A\,E} = 25$ for each member. In any ordinary truss of the same material throughout, E is constant and $\frac{L}{A}$ a variable.

[7] "Williot Equations for Statically Indeterminate Structures," by Charles A. Ellis, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 580.

[8] "Simplified Analysis of Indeterminate Frames," by Charles A. Ellis, *Engineering News-Record*, April 26, 1934, p. 534.

[9] "Simplified Analysis of Indeterminate Frames," by Charles A. Ellis, *ibid.*, April 26, 1934, p. 534, Eq. 2.

In this case the stress in each member should be multiplied by $\frac{L}{A}$ for the member, to give a quantity proportional to the strain. In this problem each stress is proportional to the strain, and the factor 25, which would appear in all terms of the Williot equation, may be canceled. Substituting in Eq. 73 (in which $\operatorname{cosec}\theta = 5/4$ and $\cot\theta = 3/4$):
For the left panel—

$$\left(-\tfrac{5}{6} P + 10\,a\right) \tfrac{5}{4} = \left(\tfrac{1}{2} P - 6\,a\right) \tfrac{3}{4} + \tfrac{2}{3} P - 8\,a - 4\,b \ldots\ldots (74a)$$

for the middle panel—

$$\left(\tfrac{5}{12} P + 10\,b\right) \tfrac{5}{4} = \left(-\tfrac{1}{4} P - 6\,b\right) \tfrac{3}{4} + \tfrac{1}{3} P - 4\,a - 8\,b - 4\,c \ldots (74b)$$

for the right panel—

$$\left(\tfrac{5}{12} P + 10\,c\right) \tfrac{5}{4} = \left(-\tfrac{1}{4} P - 6\,c\right)\tfrac{3}{4} - \tfrac{2}{3} P - 4\,b - 8\,c \ldots\ldots (74c)$$

Eqs. 74 reduce to

$$100\,a + 16\,b = \tfrac{50}{6} P \ldots\ldots\ldots\ldots\ldots\ldots (75a)$$

$$16\,a + 100\,b + 16\,c = -\tfrac{9}{6} P \ldots\ldots\ldots\ldots\ldots (75b)$$

and

$$16\,b + 100\,c = -\tfrac{33}{6} P \ldots\ldots\ldots\ldots\ldots\ldots (75c)$$

whence—

$$\begin{aligned} a &= 0.0866\,P, & 5\,a &= 0.433\,P \\ b &= -\,0.0206\,P, & 5\,b &= -\,0.103\,P \\ c &= -\,0.0517\,P, & 5\,c &= -\,0.2585\,P \end{aligned}$$

—which agrees with the author's solution for the stresses in the redundant members.

Eqs. 75 have one peculiarity in common with slope-deflection equations in that the coefficient of one unknown quantity in each equation is large in comparison with the coefficients of the other unknowns in the same equation. The unknown values in equations having this peculiarity converge rapidly when solved by a series of approximations. The solution of slope-deflection equations by approximations is analogous to the approximations made in moment distribution. However, when only a few equations are involved, the ordinary methods of solution are quicker.

The rectangular arch which the author shows in section 13 may be solved much more quickly by any one of the first three methods of the principle of deflections previously mentioned.

The author is aware, of course, that his solutions are long and considerably involved in comparison with usual methods. He states that the purpose of his paper is to present some of the principles involved in using orthogonal forces. What the writer would very much like to see is a problem or two for which this method is shorter and therefore superior to usual methods; or perhaps a problem, for which other methods are impotent, that may be solved by this method alone.

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DISCUSSIONS

ANALYSIS OF THE GENERAL TWO-DIMENSIONAL FRAMEWORK

Discussion

BY JAROSLAV POLIVKA

JAROSLAV POLIVKA,[15] M. AM. SOC. C. E.[15a]—An interesting and original general method of analyzing a two-dimensional framework, based on the superposition principle, is presented by the author. The method has the same advantages and disadvantages as some of the other methods of superposition, such as the method of moment distribution by successive approximation. Superposition methods require more accuracy because of the many calculations involved and extreme caution is necessary to avoid confusion in sign conventions.

A greater simplification is attained by methods permitting a better visualization of the elastic deformations under the given load, such as, for example, column analogy, the method of fixed points, and a method based on the ellipse of elasticity.

The author's method and the method of characteristic (fixed) points are similar in some ways. To demonstrate the common features of both methods, and for purposes of checking, the writer has solved the problem illustrated in Fig. 13, using the method of characteristic points, as shown in Fig. 19. The results were checked by the slope deflection method, involving the following equations:

$$\left.\begin{aligned} 7\,B + 2\,C - 18\,R &= 0 \\ 9\,C + B + 2\,E + 46\,R &= 0 \\ 5\,E + C + 5\,R &= 80 \\ 27\,B - 138\,C - 30\,E - 6{,}526\,R &= 11{,}520 \end{aligned}\right\} \qquad (61)$$

Solving: $B = -\dfrac{605{,}460}{821{,}199}$; $C = +\dfrac{6{,}181{,}120}{821{,}199}$; $E = +\dfrac{13{,}570{,}720}{821{,}199}$; and

NOTE.—This paper by Yves Nubar, M. Am. Soc. C. E., was published in April, 1944, *Proceedings*. Discussion on this paper appeared in *Proceedings*, as follows: June, 1944, by Leon Beskin.

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[15a] Received by the Secretary May 27, 1944.

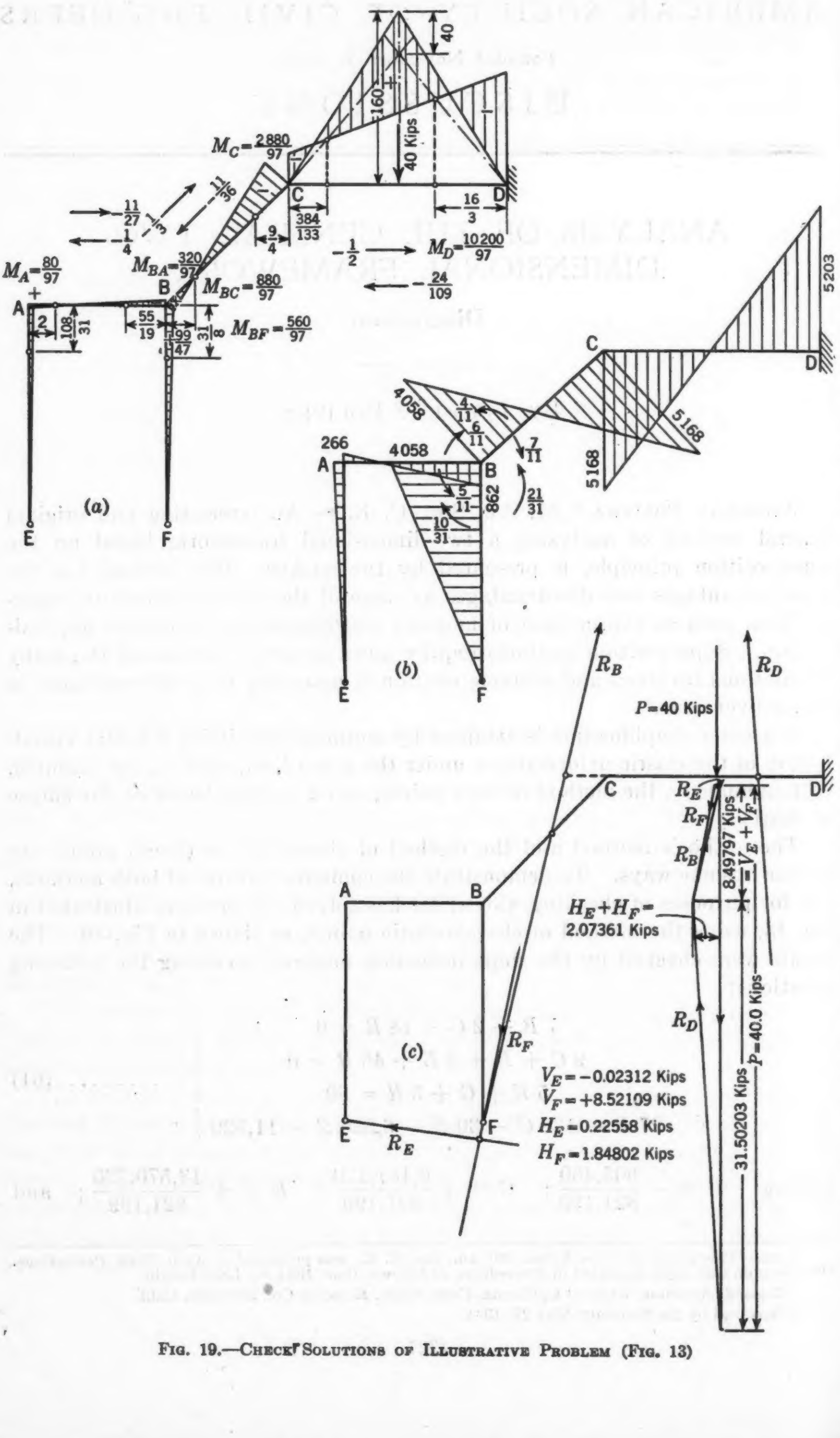

FIG. 19.—CHECK SOLUTIONS OF ILLUSTRATIVE PROBLEM (FIG. 13)

$R = -\dfrac{1{,}667{,}760}{821{,}199}$. The slope deflection equations are:

$$\left.\begin{aligned} &M_{AE} = \frac{3}{4}B - \frac{9}{2}R;\quad M_{AB} = B + \frac{C}{2};\quad M_{BA} = C + \frac{B}{2};\\ &M_{BF} = 2\left(\frac{3}{4}C - \frac{9}{2}R\right);\quad M_{BC} = 2\left(C + \frac{E}{2} + 16\,R\right);\\ &M_{CB} = 2\left(E + \frac{C}{2} + 16\,R\right);\quad M_{CD} = 3\,(E - 9\,R) - 80;\\ &\text{and}\quad M_{DC} = 3\left(\frac{E}{2} - 9\,R\right) + 80 \end{aligned}\right\}\ldots(62)$$

The shear equations are:

$$\left.\begin{aligned} &M_{BA} + M_{AE} = 16\,H_E;\quad M_{BF} + M_{FB} = 16\,H_F;\\ &M_{BC} + M_{CB} = 9\,(H_E + H_F) - 9\,(V_E + V_F);\\ &\text{and}\quad M_{CD} + M_{DC} = -\,16\,(V_E + V_F) + 8 \times 40 \end{aligned}\right\}\ldots\ldots(63)$$

Furthermore,

$$\left.\begin{aligned} &\frac{16}{9}(M_{BC} + M_{CB}) = M_{AE} + M_{BF} + M_{CD} + M_{DC} - 320;\quad \text{and}\\ &36\,M_{AE} + 36\,M_{BF} + 36\,M_{CD} + 36\,M_{DC}\\ &\quad - 64\,M_{BC} - 64\,M_{CB} = 11{,}520 \end{aligned}\right\}\ldots(64)$$

and the joint equations are:

$$\left.\begin{aligned} &4\,M_{AE} + 4\,M_{AB} = 0;\quad 2\,M_{BA} + 2\,M_{BF} + 2\,M_{BC} = 0;\\ &\text{and}\quad M_{CB} + M_{CD} = 0 \end{aligned}\right\}\ldots\ldots(65)$$

The solution (computation machine values) yields:

$$M_{AE} = -\,M_{AB} = +\,\frac{2{,}964{,}000}{821{,}199} = +\,3.60;\quad M_{BA} = +\,\frac{3{,}153{,}840}{821{,}199} = +\,3.84;$$

$$M_{BF} = +\,\frac{24{,}281{,}520}{821{,}199} = +\,29.56;\quad M_{BC} = -\,\frac{27{,}435{,}360}{821{,}199} = -\,33.40;$$

$$M_{CB} = -\,M_{CD} = \frac{20{,}045{,}760}{821{,}199} = -\,24.41;$$

$$\text{and}\quad M_{DC} = +\,\frac{131{,}081{,}520}{821{,}199} = +\,159.62.$$

The same results were obtained by using the method of ellipse of elasticity, substituting for the frame EABF an imaginary ellipse of support at joint B, and combining this ellipse of support with the ellipse of the section BCD. The graphic results of this analysis are shown in Fig. 19(c). Comparing both methods the following statements may be made:

(1) The values C in the author's paper are the carry-over factors of the individual members of the framework (Fig. 19(a)): $C_{1B} = -\dfrac{1}{4}$, $C_{1A} = -\dfrac{11}{27}$, $C_{2C} = -\dfrac{11}{36}$, $C_{3D} = -\dfrac{24}{109}$, etc.

(2) The coefficients of $E\,\epsilon_6$ in Eqs. 42, 43, 44, and 45 are the moments due to axial strain $E\,\epsilon_6$ of the imaginary member 6 (Fig. 13) and are proportional to the moments of the framework when subjected to a horizontal displacement (translation) along AB equal to Δ_B.

(3) The other values of Eqs. 42 and 45 are the bending moments of the framework subjected to the given load when the lateral displacement Δ_B is equal to zero.

Corrections for *Transactions:* In April, 1944, *Proceedings*, on page 453, change "20.55" in Eq. 44 to "25.81"; "10.20" in Eq. 45*a* to "5.21"; and "0.78" in Eqs. 45*b* and 45*c* to "2.03"; on page 454, change "1.41" to "1.37" in lines 6, 9 (two places), and 16; change "26" to "24.4" in line 9; change "161" to "159.5" in line 10; and change "3,880" to "3,994" in lines 17 and 19 (two places). See also errata in June *Proceedings*, on page 920.

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DISCUSSIONS

SIMULTANEOUS CONSOLIDATION OF CONTIGUOUS LAYERS OF UNLIKE COMPRESSIBLE SOILS

Discussion

BY GEORGE W. GLICK

GEORGE W. GLICK,[21] ESQ.[21a]—The analysis by Professor Gray of consolidation in a two-layer system gives the answer to a most difficult problem and is a very worth-while contribution to the study of soil mechanics. As has been shown, the application of the formulas is long and tedious; yet, within limits, certain approximations can be used.

The problem of drainage through an incompressible porous medium is related to the problem of consolidation of a two-layer system with single drainage. This can be seen readily when the thickness of the outer layer becomes very thin, as the solutions for the consolidation of the inner layers become identical.

This type of problem was presented first a number of years ago during the testing of silty soils in the usual consolidation devices. At that time, it was realized that stones of high permeability were desirable, but the magnitude of the error that could occur from improperly chosen drainage stones was unknown until Professor Gray solved the problem in 1938.[22]

The author's remarks on the influence of the drainage medium on the shearing strength of consolidating soils are most pertinent.

The solution of the problem of the two-layer system with double drainage is most interesting and raises some unusual speculations. When one considers the solution as applied to two layers of homogeneous soil, it can be concluded that this is a solution of the familiar single-layer problem with the time factor referenced to the thickness of one of the two layers. In other words, the time

NOTE.—This paper by Hamilton Gray, Jun. Am. Soc. C. E., was published in February, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1944, by Edward S. Barber, and Jacob Feld.

[21] Staff Engr., Moran, Proctor, Freeman & Mueser, New York, N. Y.

[21a] Received by the Secretary June 13, 1944.

[22] "Research on Consolidation of Fine-Grained Soils," by Hamilton Gray, thesis presented to the faculty of the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in partial fulfilment of the requirements for the degree of Doctor of Science in the field of Soil Mechanics, April, 1938. On file for reference in Engineering Societies Library, New York, N. Y.

factor for a single-layer system is multiplied by the square of the ratio of the total thickness to one of the layers to secure the time factor for that layer.

For homogeneous layers, the values of both σ and μ are unity. When only σ has the value of unity, the system might be considered pseudo-homogeneous as a stretching transformation can be applied to one of the layers to make a homogeneous system. In this manner, the time-consolidation coefficients for a single-layered homogeneous system can be used, thereby saving the labor of solving the equations for the two-layer system. When σ is unity, any number of layers can be combined by using their transformed thicknesses. The same type of reasoning can be applied to the two-layer system with single drainage.

From the solution of the two-layer system with double drainage, a solution for a single layer having one side with restricted drainage and one side with free drainage can be deduced by assuming one layer to be very thin.

When the value of σ differs from unity, the use of a stretching transformation introduces an error of unknown magnitude. It is a safe assumption that, when σ is less than 2, the error will be less than that for the author's case in which the time factor is about 13% too small at 90% consolidation. It would be interesting to know whether the writer's suggested transformation is applicable to other stress distributions.

The author is to be congratulated for his efforts in solving one of the most common, yet annoying, problems in this new science.

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DISCUSSIONS

HYDRAULIC MODEL INVESTIGATION OF MITER GATE OPERATION

Discussion

BY ISAAC DEYOUNG, AND F. W. EDWARDS

ISAAC DEYOUNG,[4] M. AM. SOC. C. E.[4a]—Tests for determining the force required to swing the mitering gates of the locks at St. Marys Falls Canal in Michigan were made in 1941. Although the method of testing was not highly refined, the results, in some respects, are similar to those determined by the more elaborate tests conducted with the model mechanism of the Panama Locks.

The gate engines were not tested as usual, because of the difficulty of inserting a measuring device for determining the power to move these gates. The gates tested are operated by cables reaved over spiral drums with the cables attached to the gates near the toe of the gates and close to the center of gravity of the mass of water through which the gates swing.

Power for swinging the gates was obtained from adjacent capstans, and the tension in the cable was measured on a hydraulic weighing machine.

One of the tests was made on the intermediate lower gates of the Davis Lock at St. Marys Falls Canal. The tests were made on a single-leaf operation with the other leaf fixed in its recess. The time of a complete swing of the leaf was 1½ min, about the usual time for actual operation. The operating speed of the capstan through which the power was applied was constant, resulting in a uniform intake of rope which was attached to the gate fastening. The depth of submergence was 34 ft.

The width of the Davis Lock is 80 ft. The leaves of all the operating gates are 44.56 ft wide, and the intermediate gates are about 52 ft high. The gate swings through an arc of 68° 12′, and the operating cable is attached to the leaf at 39.62 ft from the center of rotation. The weight of each leaf is about 173 tons.

The results of the tests are shown in Fig. 11, representing the average of three test runs, for the opening of gate only. The maximum pull is about 61°

NOTE.—This paper by Maurice N. Amster, Assoc. M. Am. Soc. C. E., was published in March, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1944, by Edward Soucek.

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[4a] Received by the Secretary July 10, 1944.

from the position of the gate in the recess and is in close agreement with that obtained in the model tests at this point. Since the opposing leaf is in its recess, the movement of water seems to be throttled between the sill and the sill cushion of the leaf tested. There should be liberal depths under the gate

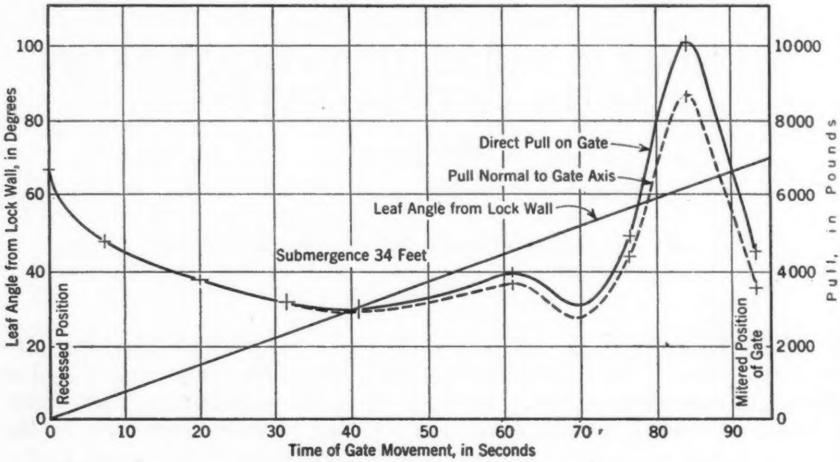

FIG. 11.—TYPICAL OPERATING GRAPH, DAVIS LOCK GATES, ST. MARYS FALLS CANAL

in the pit over which the gate swings, as well as in the gate recess in order to give a more effective path for the ingress and egress of the water back of the gate.

The clearance under the gate of the Davis Lock when in its recess is 3.5 ft and that between back of gate and recess wall is about 5 in. The operating graph of the Davis Lock gate shows that the force required to swing the gate from the open position is much larger than that required for the model tests. The low torque required in the model tests to start from the open position is surprisingly small and indicates an effective way for the water to reach the back of the gate in the recess.

Mr. Amster computed the mechanical advantage of the crank-and-strut linkage by trigonometry. The dead center positions of the crank coincide with the extreme positions of the gate, effecting theoretically an infinite mechanical advantage at those positions. The type of graph in Fig. 12 shows more clearly the relative torque applied to the gate at the different positions of its swing.

In the model and prototype the crank arm and the connecting rod form a straight line in both open and closed positions of the lock gate. With constant speed of gear input shaft, the rate of angular movement of the gate leaf swinging in either direction begins at zero, accelerating to a maximum at about 42° from the recessed position and then reduces to zero at the end of the swing. What is measured in the model tests is the torque required at the gate leaf which is taken as being directly proportional to the rate of swing, giving results of tests that would not apply if the gate were swung by an operating mechanism having

different kinematic conditions between power input to shaft and force applied to the gate leaf.

The method used to measure the torque in the gate is applicable to model study of any lock gate of similar type. The results obtained, however, are

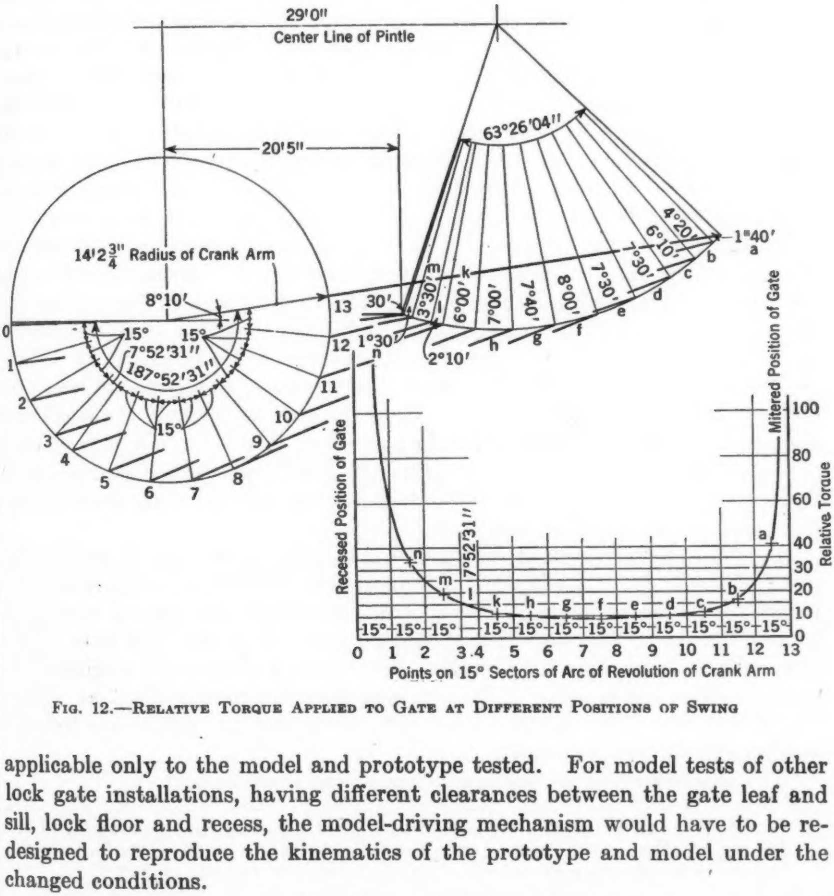

FIG. 12.—RELATIVE TORQUE APPLIED TO GATE AT DIFFERENT POSITIONS OF SWING

applicable only to the model and prototype tested. For model tests of other lock gate installations, having different clearances between the gate leaf and sill, lock floor and recess, the model-driving mechanism would have to be redesigned to reproduce the kinematics of the prototype and model under the changed conditions.

It is interesting to note, from Fig. 2, that the seal between the gate and sill is near the upstream face of the gate leaf and that clearance between bottom of gate and top of seal is something less than 11 in. As a consequence, the gate in opening from the closed position is swinging over the sill with very little clearance for water to pass under the gate from the upstream to the downstream side. Consequently, water on the upstream side must be pushed into the lock chamber by the gate until the space between leaves at the miter end becomes large enough to equalize levels on upstream and downstream sides. This would account for the high peak torque at approximately 53° in the opening operation.

F. W. EDWARDS,[5] M. AM. SOC. C. E.[5a]—The use of an hydraulic model for the solution of a new type of problem is demonstrated in this paper. The data have been presented in a way which should be particularly valuable to designers. Mr. Amster deserves credit for his effort toward this end.

The curves in Fig. 6 are interesting from a design standpoint. An increase in gate operating time reduces greatly the peak torque near the mitered position, but has less effect on the peak near the recessed position. This is true for the smaller submergences as well as for the base test conditions. Where reduction in operating time is not an important factor, the time should be increased until the maximum torques near the recessed position and near the mitered position are equal. It is unfortunate that the tests were not extended to include gate operation for periods longer than 2.75 min. However, from extrapolation of data in Fig. 6(*a*), it appears that an operating time of about 4 min would reduce the peak torque to the practical minimum at both critical positions of the gate. This maximum torque for 4-min operation would be less than 10% of the peak for an operating time of 1.75 min and would have a pronounced effect on the required capacity of the gate operating machinery.

The curves in Fig. 7 illustrate a similar effect on the peak torque at the two positions for nonsynchronous operation. In this case, an increase in lead time causes a reduction of peak torque. However, it appears from the single-leaf operation that the peak torque near the mitered position could not be reduced to a value corresponding to the maximum obtained near the recessed position. Apparently, a 25-sec lead time, just outside the range of the tests, would reduce the torque to the practical minimum.

It is not entirely accurate to designate the prototype tests as verification tests. Prototype tests which are conducted specifically for verification of a model should be made with special care. Although the general conditions were indicated by the author, it should be emphasized that the tests on the existing lock gates were very rough. They were not intended originally for verification of the model. It is surprising that the results check as well as Fig. 8 indicates. The model, rather than the prototype, results are believed to represent the correct values.

The author states (see heading, "Conclusions from Tests"),

> "* * * but the best load distribution for simultaneous operation of the leaves, whether synchronized or not, can be obtained by the use of an operating cycle involving low speeds at the extremes of travel—* * *."

This part of conclusion 2 is not based entirely on the model results. In fact, the variation of the kinematic characteristics (except speed) of the gate operation was not tested. It would appear from the torque curves that, for the base operation time of 2 min, a faster movement of the gate could be used near the recessed position without affecting the required capacity of the gate operation machinery. This could be accomplished by establishing the end position of the crank arm away from dead center.

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[5a] Received by the Secretary July 31, 1944.

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DISCUSSIONS

ECONOMICAL CANAL CROSS SECTIONS

Discussion

BY V. A. ENDERSBY

V. A. ENDERSBY,[5] M. AM. SOC. C. E.[5a]—The conclusions, arrived at analytically by Professor Streeter and substantiated by field studies made by the writer, are of particular interest in connection with flexible types of canal linings.

Some Field Observations.—The problem of an optimum canal section arose in connection with the investigation of the possible use of asphaltic lining membranes to fit the needs of laterals and other small canals that cannot carry the capital costs of cement or asphalt concrete linings of the thicker types (2 in. to 4 in.). Since the proposed thin linings can be expected only to serve as impermeabilizing agents and not as slope stabilizers, except in a minor degree, the matter of self-stabilizing canal sections is of primary importance. The thin, flexible linings will prevent erosion, weathering, and raveling of the surface soil, etc., but will give no appreciable resistance to hydrostatic pressures or mass soil movements.

The observations of ditches in use revealed a tendency for sections that were constructed as trapezoids to become modified by nature, in four different ways:

Case 1.—Cohesive soil of an unstable type will bulge into the canal near the bottom, and will slump near the top. This bulge slumps further as it becomes soaked, tending to assume a talus formation.

Case 2.—Granular, noncohesive soil erodes below the water line, thus releasing slides that fill in and overrun the eroded cavity.

Case 3.—Granular, cohesive soil, resistant to erosion, often weathers and ravels above the water line, moves down the slope, and forms a talus.

Case 4.—In combination with case 1, 2, or 3, vegetable growth tends to build out a berm at the water line. Since all cases result in rounded bottom corners, this combination of factors tends to form a semicircular cross section; and in fact sections are often found which are nearly perfect semicircles.

NOTE.—This paper by Victor L. Streeter, Assoc. M. Am. Soc. C. E., was published in May, 1944, *Proceedings*.

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[5a] Received by the Secretary August 8, 1944.

All other factors being equal, a section, originally build to the form most acceptable to nature, will be the most likely to escape natural changes. A rounded section is mechanically stable against gravitational forces because it forms a series of inverted elementary arches. The weakness of sharp angles in any structure, as compared with that of rounded forms, has long been established in relation to rigid structures and in fact to tunnel work. If a section that is fundamentally stable is constructed, the problem of maintaining it is merely that of preventing changes in the mechanical properties of the materials composing it. In this case the sole agent materially active in producing such changes is moisture, either by increase or reduction of content, or by erosion. Thus, the sole need for meeting the problem is one of adequate waterproofing.

Application to Thin or Flexible Linings.—Professor Streeter develops his theory in relation to the tendency of fresh concrete to slump on a slope. The fundamental principles are applicable to granular materials such as sand, to granular-plastic materials such as plastic soils, and to bituminous linings. The latter are granular-plastic in nature rather than rigid, especially while being placed hot, or when exposed to very hot weather.

It also happens that a great part of such flexible linings as have been placed has been compacted by transverse rolling. To roll transversely, it is necessary to round the corners to some degree; and, although this may not be a consideration when more suitable means of compaction are employed, the fundamental structural considerations remain.

During the recent construction of an asphalt-lined canal for military uses, the lining was placed by machinery in a trapezoidal section, but the problem of the rounded corner was approached by the subsequent addition of fillets, indicating an appreciation, by the authorities concerned, of the underlying structural considerations.

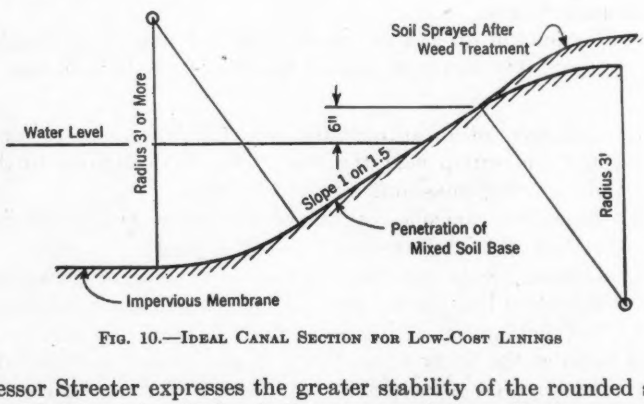

FIG. 10.—IDEAL CANAL SECTION FOR LOW-COST LININGS

Professor Streeter expresses the greater stability of the rounded section in Eq. 2*a* and in Fig. 2, in which σ is the steepness index. In the case of a trapezoidal canal, y_0 becomes the full depth of the canal and θ_0 becomes zero except in the case where $\int (t\,\gamma_0 \sin\theta - F)\,dl$ is zero or minus. In other words, the trapezoidal section gives the maximum slumping force; and, even where the

frictional resistance is greater than the accumulated slumping force, the rounded section gives greater resistance to possible ground movements under the lining.

The Ogee Section.—The logical section devised by the writer was determined both by the foregoing considerations as to the economy of the wetted perimeter, and by the special needs of a thin, flexible lining, and is shown in Fig. 10. The total thickness of base and membrane should be not less than 1 in. except when the section is located on firm and cohesive soil. The reversed curve at the berm is intended to prevent edge damage to the lining from stock, etc., and to prevent the infiltration of surface water behind it. The wetted permeter may equally well be any one of the sections discussed by Professor Streeter—circular, parabolic, or catenary.

Summary.—Professor Streeter's conclusions appear to be sound from the points of view of the stability of fresh concrete on the canal slopes, of economy of construction, and of hydraulic efficiency. Furthermore, they conform to the section which nature appears to produce for reasons of inherent stability, and have a special importance where thin, plastic, or flexible linings are used.